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Where Minds Pollinate

Class 11
Mathematics

Chapter 8
Exercise 8.1
Notes

National Book Foundation

Unit # 08
Fundametals of Trigonometry

Exercise # 8.1

Q. Find value of $\cos(\alpha \pm \beta)$ and $\sin(\alpha \pm \beta)$, $\tan(\alpha \pm \beta)$

i) $\alpha = 180^\circ$, $\beta = 60^\circ$

Sol:-

Given $\alpha = 180^\circ$, $\beta = 60^\circ$

$\because \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(180^\circ + 60^\circ) = \cos(180^\circ)\cos(60^\circ) - \sin(180^\circ)\sin(60^\circ)$

$\cos(180^\circ + 60^\circ) = -1(1/2) - 0(\sqrt{3}/2)$

$\cos(180^\circ + 60^\circ) = -1/2$.

Now, $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(180+60)^\circ = \cos(180^\circ)\cos(60^\circ) + \sin(180^\circ)\sin(60^\circ)$

$\cos(180^\circ - 60^\circ) = -1/2$

also, $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(180^\circ + 60^\circ) = \sin(180^\circ)\cos(60^\circ) + \cos(180^\circ)\sin(60^\circ)$

$= 0(1/2) + (-1)(\sqrt{3}/2)$

$\sin(180^\circ + 60^\circ) = -\sqrt{3}/2$

also, $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin(180^\circ-60^\circ) = \sin(180^\circ)\cos(60^\circ) - \cos(180^\circ)\sin(60^\circ)$

$\sin(180^\circ-60^\circ) = 0(1/2) - (-1)(\sqrt{3}/2)$

$\sin(180^\circ-60^\circ) = +\sqrt{3}/2$

Now for tan.

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(180^\circ+60^\circ) = \frac{\tan(180^\circ)+\tan(60^\circ)}{1-\tan(180^\circ)\tan(60^\circ)}$$

$$= \frac{0+\sqrt{3}}{1-(0)(\sqrt{3})} \Rightarrow \frac{\sqrt{3}}{1}$$

$\tan(180^\circ+60^\circ) = \sqrt{3}$

also, $$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$\tan(180^\circ-60^\circ) = \frac{\tan(180^\circ)-\tan(60^\circ)}{1+\tan(180^\circ)(\tan(60^\circ)}$$

$\tan(180^\circ-60^\circ) = -\sqrt{3}$

ii) $\alpha = 60^\circ, \beta = 90^\circ$

Sol:-

$$\alpha = 60^\circ, \quad \beta = 90^\circ$$

$\because \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\cos(60^\circ \pm 90^\circ) = \cos(60^\circ)\cos(90^\circ) \mp \sin(60^\circ)\sin(90^\circ)$

$= \frac{1}{2}(0) \mp \frac{\sqrt{3}}{2}(1)$

$\cos(60^\circ \pm 90^\circ) = \mp \frac{\sqrt{3}}{2}$

Now, $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\sin(60^\circ \pm 90^\circ) = \sin(60^\circ)\cos(90^\circ) \pm \cos(60^\circ)\sin(90^\circ)$

$= \frac{\sqrt{3}}{2}(0) \pm \frac{1}{2}(1)$

$\sin(60^\circ \pm 90^\circ) = \pm \frac{1}{2}$

lastly, $\tan(\alpha \pm \beta) = \dfrac{\sin\alpha\cos\beta \pm \cos\alpha\sin\beta}{\cos\alpha\cos\beta \mp \sin\alpha\sin\beta}$

$\tan(60^\circ \pm 90^\circ) = \dfrac{\sin(60^\circ)\cos(90^\circ) \pm \cos(60^\circ)\sin(90^\circ)}{\cos(60^\circ)\cos(90^\circ) \mp \sin(60^\circ)\sin(90^\circ)}$

$= \dfrac{\sin 60(0) \pm 1(\cos(60))}{\cos 60(0) \mp 1(\sin 60^\circ)} \Rightarrow \dfrac{\pm \frac{1}{\cancel{2}}}{\mp \frac{\sqrt{3}}{\cancel{2}}} \rightarrow \dfrac{\pm 1}{\mp \sqrt{3}}$

$\tan(90^\circ + 60^\circ) = -\dfrac{1}{\sqrt{3}}, \quad \tan(90 - 60) = \dfrac{1}{\sqrt{3}}$

iii) $\alpha = 180°$, $\beta = 30°$

Sol:-

$\alpha = 180°$, $\beta = 30°$

$\because \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\cos(180° \pm 30°) = \cos(180°)\cos(30°) - \sin(180°)(\sin(30°))$

