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Where Minds Pollinate

Class 11
Mathematics

Chapter 8

Exercise 8.2

Notes

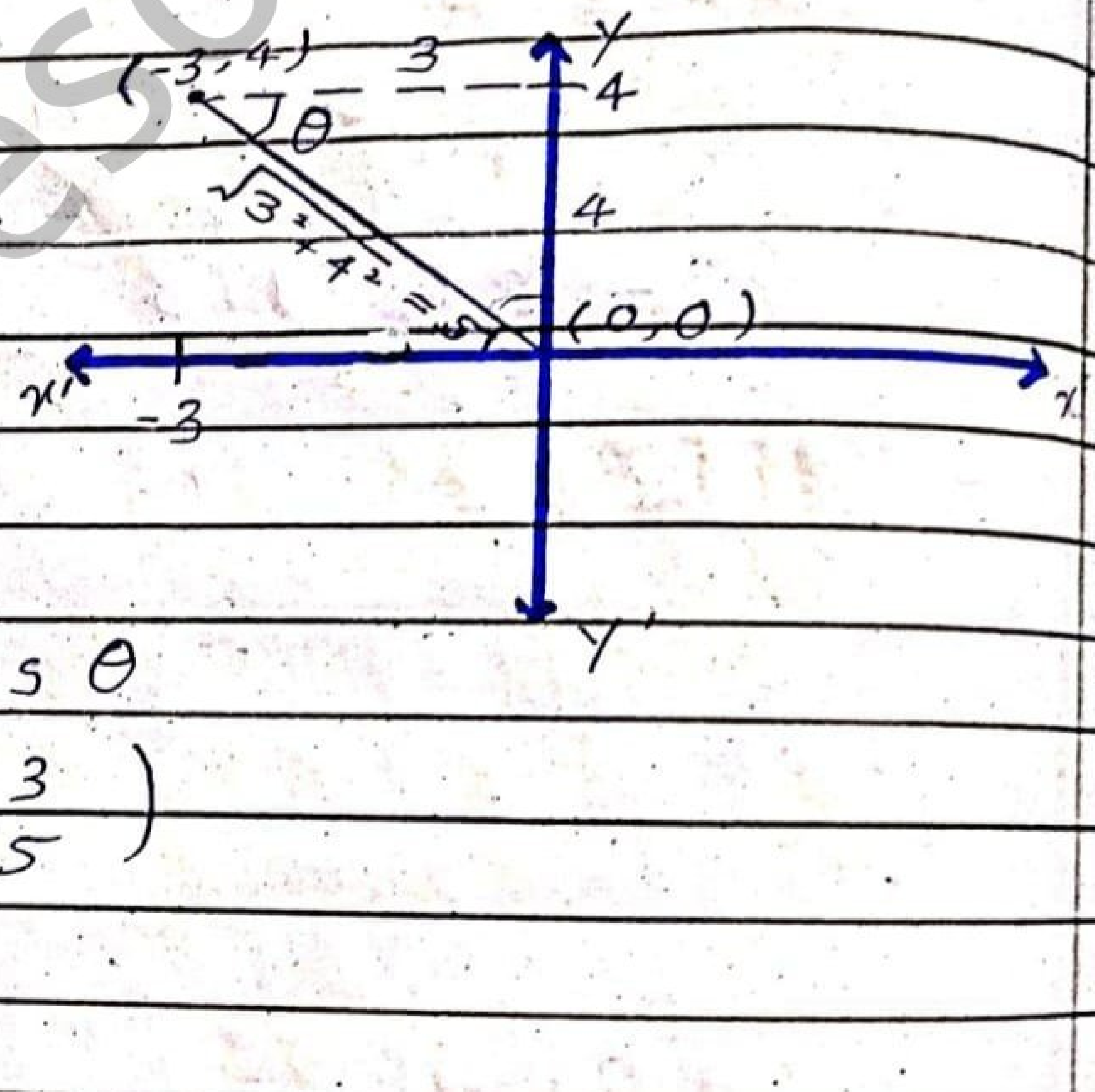
National Book Foundation

Exercise # 8.2

Q1. Suppose $P(-3, 4)$ lies on terminal side of θ when θ is plotted in Qud. II. Find $\cos 2\theta$ and $\sin 2\theta$. Determin the Quad. in which 2θ lies.

Sol:-

Given point $P(-3, 4)$



$\therefore \sin\theta = \frac{4}{5}$

$\cos\theta = -\frac{3}{5}$

We know that

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)$

$\sin 2\theta = -\frac{24}{25}$

also, $\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 2\left(-\frac{3}{5}\right)^2 - 1$