$= -1(\sqrt{3}/2) \mp 0(1/2)$

$\cos(180° \pm 30°) = \mp\sqrt{3}/2$

Now, $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\sin(180° \pm 30°) = \sin(180°)\cos(30°) \pm \cos(180°)\sin(30°)$

$\sin(180° \pm 30°) = 0(\sqrt{3}/2) \pm (-1)(1/2)$

$\sin(180° \pm 30°) = \mp 1/2$

also, $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$\tan(180° \pm 30°) = \frac{\tan(180°) \pm \tan(30°)}{1 \mp \tan(180°)\tan(30°)}$$

$$= \frac{0 \pm \sqrt{3}/3}{1 \mp (0)(\sqrt{3}/3)} \Rightarrow \pm\frac{\sqrt{3}}{3}$$

$$\tan(180° \pm 30°) = \pm\frac{\sqrt{3}}{3}$$

iv) $\alpha = \pi$, $\beta = \frac{2\pi}{3}$

Sol:-

$\alpha = \pi \Rightarrow 180°$ $\{\pi = 180°\}$

and $\beta = \frac{2\pi}{3} \Rightarrow 120°$

$\because \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\cos(180° \pm 120°) = \cos(180°)\cos(120°) \mp \sin(180°)\sin(120°)$

$\cos(180° \pm 120°) = -1(-1/2) \mp 0(\sqrt{3}/2)$

$\cos(180° \pm 120°) = 1/2$

Now, $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\sin(180° \pm 120°) = \sin(180)\cos(120) \pm \cos(180)\sin(120)$

$= 0(-1/2) \pm (-1)(\sqrt{3}/2)$

$\sin(180° \pm 120°) = \mp\sqrt{3}/2$

also $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\tan(180 \pm 120)° = \frac{\tan(180°) \pm \tan(120°)}{1 \mp \tan(180)\tan(120)}$

$= \frac{0 \pm (-\sqrt{3})}{1 \mp (0)(-\sqrt{3})} \Rightarrow \mp\sqrt{3}$

$\tan(180° \pm 120°) = \mp\sqrt{3}$

v) $\alpha = \frac{4\pi}{3}, \beta = \frac{\pi}{6}$

Sol:-

$$\alpha = \frac{4\pi}{3} \Rightarrow 240^\circ$$

also $\beta = \frac{\pi}{6} \Rightarrow 30^\circ$

$\because \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$\cos\left(\frac{4\pi}{3} \pm \frac{\pi}{6}\right) = \cos(240)\cos(30) \mp \sin(240)\sin(30)$$

$$= -1/2(\sqrt{3}/2) \mp (-\sqrt{3}/2)(1/2)$$

$$= \frac{-\sqrt{3}}{4} \pm \frac{\sqrt{3}}{4}$$

$\cos\left(\frac{4\pi}{3} + \frac{\pi}{6}\right) = 0$, $\cos\left(\frac{4\pi}{3} - \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

Now, $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$$\sin\left(\frac{4\pi}{3} \pm \frac{\pi}{6}\right) = \sin(240)\cos(30) \pm \cos(240)\sin(30)$$

$$= -\sqrt{3}/2(\sqrt{3}/2) \pm (-1/2)(1/2)$$

$$= \frac{-3}{4} \mp \frac{1}{4}$$

$\sin\left(\frac{4\pi}{3}+\frac{\pi}{6}\right) = -1$, $\sin\left(\frac{4\pi}{3}-\frac{\pi}{6}\right) = \frac{-1}{2}$

and $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$

$\tan\left(\frac{4\pi}{3} \pm \frac{\pi}{6}\right) = \dfrac{\tan(240) \pm \tan(30)}{1 \mp \tan(240)\tan(30)}$

$= \dfrac{\sqrt{3} \pm \sqrt{3}/3}{1 \mp \sqrt{3}(\sqrt{3}/3)}$

$\tan\left(\frac{4\pi}{3}+\frac{\pi}{6}\right) = \infty$, $\tan\left(\frac{4\pi}{3}-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$

vi) $\alpha = \frac{7\pi}{4}$, $\beta = \frac{3\pi}{4}$

Sol:-

$\alpha = \frac{7\pi}{4} \Rightarrow 315°$

and $\beta = \frac{3\pi}{4} \Rightarrow 135°$

$\because \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\cos(315° \pm 135°) = \cos(315)\cos(135) \mp \sin(315)\sin(135)$