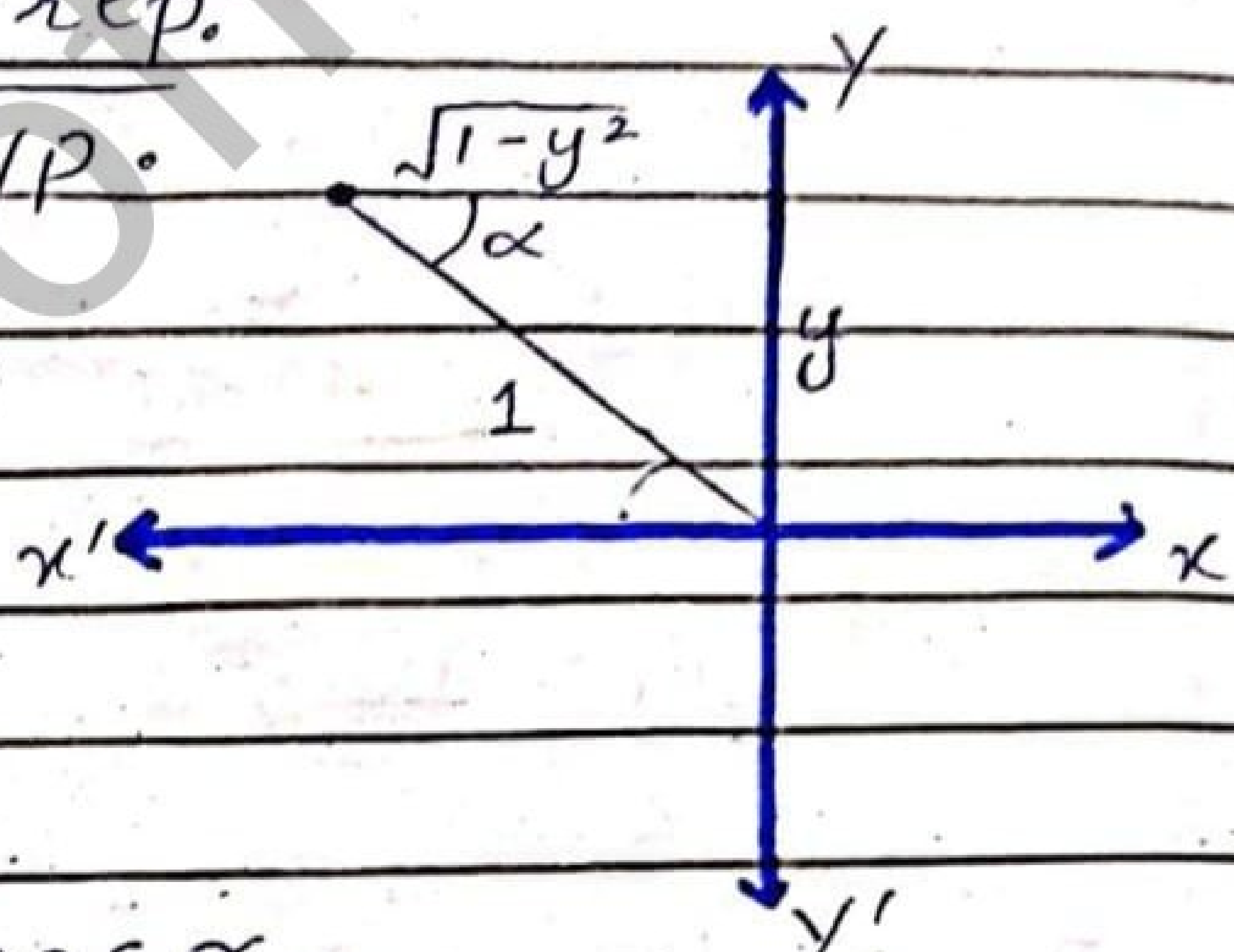
$\cos 2\theta = -\frac{7}{25}$

$\therefore$ 2θ lies in Quad. III

Q2. If $\sin\alpha = y$ and α lies in Quad. II. Find expression for $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$ in terms of y.

Sol:-

$$\sin\alpha = y = \frac{y}{1} \Rightarrow \frac{\text{Prep.}}{\text{Hyp.}}$$



$$\therefore \cos\alpha = -\sqrt{1-y^2}$$

also $\tan\alpha = \dfrac{-y}{\sqrt{1-y^2}}$

So, $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$\therefore \sin 2\alpha = 2(y)(-\sqrt{1-y^2})$$

$$\sin 2\alpha = -2y\sqrt{1-y^2}$$

also, $\cos 2\alpha = 2\cos^2\alpha - 1$

$$\cos 2\alpha = 2(-\sqrt{1-y^2})^2 - 1$$

$$\cos 2\alpha = -1 + 2 - 2y^2$$

$$\cos 2\alpha = 1 - 2y^2$$

$$\therefore \tan 2\alpha = \frac{-2y\sqrt{1-y^2}}{1-2y^2}$$

Q3. Use half angle formula to find exact value of $\cos 15°$.

Sol:-

$$= \cos 15°$$

OR $$= \cos\left(\frac{30°}{2}\right)$$

$$\therefore \quad = \pm\sqrt{\frac{1+\cos 30°}{2}}$$

$$= \pm\sqrt{\frac{1+\sqrt{3}/2}{2}} \Rightarrow \pm\sqrt{\frac{2+\sqrt{3}/\cancel{2}}{\cancel{2}}}$$

$$= \pm\sqrt{\frac{2+\sqrt{3}}{4}} \Rightarrow \cos 15° = \pm 0.9659$$

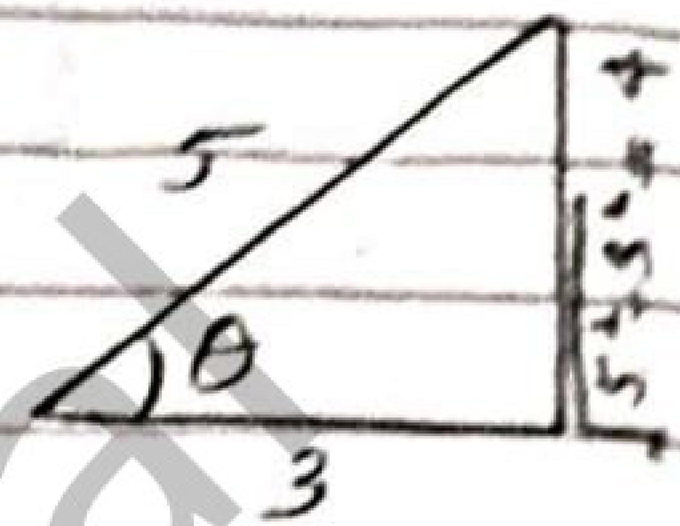
Q4. Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, $\sin\frac{\theta}{2}$, $\cos\frac{\theta}{2}$, $\tan\frac{\theta}{2}$, when :-

i) $\cos\theta = \frac{3}{5}$ where, $0 < \theta < \pi/2$

Sol:-

Given, $\cos\theta = \frac{3}{5}$, (Qud. I)

$\therefore \sin\theta = \frac{4}{5}$



also, $\tan\theta = \frac{4}{3}$

Now, $\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 2\left(\frac{3}{5}\right)^2 - 1$

$\cos 2\theta = -\frac{7}{25}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$\sin 2\theta = \frac{24}{25}$

$\therefore \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} \Rightarrow -\frac{24}{7}$

also, $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+3/5}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \pm\frac{2\sqrt{5}}{5}$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-3/5}{2}}$$

$$\sin\frac{\theta}{2} = \pm\frac{\sqrt{5}}{5}$$

$$\therefore \tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta/2)}{\cos(\theta/2)}$$

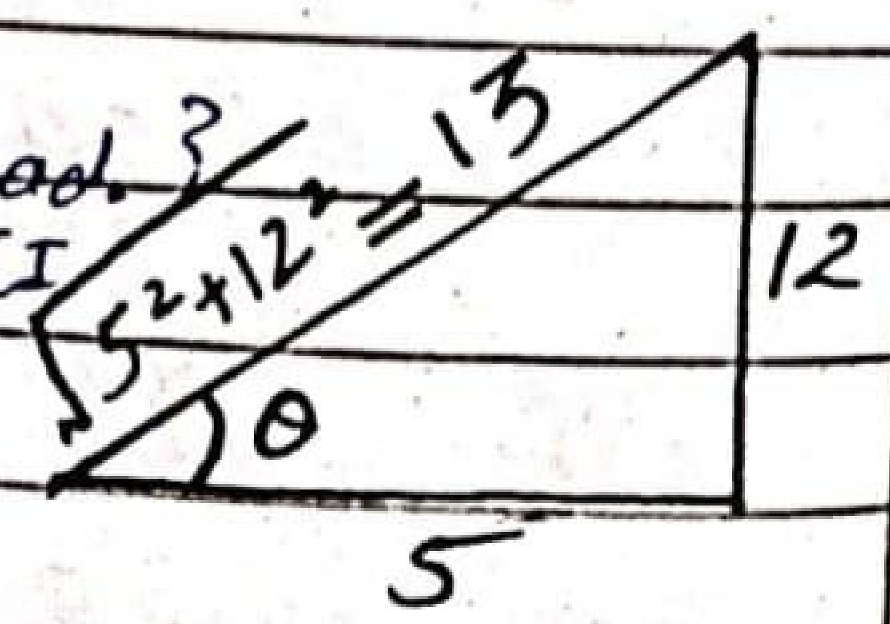
$$\tan\left(\frac{\theta}{2}\right) = \frac{\sqrt{5}}{2\sqrt{5}} \Rightarrow 1/2$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{2}$$

ii) $\tan\theta = \frac{12}{5}$ where $\pi < \theta < \frac{3\pi}{2}$

Sol:-

Given, $\tan\theta = \frac{12}{5}$ {Quad. III}



$$\therefore \sin\theta = -\frac{12}{13}$$

$$\cos\theta = -\frac{5}{13}$$

$$\therefore \quad \cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 2\left(-\frac{5}{13}\right)^2 - 1$$

$$\cos 2\theta = -\frac{119}{169}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(-\frac{12}{13}\right)\left(-\frac{5}{13}\right)$$

$$\sin 2\theta = \frac{120}{169}$$

$$\therefore \quad \tan 2\theta = -\frac{120}{119}$$

Now, $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$$= \pm\sqrt{\frac{1 - 5/13}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\frac{2\sqrt{13}}{13}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$= \pm\sqrt{\frac{1+5/13}{2}}$$