$= \sqrt{2}/2\,(-\sqrt{2}/2) \mp (-\sqrt{2}/2)(\sqrt{2}/2)$

$= -1/2 \pm 1/2$

$\cos\left(\frac{7\pi}{4} + \frac{3\pi}{4}\right) = 0$, $\cos\left(\frac{7\pi}{4} - \frac{3\pi}{4}\right) = -1$

Now, $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\sin\left(\frac{7\pi}{4} \pm \frac{3\pi}{4}\right) = \sin(315)\cos(135) \pm \cos(315)\sin(135)$

$= -\sqrt{2}/2\,(-\sqrt{2}/2) \pm \sqrt{2}/2\,(+\sqrt{2}/2)$

$= \frac{1}{2} \pm \frac{1}{2}$

$\sin\left(\frac{7\pi}{4} + \frac{3\pi}{4}\right) = 1$, $\sin\left(\frac{7\pi}{4} - \frac{3\pi}{4}\right) = 0$

and, $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$$\tan\left(\frac{7\pi}{4} \pm \frac{3\pi}{4}\right) = \frac{\tan(315) \pm \tan(135)}{1 \mp \tan(135)\tan(315)}$$

$$= \frac{-1 \mp 1}{1 \mp (-1)(-1)}$$

$$\tan\left(\frac{7\pi}{4} + \frac{3\pi}{4}\right) = \infty, \tan\left(\frac{7\pi}{4} - \frac{3\pi}{4}\right) = 0$$

Q2.a. Find value of $\cos 15°$ using $\cos(45° - 30°)$

Sol:-

$\cos(45° - 30°) = \cos 15°$

$\cos(45° - 30°) = \cos(45)\cos(30) + \sin(45)\sin(30)$

OR, $\cos(15°) = \sqrt{2}/2\,(\sqrt{3}/2) + \sqrt{2}/2\,(1/2)$

$= \sqrt{6}/4 + \sqrt{2}/4$

$$\cos(15°) = \frac{\sqrt{6} + \sqrt{2}}{4}$$

b. Use the value of $\cos 15°$ found in a. to find $\cos 165°$ using $\cos(180° - 15°)$

Sol:-

$\cos(180° - 15°) = \cos 165°$

$\cos(180° - 15°) = \cos(180°)\cos(15°) + \sin(180°)\sin(15°)$

$\cos 165° = -1(\sqrt{6} + \sqrt{2}/4) + 0(\sin 15°)$

$\cos 165° = -\dfrac{\sqrt{6} - \sqrt{2}}{4}$

c. Use the value of $\cos 15°$ found in a. to find $\cos 345°$ using $\cos(360° - 15°)$

Sol:-

$\cos(360° - 15°) = \cos 345°$

$\cos(360° - 15°) = \cos(360)\cos(15) + \sin(360)\sin(15)$

$= 1\left(\dfrac{\sqrt{6} + \sqrt{2}}{4}\right) + 0(\sin 15°)$

$\cos(345°) = \dfrac{\sqrt{6} + \sqrt{2}}{4}$

d. Use $\cos A = \sin(90° - A)$ to find value of $\sin 75°$ and $\tan 75°$

Sol:-

$\sin 75° = \sin(90° - 15°)$

$\sin(90° - 15°) = \sin(90°)\cos(15°) - \cos(90°)\sin 15°$

OR,

$\sin 75° = 1\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + 0(\sin 15°)$

$\sin 75° = \frac{\sqrt{6}+\sqrt{2}}{4}$

Now, $\tan 75° = \tan(30°+45°)$

$\tan(30°+45°) = \frac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°}$

$= \frac{\sqrt{3}/3 + 1}{1 - \sqrt{3}/3 (1)} \Rightarrow \frac{3+\sqrt{3}/\cancel{3}}{3-\sqrt{3}/\cancel{3}}$

$\tan 75° = 2+\sqrt{3}$

Q3.a. Find value of $\cos 120°$ using $\cos(180°-60°)$ and $\cos(90°+30°)$.

Sol:-

$\cos(180°-60°) = \cos 120°$

$\cos(180°-60°) = \cos 180° \cos 60° + \sin 180° \sin 60°$

$= -1(1/2) + 0(\sin 60°)$

$\cos(180°-60°) = -1/2 \Rightarrow \cos 120°$

also $\cos 120° = \cos(90° + 30°)$

$\cos(90°+30°) = \cos 90° \cos 30° - \sin 90° \sin 30°$
$= 0(\cos 30°) - 1(1/2)$

$\cos(90°+30°) = -1/2 \Rightarrow \cos 120°$

b. Find value of cos 120° and tan 120°

Sol:-
$\cos 120° = \cos(90° + 30°)$

$\cos(90°+30°) = \cos 90° \cos 30° - \sin 90° \sin 30°$
$= 0(\cos 90°) - 1(1/2)$
$\cos(90°+30°) = -1/2 \Rightarrow \cos 120°$

and $\tan(120°) = \tan(180° - 60°)$

$\tan(180°-60°) = \dfrac{\tan 180° - \tan 60°}{1 + \tan 180° \tan 60°}$

$= \dfrac{0 - \sqrt{3}}{1 + 0(\sqrt{3})} \Rightarrow \dfrac{-\sqrt{3}}{1}$

$\tan 120° = -\sqrt{3}$

c. Find value of $\cos 75°$ using $\cos(120° - 45°)$.

Sol:-

$$\cos 75° = \cos(120° - 45°)$$

$$\cos(120° - 45°) = \cos 120 \cos 45 + \sin 120 \sin 45$$

$$= -\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\cos 75° = \frac{\sqrt{6} - \sqrt{2}}{4}$$

d. Use value of $\cos 75°$ found in c. to find $\cos 105°$ using $\cos(180° - 75°)$.

Sol:-

$$\cos 105° = \cos(180° - 75°)$$

$$\cos(180° - 75°) = \cos 180° \cos 75° + \sin 180° \sin 75°$$

$$= -1\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) + 0(\sin 75°)$$

$$\cos 105° = \frac{\sqrt{2} - \sqrt{6}}{4}$$

e. Use value of $\cos 75°$ found c. to find $\cos 285°$ usin $\cos(360° - 75°)$.

Sol:-

$$\cos 285° = \cos(360° - 75°)$$

$$\cos(360° - 75°) = \cos 360° \cos 75° + \sin 360° \sin 75°$$

$$= 1\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) + 0(\sin 75°)$$

$$\cos 285° = \frac{\sqrt{6} - \sqrt{2}}{4}$$

f. Find value of $\sin 15°$

Sol:-

$$\sin 15° = \sin(90° - 75°)$$

$$\sin(90° - 75°) = \sin 90° \cos 75° - \cos 90° \sin 75°$$

$$= 1\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) - 0(\sin 75°)$$

$$\sin 15° = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Q.4 Rewrite in single expression.

i) $\sin\left(\frac{\theta}{3}\right)\cos\left(\frac{\theta}{6}\right)+\cos\left(\frac{\theta}{3}\right)\sin\left(\frac{\theta}{6}\right)$

Sol:-

$=\sin\left(\frac{\theta}{3}\right)\cos\left(\frac{\theta}{6}\right)+\cos\left(\frac{\theta}{3}\right)\sin\left(\frac{\theta}{6}\right)$

using formula.

$=\sin\left(\frac{\theta}{3}+\frac{\theta}{6}\right)\Rightarrow\sin\left(\frac{2\theta+\theta}{6}\right)$