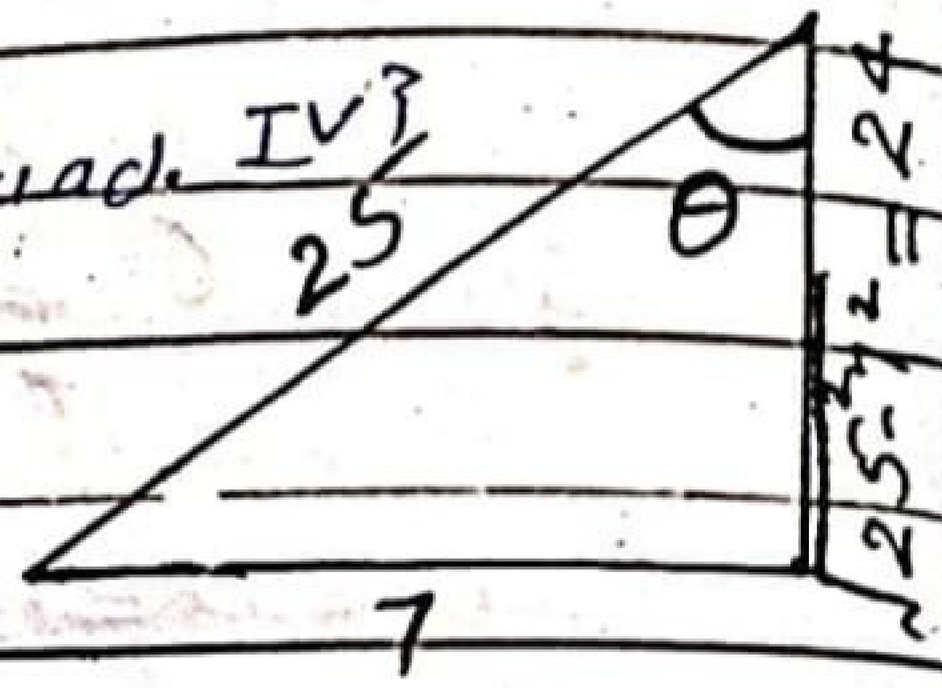
$$\sin\left(\frac{\theta}{2}\right) = \pm \frac{3\sqrt{13}}{13}$$

$$\therefore \tan\left(\frac{\theta}{2}\right) = \frac{3\cancel{\sqrt{13}}}{2\cancel{\sqrt{13}}} = \pm\frac{3}{2}$$

iii) $\sin\theta = -\frac{7}{25}$ where $\frac{3\pi}{2} < \theta < 2\pi$

Sol:-

Given:- $\sin\theta = -\frac{7}{25}$ {Quad. IV}



$$\therefore \cos\theta = \frac{24}{25}$$

and, $\tan\theta = -\frac{7}{24}$

So, $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2\left(\frac{24}{25}\right)^2 - 1$$

$$\cos 2\theta = \frac{527}{625}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(-\frac{7}{25}\right)\left(\frac{24}{25}\right)$$

$$\sin 2\theta = -\frac{336}{625}$$

$\therefore \tan 2\theta = -\frac{336}{527}$

Now, $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}} \Rightarrow \pm\sqrt{\frac{1+24/25}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \pm\frac{7\sqrt{2}}{10}$

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} \Rightarrow \pm\sqrt{\frac{1-24/25}{2}}$

$\sin\left(\frac{\theta}{2}\right) = \sqrt{2}/10$

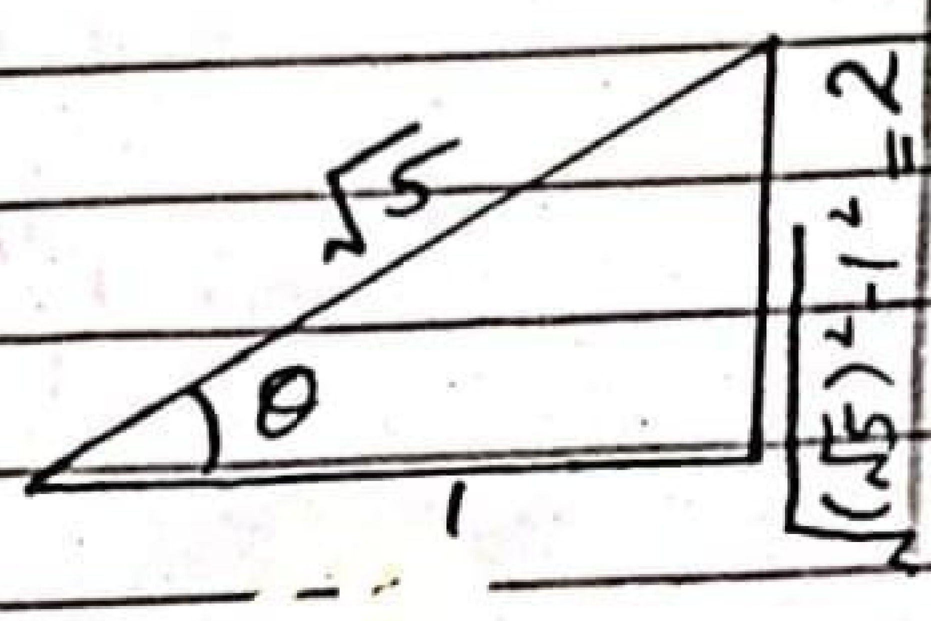
$\therefore \tan\left(\frac{\theta}{2}\right) = \frac{\cancel{\sqrt{2}}}{7\cancel{\sqrt{2}}} \Rightarrow \frac{1}{7}$

iv) $\sec\theta = \sqrt{5}$ where $\frac{3\pi}{2} < \theta < 2\pi$

Sol:-

Given:- $\sec\theta = \sqrt{5} \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}$ {Q→IV}

$\therefore \sin\theta = \frac{2}{\sqrt{5}}$



So, $\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$

$$\cos 2\theta = -3/5$$

also, $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(\frac{-2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right)$$

$$\sin 2\theta = -\frac{4}{5}$$

$$\therefore \tan 2\theta = \frac{4}{3}$$

Now, $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$$= \pm\sqrt{\frac{1+1/\sqrt{5}}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{5+\sqrt{5}}{10}}$$

$$\sin\left(\frac{\theta}{2}\right) = +\sqrt{\frac{1-\cos\theta}{2}}$$

$$= \pm\sqrt{\frac{1-1/\sqrt{5}}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{5-\sqrt{5}}{10}}$$