$=\sin\left(\frac{\theta}{2}\right)$

ii) $\cos 7\theta\cos 2\theta+\sin 7\theta\sin 2\theta$

Sol:-

$=\cos 7\theta\cos 2\theta+\sin 7\theta\sin 2\theta$

$=\cos(7\theta-2\theta)\Rightarrow\cos(5\theta)$

iii) $\cos 6\theta \cos 3\theta - \sin 6\theta \sin 3\theta$

Sol:-

$= \cos 6\theta \cos 3\theta - \sin 6\theta \sin 3\theta$

$= \cos(6\theta + 3\theta) \Rightarrow \cos(9\theta)$

iv) $\sin 138^\circ \cos 46^\circ - \cos 138^\circ \sin 46^\circ$

Sol:-

$= \sin 138^\circ \cos 46^\circ - \cos 138^\circ \sin 46^\circ$

$= \sin(138^\circ - 46^\circ) \Rightarrow \sin 92^\circ$

v) $\dfrac{\tan 75^\circ - \tan 45^\circ}{1 + \tan 75^\circ \tan 45^\circ}$

Sol:-

$= \dfrac{\tan 75^\circ - \tan 45^\circ}{1 + \tan 75^\circ \tan 45^\circ}$

$= \tan(75^\circ - 45^\circ)$

$= \tan 30^\circ$

vi) $\dfrac{\tan 4\pi/3 + \tan 2\pi/3}{1 - \tan 4\pi/3 \tan 2\pi/3}$

Sol:-

$$= \frac{\tan 4\pi/3 + \tan 2\pi/3}{1 - \tan 4\pi/3 \tan 2\pi/3}$$

$$= \tan\left(\frac{4\pi}{3} + \frac{2\pi}{3}\right)$$

$$= \tan\left(\frac{6\pi}{3}\right) \Rightarrow \tan(2\pi)$$

$$= \tan 2\pi$$

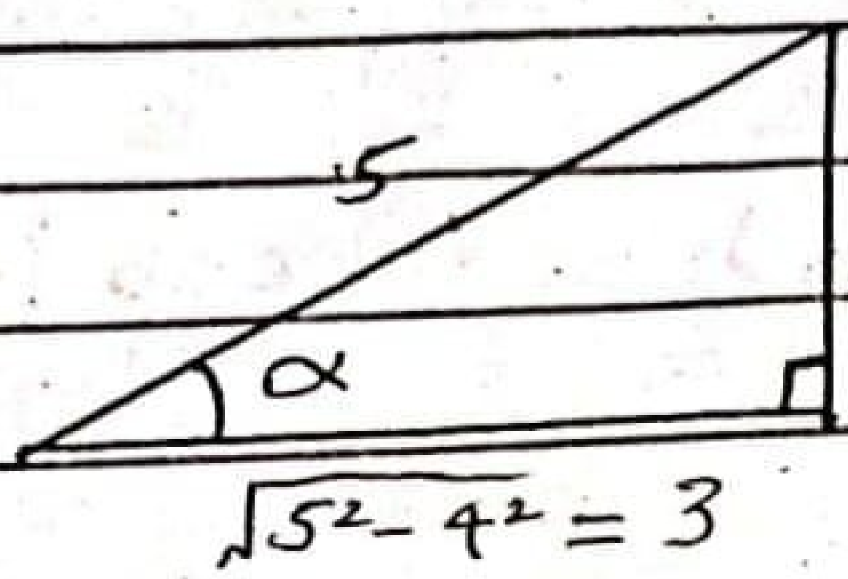
Q5. For $\sin\alpha = \frac{4}{5}$, $\tan\beta = -\frac{5}{12}$ with terminal side of in Quad. II, find $\cos(\alpha \pm \beta)$.

Sol:-

Given:- $\sin\alpha = \frac{4}{5} =$

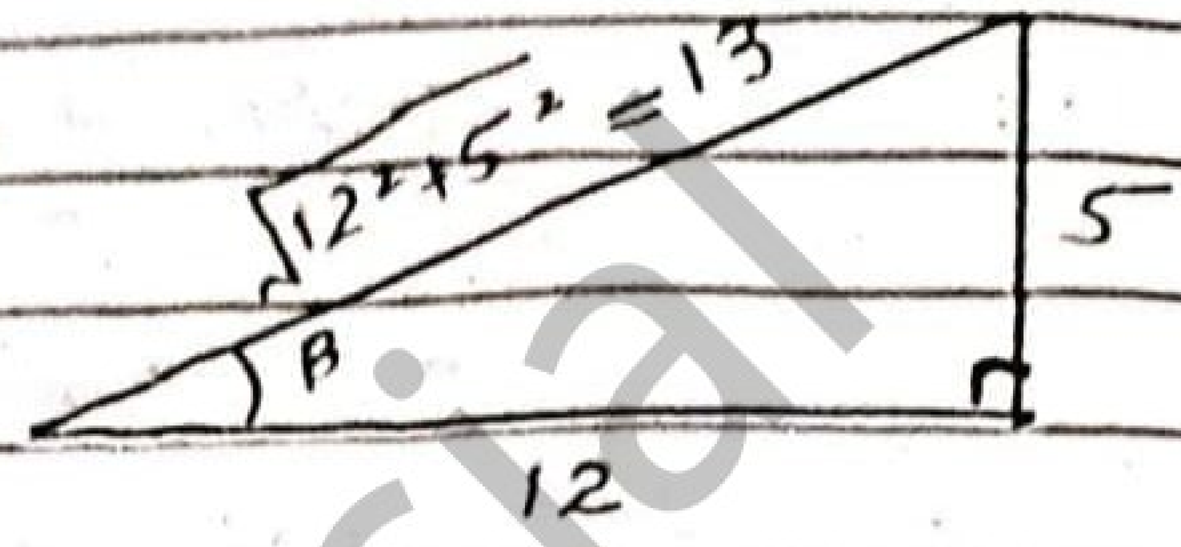
and $\tan = -\frac{1(5)}{12}$ {in Quad. II}

for $\sin\alpha = \frac{4}{5}$.