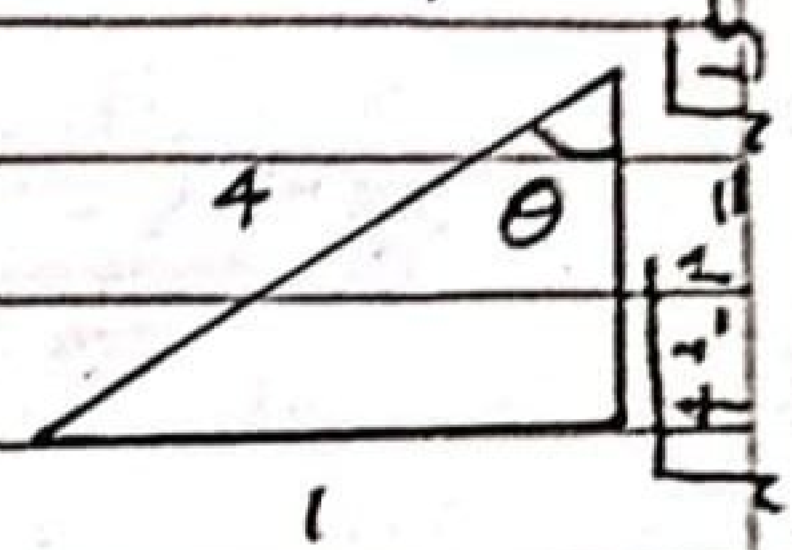
$$\therefore \tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{5-\sqrt{5}}{5+\sqrt{5}}}$$

v) $\csc\theta = 4$ where $\frac{\pi}{2} < \theta < \pi$

Sol:-

Given $\csc\theta = 4 \Rightarrow \sin\theta = \frac{1}{4}$ $\{\theta \rightarrow II\}$

$$\therefore \cos\theta = -\frac{\sqrt{15}}{4}$$



So, $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2\left(-\frac{\sqrt{15}}{4}\right) - 1$$

$$\cos 2\theta = \frac{7}{8}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(\frac{1}{4}\right)\left(-\frac{\sqrt{15}}{4}\right)$$

$$\sin 2\theta = -\frac{\sqrt{15}}{8}$$

$$\tan 2\theta = -\frac{\sqrt{15}}{7}$$

Now, $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$$= \pm\sqrt{\frac{1+\sqrt{15}/4}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{4+\sqrt{15}}{8}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$= \pm\sqrt{\frac{1-\sqrt{15}/4}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \frac{4-\sqrt{15}}{8}$$

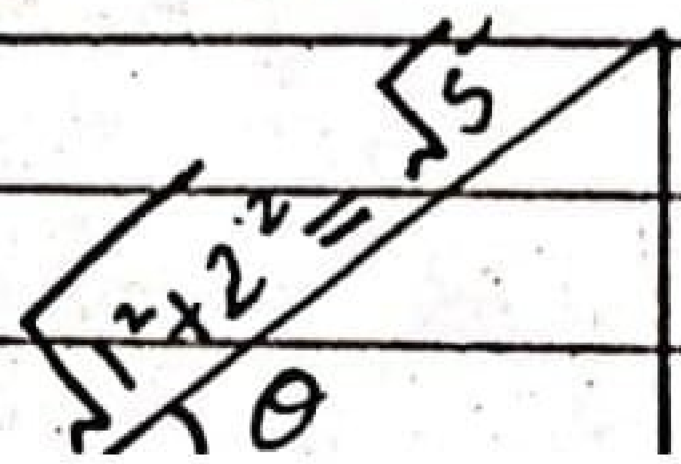
$$\therefore \tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}}$$

vi) $\cot\theta = -\frac{1}{2}$ where $\frac{\pi}{2} < \theta < \pi$

Sol:-

$$\cot\theta = -\frac{1}{2} \Rightarrow \tan\theta = -2$$

$$\therefore \cos\theta = -\frac{1}{\sqrt{5}}$$



$$\sin\theta = \frac{2}{\sqrt{5}}$$

So, $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2\left(-\frac{1}{\sqrt{5}}\right)^2 - 1$$

$$\cos 2\theta = -3/5$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(\frac{2}{\sqrt{5}}\right)\left(-\frac{1}{\sqrt{5}}\right)$$

$$\sin 2\theta = -4/5$$

$$\therefore \tan 2\theta = 4/3$$

Now, $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$$= \pm\sqrt{\frac{1-1/\sqrt{5}}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{5-\sqrt{5}}{10}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} \Rightarrow \pm\sqrt{\frac{1+1/\sqrt{5}}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{5+\sqrt{5}}{10}}$$

$$\therefore \tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{5+\sqrt{5}}{5-\sqrt{5}}}$$

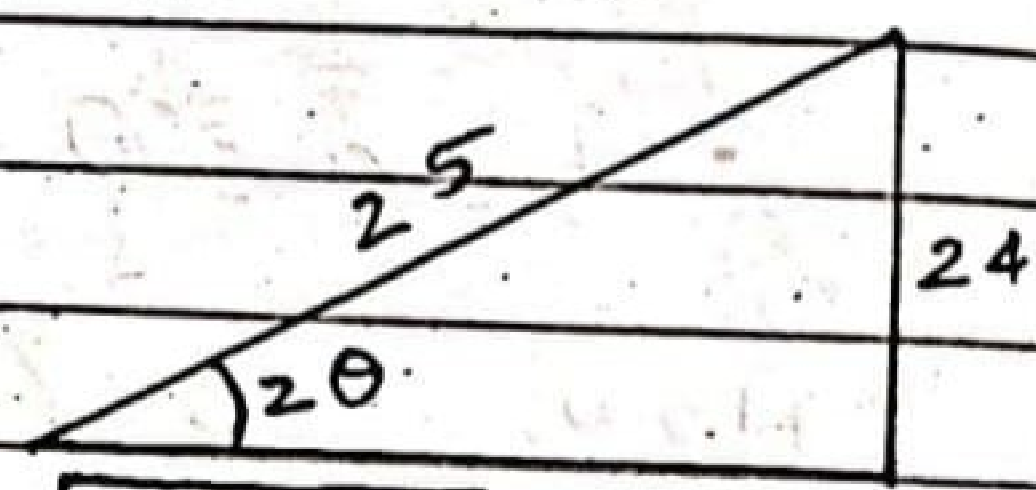
Q5. Find exact value of $\sin\theta$, $\cos\theta$ and $\tan\theta$ using information given below:-

i) $\sin 2\theta = \frac{24}{25}$, 2θ lies in Q.II.