$\therefore \cos\alpha = -\frac{3}{5}$

also for $\tan\beta = \frac{-5}{12}$



$\therefore \quad \cos\beta = -\frac{12}{13}$

$\sin\beta = \frac{5}{13}$

So, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{-3}{5}\left(\frac{-12}{13}\right) - \frac{4}{5}\left(\frac{5}{13}\right)$

$= \frac{36}{65} - \frac{4}{13}$

$\cos(\alpha+\beta) = \frac{16}{65}$

and $\cos(\alpha-\beta) = \frac{36}{65} + \frac{4}{13}$

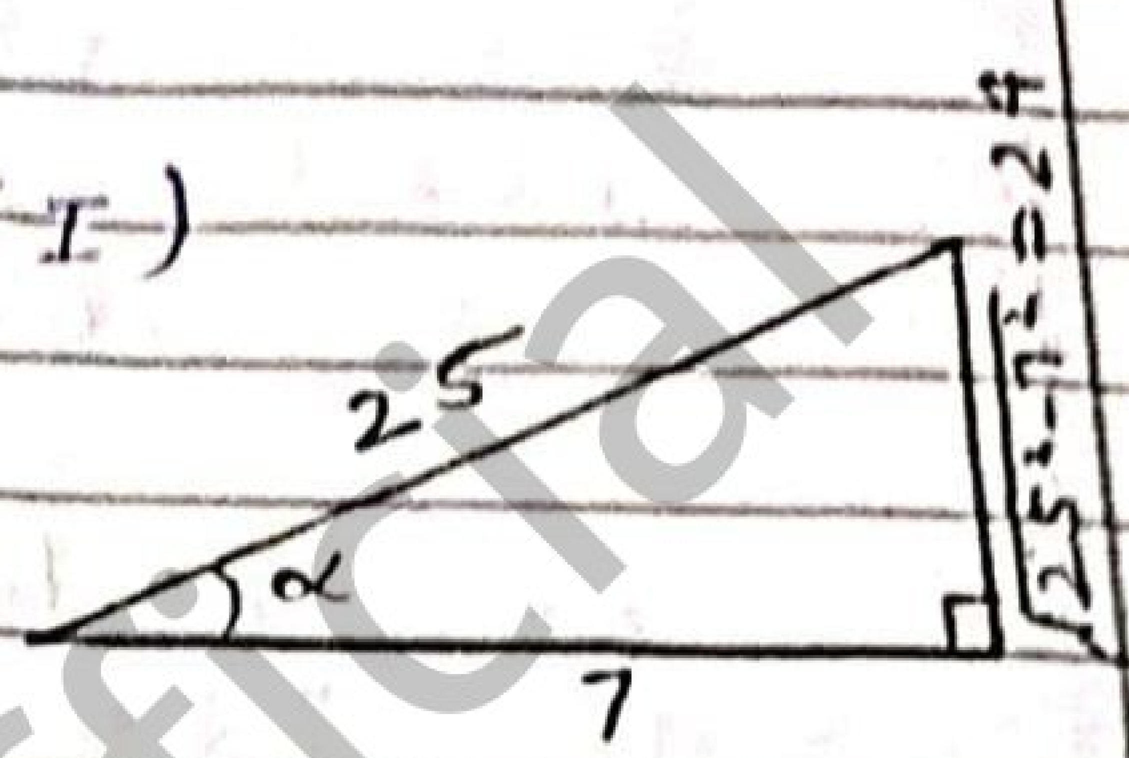
$\cos(\alpha-\beta) = \frac{56}{65}$

Q6. For $\cos\alpha = -\frac{7}{25}$ with terminal side in Qud. II and $\cot\beta = \frac{15}{8}$ with terminal side in Quad. III, find

i) $\sin(\alpha-\beta)$ ii) $\cos(\alpha-\beta)$ iii) $\tan(\alpha-\beta)$

Sol:-

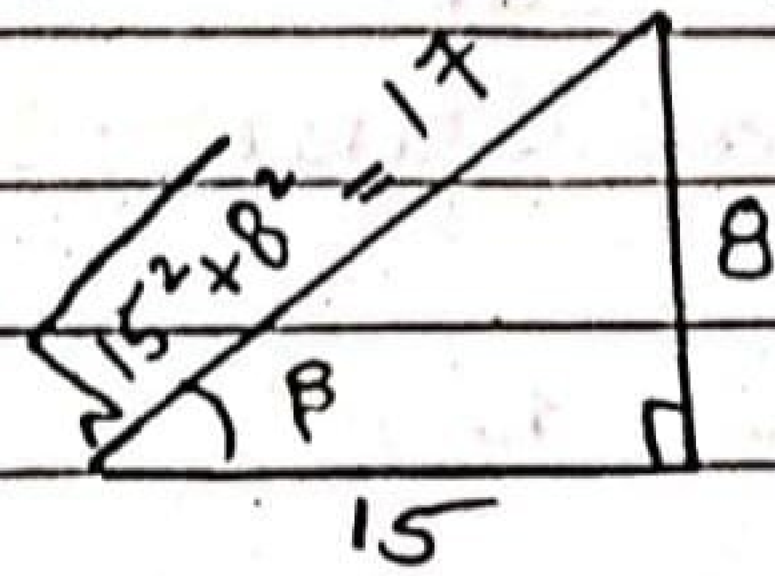
for $\cos\alpha = -\frac{7}{25}$ {Quad. II}



$\therefore \sin\alpha = \frac{24}{25}$

$\tan\alpha = -\frac{24}{7}$

Now for $\cot\beta = \frac{15}{8} \Rightarrow \tan\beta = \frac{8}{15}$ {Qud. III}



$\therefore \cos\beta = -\frac{15}{17}$

$\sin\beta = -\frac{8}{17}$

Now, $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{24}{25}\left(-\frac{15}{17}\right) - \left(-\frac{7}{25}\right)\left(-\frac{8}{17}\right)$

$\sin(\alpha-\beta) = -\frac{416}{425}$

also, $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{-7}{25}\right)\left(\frac{-15}{17}\right) + \left(\frac{24}{25}\right)\left(\frac{-8}{17}\right)$

$\cos(\alpha-\beta) = -87/425$

Now, $\tan(\alpha-\beta) = \dfrac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$

$\therefore \quad = \dfrac{-416/425}{-87/425}$

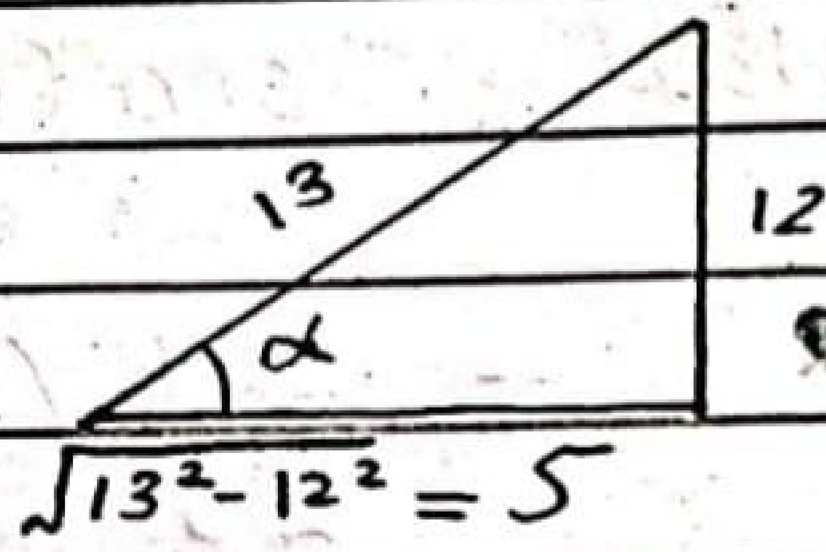
$\tan(\alpha-\beta) = \dfrac{416}{87}$

Q7. For $\sin\alpha = \dfrac{12}{13}$ and $\tan\beta = \dfrac{4}{3}$, α and β are acute angles. Find:-