Sol:-

$$\sin 2\theta = \frac{24}{25} \quad \{Q.II\}$$

$$\therefore \cos 2\theta = -\frac{7}{25}$$



$\sqrt{25^2-24^2} = 7$

Now,
$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos\theta = \pm\sqrt{\frac{\cos 2\theta + 1}{2}}$$

$$\cos\theta = \pm\sqrt{\frac{-7/25+1}{2}} \Rightarrow \pm\frac{3}{5}$$

$$\cos\theta = \pm\frac{3}{5}$$

also, $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$\sin\theta = \sqrt{\cos^2\theta - \cos 2\theta}$$

$$\sin\theta = \sqrt{(3/5)^2 - (-7/25)}$$

$$\sin\theta = \pm\frac{4}{5}$$

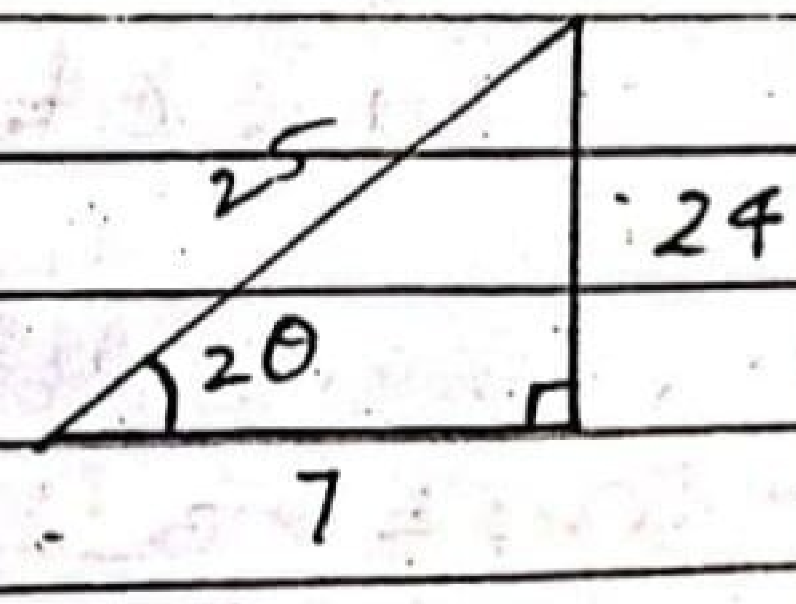
$$\therefore \tan\theta = \pm\frac{4}{3}$$

ii) $\cos 2\theta = -\frac{7}{25}$, 2θ in Quad. III

Sol:-

$$\cos 2\theta = -\frac{7}{25} \quad \{Q.\ III\}$$

$$\therefore \sin 2\theta = -\frac{24}{25}$$



Now, $\cos\theta = \pm\sqrt{\frac{\cos 2\theta + 1}{2}}$

$$= \pm\sqrt{\frac{-7/25 + 1}{2}}$$

$$\cos\theta = \pm\frac{3}{5}$$

also, $\sin\theta = \pm\sqrt{\cos^2\theta - \cos 2\theta}$

$$\sin\theta = \pm\sqrt{(3/25)^2 + 7/25}$$

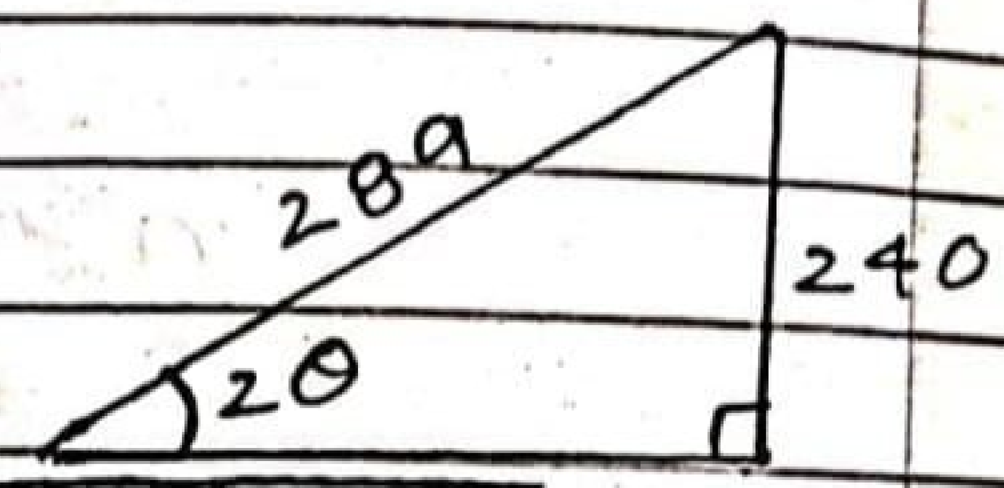
$$\sin\theta = \pm\frac{4}{5}$$

$$\therefore \tan\theta = \pm\frac{4}{3}$$

iii) $\sin 2\theta = -\frac{240}{289}$, 2θ lies in Q. III

Sol:-

$$\sin 2\theta = -\frac{240}{289}$$



$$\sqrt{289^2 - 240^2} = 161$$

$$\therefore \cos 2\theta = -\frac{161}{289}$$

Now, $\cos\theta = \pm\sqrt{\frac{\cos 2\theta + 1}{2}}$

$$= \pm\sqrt{\frac{-161/289 + 1}{2}}$$

$$\cos\theta = +\frac{8}{17}$$

also, $\sin\theta = \pm\sqrt{\cos^2\theta - \cos 2\theta}$

$= \pm\sqrt{(8/17)^2 + 161/289}$

$\sin\theta = \pm\frac{15}{17}$

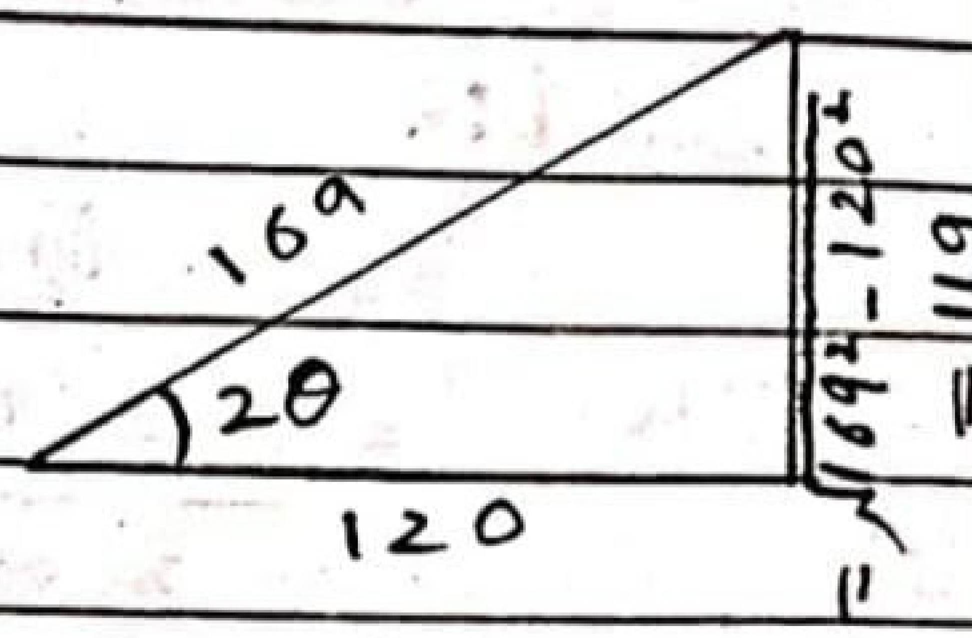
$\therefore \tan\theta = \pm\frac{15}{8}$

iv) $\cos 2\theta = \frac{120}{169}$, 2θ lies in Q. IV

Sol:-

$\cos 2\theta = \frac{120}{169}$ {Q. IV}

$\therefore \sin 2\theta = -\frac{119}{169}$



Now, $\cos\theta = \pm\sqrt{\frac{\cos 2\theta + 1}{2}}$

$= \pm\sqrt{\frac{120/169 + 1}{2}}$

$\cos\theta = \pm\frac{17\sqrt{2}}{26}$

also, $\sin\theta = \pm\sqrt{\cos^2\theta - \cos 2\theta}$

$$\sin\theta = \pm\sqrt{\left(17\sqrt{2}/26\right)^2 - 120/169}$$

$$\sin\theta = \pm\frac{7\sqrt{2}}{26}$$

$$\therefore \tan\theta = \pm\frac{7}{17}$$

Q6. Use double angle identies to find exact value of :-

i) $\sin 15^\circ \cos 15^\circ$

Sol:-

$$= \sin 15^\circ \cos 15^\circ$$

OR
$$= \frac{2\sin(15^\circ)\cos(15^\circ)}{2}$$

$$\because \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\therefore = \frac{\sin 2(15)^\circ}{2}$$

$$= \frac{\sin 30^\circ}{2}$$

$= \frac{1/2}{2} \Rightarrow \frac{1}{4}$

$\therefore \sin 15^\circ \cos 15^\circ = \frac{1}{4}$

ii) $\cos^2 15^\circ - \sin^2 15^\circ$

Sol:-

$= \cos^2 15^\circ - \sin^2 15^\circ$

$\because \cos 2\beta = \cos^2\beta - \sin^2\beta$

$\therefore = \cos 2(15)^\circ$

$= \cos 30^\circ \Rightarrow \frac{\sqrt{3}}{2}$

$\therefore \cos^2 15^\circ - \sin^2 15^\circ = \frac{\sqrt{3}}{2}$

iii) $1 - 2\sin^2\left(\frac{\pi}{8}\right)$

Sol:-

$= 1 - 2\sin^2\left(\frac{\pi}{8}\right)$

OR $= 1 - 2\sin^2(22.5^\circ)$

$\because \cos 2\alpha = 1 - 2\sin^2\alpha$

So,

$= \cos 2(22.5)^\circ$

$= \cos(45^\circ)$

$= \dfrac{\sqrt{2}}{2}$

$\therefore 1 - 2\sin^2\left(\dfrac{\pi}{8}\right) = \dfrac{\sqrt{2}}{2}$

iv) $2\cos^2(\pi/12) - 1$

Sol:-

$= 2\cos^2(\pi/12) - 1$

$= 2\cos^2(15)^\circ - 1$

$= \cos 2(15^\circ)$ $\{\because \cos 2\alpha = 2\cos^2\alpha - 1$

$= \sqrt{3}/2$

v) $2\tan(\pi/12) / 1 - \tan^2(\pi/12)$

Sol:-

$= \dfrac{2\tan(15^\circ)}{1 - \tan^2(15^\circ)} \Rightarrow \tan 2(15^\circ)$

$= \dfrac{\sqrt{3}}{3}$

Q6. Rewrite in terms of cosine of power 1.

i) $\sin^2\alpha\cos^2\alpha$

Sol:-

$$= \sin^2\alpha\cos^2\alpha$$

$$\because \sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$$

OR $\sin^2\alpha = \frac{1-\cos2\alpha}{2}$

also, $\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$

OR $\cos^2\alpha = \sqrt{\frac{1+\cos2\alpha}{2}}$

$$\therefore \sin^2\alpha\cos^2\alpha = \frac{1-\cos2\alpha}{2} \cdot \frac{1+\cos2\alpha}{2}$$

$$= \frac{1}{4}\left[\cancel{1-\cos^2 2\alpha}\right]$$

$$\because \cos2\alpha = \sqrt{\frac{1+\cos4\alpha}{2}} \Rightarrow \cos^2 2\alpha = \frac{1+\cos4\alpha}{2}$$

$$\therefore = \frac{1}{4}\left[1 - \frac{1+\cos 4\alpha}{2}\right]$$

$$= \frac{1}{4}\left[\frac{2-1-\cos 4\alpha}{2}\right]$$

$$= \frac{1}{8}\left[1-\cos 4\alpha\right]$$

ii) $\sin^4\alpha \cos^2\alpha$

Sol:-

$$= \sin^4\alpha \cos^2\alpha$$

$$\because \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} \Rightarrow \sin^2\varphi = \frac{1-\cos 2\varphi}{2}$$

also, $\cos^2\varphi = \dfrac{1+\cos 2\varphi}{2}$

$$\therefore = \left[\frac{1-\cos 2\alpha}{2}\right]^2 \left[\frac{1+\cos 2\alpha}{2}\right]$$

$$= \left[\frac{1-\cos 2\alpha}{2}\right]\left[\frac{1-\cos^2 2\alpha}{4}\right]$$

$$= \frac{1}{8}\left[1-\cos 2\alpha\right]\left[1-\cos^2 2\alpha\right]$$

$$\Rightarrow \frac{1}{64}\left[\frac{18+1+\cos 8\alpha+12\cos 4\alpha-16-16\cos 4\alpha}{2}\right]$$

$$=\frac{1}{128}\left[3+\cos 8\alpha-4\cos 4\alpha\right]$$

$$=\frac{1}{8}\left[1-\cos 2\alpha\right]\left[1-\frac{1+\cos 4\alpha}{2}\right]$$

$$=\frac{1}{8}\left[1-\cos 2\alpha\right]\left[\frac{2-1-\cos 4\alpha}{2}\right]$$

$$=\frac{1}{16}\left[1-\cos 2\alpha\right]\left[1-\cos 4\alpha\right]$$

$$=\frac{1}{16}\left[1-\cos 4\alpha-\cos 2\alpha+\cos 2\alpha\cos 4\alpha\right]$$

iii) $\sin^4\alpha\cos^4\alpha$

Sol:-

$$=\sin^4\alpha\cos^4\alpha$$

$$=\left[\frac{1-\cos 2\alpha}{2}\right]^2\left[\frac{1+\cos 2\alpha}{2}\right]^2$$

$$=\frac{1}{16}\left[1+\cos^2 2\alpha-2\cos 2\alpha\right]\left[1+\cos^2 2\alpha+2\cos 2\alpha\right]$$

$$=\frac{1}{16}\left[1+\frac{1+\cos 4\alpha}{2}-2\cos 2\alpha\right]\left[1+\frac{1+\cos 4\alpha}{2}+2\cos 2\alpha\right]$$

$$=\frac{1}{16}\left[\frac{2+1+\cos 4\alpha-4\cos 2\alpha}{2}\right]\left[\frac{2+1+\cos 4\alpha+4\cos 2\alpha}{2}\right]$$

$$=\frac{1}{64}\left[3+\cos 4\alpha-4\cos 2\alpha\right]\left[3+\cos 4\alpha+4\cos 2\alpha\right]$$

$$=\frac{1}{64}\left[9+\cos^2 4\alpha+6\cos 4\alpha-16\cos^2 2\alpha\right]$$

$$=\frac{1}{64}\left[9+\frac{1+\cos 8\alpha}{2}+6\cos 4\alpha-16\left(\frac{1+\cos 4\alpha}{2}\right)\right]\rightarrow$$

Q8 verify:-

i) $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

Sol:-

L.H.S $\Rightarrow (\sin\theta + \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + \sin 2\theta$

ii) $\tan 2x = \dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x}$

Proof:-

L.H.S $\Rightarrow \dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x}$

$= \dfrac{\cancel{1} + \tan x - \cancel{1} + \tan x}{(1)^2 - (\tan x)^2}$

$= \dfrac{2\tan x}{1 - \tan^2 x} \Rightarrow \tan 2x$

$=$ R.H.S

iii) $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$

Proof:-

L.H.S $\Rightarrow \frac{\sin\theta}{1+\cos\theta}$

OR $= \frac{2\sin\theta/2\cos\theta/2}{\cancel{1}+2\cos^2\theta/2-\cancel{1}}$ $\{\because \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\}$

$= \frac{\cancel{2}\sin\theta/2\,\cancel{\cos\theta/2}}{\cancel{2}(\cos\theta/2)\cancel{(\cos\theta/2)}} \Rightarrow \frac{\sin\theta/2}{\cos\theta/2}$

$= \tan\left(\frac{\theta}{2}\right) \Rightarrow$ R.H.S

iv) $\csc 2\alpha = \frac{\tan\alpha+\cot\alpha}{2}$

Sol:-

L.H.S $\Rightarrow \frac{\tan\alpha+\cot\alpha}{2}$

$= \frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}}{2} \Rightarrow \frac{\frac{\sin^2\alpha+\cos^2\alpha}{\cos\alpha\sin\alpha}}{2}$

$$= \frac{1}{2\cos\alpha\sin\alpha} \Rightarrow \frac{1}{\sin 2\alpha}$$

$$= \csc 2\alpha \Rightarrow R.H.S$$

iv) $8\sin^4\theta = 3 + \cos 4\theta - 4\cos 2\theta$

Proof:-

$L.H.S \Rightarrow 8\sin^4\theta$

$= 8(\sin^2\theta)^2$

$= 8\left[\frac{1-\cos 2\theta}{2}\right]^2$

$= {}^{2}\cancel{8}\left[\frac{1+\cos^2 2\theta - 2\cos 2\theta}{\cancel{4}_1}\right]$

$= 2(1+\cos^2 2\theta - 2\cos 2\theta)$

$= 2 + 2\cos^2 2\theta - 4\cos 2\theta$

$= 2\left[1 + \cos^2 2\theta\right] - 4\cos 2\theta$

$= 2\left[1 + \frac{1+\cos 4\theta}{2}\right] - 4\cos 2\theta$

$= \cancel{2}\left[\frac{2+1+\cos 4\theta}{\cancel{2}}\right] - 4\cos 2\theta$

$= 3 + \cos 4\theta - 4\cos 2\theta$

$= R.H.S$

vi) $\sin 4\theta = 4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$

Proof:-

R.H.S $\Rightarrow$ $4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$

$= 4\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta)$

$= 2(2\sin\theta\cos\theta)(\cos^2\theta - \sin^2\theta)$

$= 2\sin 2\theta(\cos 2\theta)$

$= 2\sin 2\theta\cos 2\theta$

$= \sin 4\theta$

$=$ R.H.S.

vii) $\sin 2\theta = 2\cot\theta\sin^2\theta$

Proof:-

R.H.S $\Rightarrow$ $2\cot\theta\sin^2\theta$

$= \frac{2\cos\theta}{\cancel{\sin\theta}}\cancel{\sin^2\theta}$

$= 2\sin\theta\cos\theta$

$= \sin 2\theta$

$=$ R.H.S

viii) $\cos^2 2x + 4\sin^2 x \cos^2 x = 1$

Proof:-

$L.H.S \Rightarrow \cos^2 2x + 4\sin^2 x \cos^2 x$

$= (\cos^2 x - \sin^2 x)^2 + 4\sin^2 x \cos^2 x$

$= \cos^4 x + \sin^4 x - 2\cos^2 x \sin^2 x + 4\sin^2 x \cos^2 x$

$= \cos^4 x + \sin^4 x + 2\cos^2 x \sin^2 x$

$= (\cos^2 x + \sin^2 x)^2$

$= (1)^2$

$= 1 \Rightarrow R.H.S$

ix) $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$

Proof:-

$R.H.S \Rightarrow 8\cos^4\theta - 8\cos^2\theta + 1$

$= 8\left[\frac{1+\cos 2\theta}{2}\right]^2 - \overset{4}{\cancel{8}}\left[\frac{1+\cos 2\theta}{\cancel{2}\,1}\right] + 1$

$= \frac{\overset{2}{\cancel{8}}(1+\cos 2\theta)^2}{\cancel{4}\,1} - 4(1+\cos 2\theta) + 1$

$= 2(1+\cos 2\theta)^2 - 4(1+\cos 2\theta) + 1$

$= 2(1^2 + \cos^2 2\theta + 2\cos 2\theta) - 4 - 4\cos 2\theta + 1$

$= 2 + 2\cos^2 2\theta + 4\cos 2\theta - 4 - 4\cos 2\theta + 1$

$= 2\cos^2 2\theta - 1$ $\quad \{\because \cos 2\varphi = 2\cos^2\varphi - 1$ OR $\cos 4\varphi = 2\cos^2 2\varphi - 1\}$

$= \cos 4\theta$

$=$ R.H.S

x) $\sec 2x = \dfrac{\cos x}{\cos x + \sin x} + \dfrac{\sin x}{\cos x - \sin x}$

PROOF:-

L.H.S $\Rightarrow \dfrac{\cos x}{\cos x + \sin x} + \dfrac{\sin x}{\cos x - \sin x}$

$= \dfrac{\cos x(\cos x - \sin x) + \sin x(\cos x + \sin x)}{(\cos x + \sin x)(\cos x - \sin x)}$

$= \dfrac{\cos^2 x - \cancel{\cos x \sin x} + \cancel{\cos x \sin x} + \sin^2 x}{\cos^2 x - \sin^2 x}$

$= \dfrac{1}{\cos 2x} \Rightarrow \sec 2x \Rightarrow$ R.H.S.

xi) $\cos^4 x - \sin^4 x = \cos 2x$

PROOF:- L.H.S $\Rightarrow \cos^4 x - \sin^4 x$

$= (\cos^2 x)^2 - (\sin^2 x)^2$

$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$= (1)(\cos 2x)$

$= \cos 2x$

$=$ R.H.S.

xii) $\tan \frac{\beta}{2} + \cot \frac{\beta}{2} = 2 \csc \beta$

Proof:-

L.H.S $\Rightarrow \tan \frac{\beta}{2} + \cot \frac{\beta}{2}$

$= \frac{\sin \beta/2}{\cos \beta/2} + \frac{\cos \beta/2}{\sin \beta/2}$

$= \frac{\sin^2 \beta/2 + \cos^2 \beta/2}{\cos \beta/2 \sin \beta/2}$

$= \frac{1}{\cos \beta/2 \sin \beta/2} \Rightarrow \frac{2}{2 \cos \beta/2 \sin \beta/2}$

$= \frac{2}{\sin \beta} \Rightarrow 2 \csc \beta$

xiii) $\csc 2\alpha - \cot 2\alpha = \tan \alpha$

Proof:-

L.H.S $\Rightarrow \csc 2\alpha - \cot 2\alpha$

$$= \frac{1}{\sin 2\alpha} - \frac{\cos 2\alpha}{\sin 2\alpha}$$

$$= \frac{1 - \cos 2\alpha}{\sin 2\alpha}$$

$$= \frac{2\sin^2\alpha}{\sin 2\alpha} \quad \left\{\because \cos 2\varphi = 1 - 2\sin^2\varphi\right\}$$

$$= \frac{2\sin\alpha \cdot \sin\alpha}{2\sin\alpha\cos\alpha}$$

$$= \frac{\sin\alpha}{\cos\alpha} \Rightarrow \tan\alpha \Rightarrow \text{R.H.S.}$$

$$\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{2 + \sin 2x}{2}$$

Proof:-

L.H.S $\Rightarrow \dfrac{\cos^3 x - \sin^3 x}{\cos x - \sin x}$

$$= \frac{\cancel{(\cos x - \sin x)}(\cos^2 x + \cos x \sin x + \sin^2 x)}{\cancel{\cos x - \sin x}}$$

$$= \cos^2 x + \sin^2 x + \cos x \sin x$$

$$= 1 + \cos x \sin x$$

$$= 1 + \cos x \sin x \cdot \frac{2}{2}$$

$$= \frac{2 + 2\cos x \sin x}{2}$$

$$= \frac{2 + \sin 2x}{2}$$

$= R.H.S$

xv) $\frac{\sin 3\alpha}{\sin \alpha} - \frac{\cos 3\alpha}{\cos \alpha} = 2$

Proof:

$L.H.S \Rightarrow \frac{\sin 3\alpha}{\sin\alpha} - \frac{\cos 3\alpha}{\cos\alpha}$

$$= \frac{3\sin\alpha - 4\sin^3\alpha}{\sin\alpha} - \frac{4\cos^3\alpha - 3\cos\alpha}{\cos\alpha}$$

$$= \frac{\cancel{\sin\alpha}[3 - 4\sin^2\alpha]}{\cancel{\sin\alpha}} - \frac{\cancel{\cos\alpha}[4\cos^2\alpha - 3]}{\cancel{\cos\alpha}}$$

$= 3 - 4\sin^2\alpha - [4\cos^2\alpha - 3]$

$= 3 - 4\sin^2\alpha - 4\cos^2\alpha + 3$

$= 6 - 4\sin^2\alpha - 4[1 - \sin^2\alpha]$

$= 6 - \cancel{4\sin^2\alpha} - 4 + \cancel{4\sin^2\alpha}$

$= 6 - 4 \Rightarrow 2$

$=$ R.H.S

xvii) $\dfrac{\sin\theta}{1+\cos\theta} = \tan\dfrac{\theta}{2}$

Proof:-

L.H.S $\Rightarrow \dfrac{\sin\theta}{1+\cos\theta}$

$= \dfrac{2\sin\theta/2\cos\theta/2}{\cancel{1} + 2\cos^2\theta/2 - \cancel{1}}$

$= \dfrac{\cancel{2}\sin\theta/2\,\cancel{\cos\theta/2}}{\cancel{2}\cancel{\cos\theta/2}\cdot\cos\theta/2} \Rightarrow \dfrac{\sin\theta/2}{\cos\theta/2}$

$= \tan\theta/2 \Rightarrow$ R.H.S

xvi) $\dfrac{1-\cos^2\beta}{2-2\cos\beta} = \cos^2\beta/2$

Proof:-

L.H.S $\Rightarrow \dfrac{1-\cos^2\beta}{2-2\cos\beta}$

$= \dfrac{\cancel{(1-\cos\beta)}(1+\cos\beta)}{2\cancel{(1-\cos\beta)}}$

$= \dfrac{1+\cos\beta}{2} \Rightarrow \cos^2\beta/_2$

$=$ R.H.S.

xviii) $\dfrac{1-\tan^2 x/2}{1+\tan^2 x/2} = \cos x$

Proof:-

L.H.S $\Rightarrow \dfrac{1-\tan^2 x/2}{1+\tan^2 x/2}$

$= \dfrac{1-\dfrac{\sin^2 x/2}{\cos^2 x/2}}{\sec^2 x/2}$

$$= \frac{\dfrac{\cos^2 x/2 - \sin^2 x/2}{\cos^2 x/2}}{\dfrac{1}{\cos^2 x/2}}$$

$$= \cos^2 x/2 - \sin^2 x/2$$

$$= \cos x$$

$$= R.H.S.$$

xix) $\frac{\sin 2\alpha}{\sin\alpha} - \frac{\cos 2\alpha}{\cos\alpha} = \sec\alpha$

Proof:-

$$L.H.S \Rightarrow \frac{\sin 2\alpha}{\sin\alpha} - \frac{\cos 2\alpha}{\cos\alpha}$$

$$= \frac{2\cancel{\sin\alpha}\cos\alpha}{\cancel{\sin\alpha}} - \frac{\cos 2\alpha}{\cos\alpha}$$

$$= 2\cos\alpha - \frac{\cos 2\alpha}{\cos\alpha}$$

$$= \frac{2\cos^2\alpha - \cos 2\alpha}{\cos\alpha}$$

$$= \frac{\cancel{\cos 2\alpha} + 1 - \cancel{\cos 2\alpha}}{\cos\alpha} \qquad \{\because \cos 2\alpha = 2\cos^2\alpha -$$

$= 1/\cos\alpha \Rightarrow \sec\alpha$
$= R.H.S$

xx) $2\sin^2\frac{\beta}{2} + \cos\beta = 1$

Proof:-

$L.H.S \Rightarrow 2\sin^2\frac{\beta}{2} + \cos\beta$

$= 1 - \cancel{\cos\beta} + \cancel{\cos\beta}$ $\left[\because \cos 2\varphi = 1 - 2\sin^2\varphi\right]$
$= 1$
$= R.H.S$

xxi) $2\cos y \sec 2y = \frac{1}{\cos y - \sin y} + \frac{1}{\cos y + \sin y}$

Proof:-

$L.H.S \Rightarrow \frac{1}{\cos y - \sin y} + \frac{1}{\cos y + \sin y}$

$= \frac{\cos y \cancel{+ \sin y} + \cos y \cancel{- \sin y}}{(\cos y - \sin y)(\cos y + \sin y)}$

$$= \frac{2\cos y}{\cos^2 y - \sin^2 y}$$

$$= \frac{2\cos y}{\cos 2y} \Rightarrow 2\cos y \sec 2y$$

$= R.H.S$

xxii) $2\sin y \sec 2y = \frac{1}{\cos y - \sin y} - \frac{1}{\cos y + \sin y}$

Proof:-

$L.H.S \Rightarrow \frac{1}{\cos y - \sin y} - \frac{1}{\cos y + \sin y}$

$$= \frac{\cancel{\cos y} + \sin y - \cancel{\cos y} + \sin y}{\cancel{(\cos y - \sin y)(\cos y + \sin y)}}$$

$$= \frac{2\sin y}{\cos^2 y - \sin^2 y} \Rightarrow \frac{2\sin y}{\cos 2y}$$

$= 2\sin y \sec 2y$

$= R.H.S.$