i) $\sin(\alpha+\beta)$ ii) $\cos(\alpha+\beta)$ iii) $\tan(\alpha+\beta)$

Sol:-

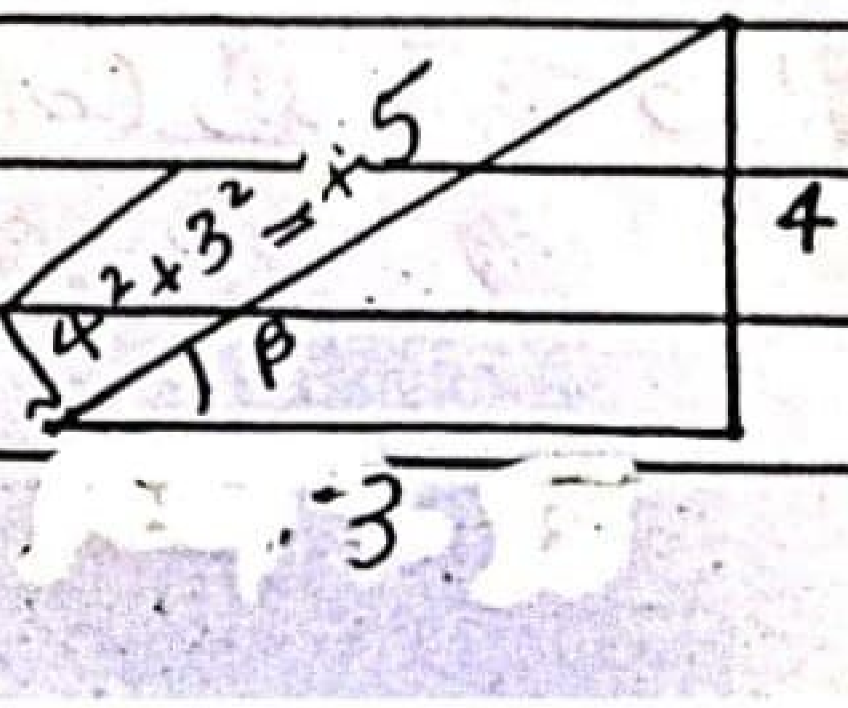
For $\sin\alpha = \dfrac{12}{13}$ {Quad. I}



$\therefore \cos\alpha = \dfrac{5}{13}$

Now for $\tan\beta = \dfrac{4}{3}$ {Quad. I}

$\therefore \cos\beta = \dfrac{3}{5}$



$\sin\beta = \dfrac{4}{5}$

So, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \left(\frac{5}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$$

$$\cos(\alpha+\beta) = -\frac{33}{65}$$

Now, $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$\therefore = \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$$

$$\sin(\alpha+\beta) = \frac{56}{65}$$

also, $\tan(\alpha+\beta) = \dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$$= \frac{56/65}{-33/65}$$

$$\tan(\alpha+\beta) = -\frac{56}{33}$$

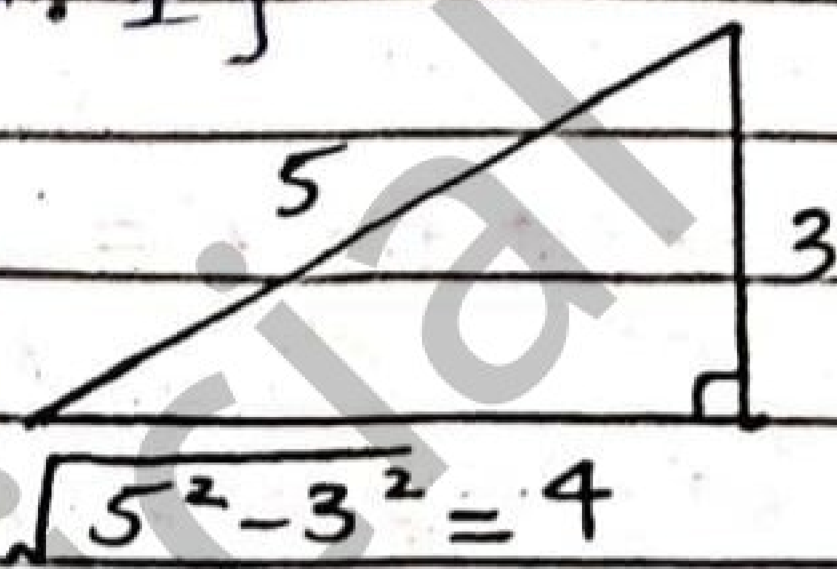
Q8. If $\sin\alpha = \frac{3}{5}$, where $0 < \alpha < \frac{\pi}{2}$ and

$\cos\beta = \frac{12}{13}$, where $\frac{3\pi}{2} > \beta > \pi$ find:-

i) $\csc(\alpha+\beta)$ ii) $\sec(\alpha+\beta)$ iii) $\cot(\alpha+\beta)$

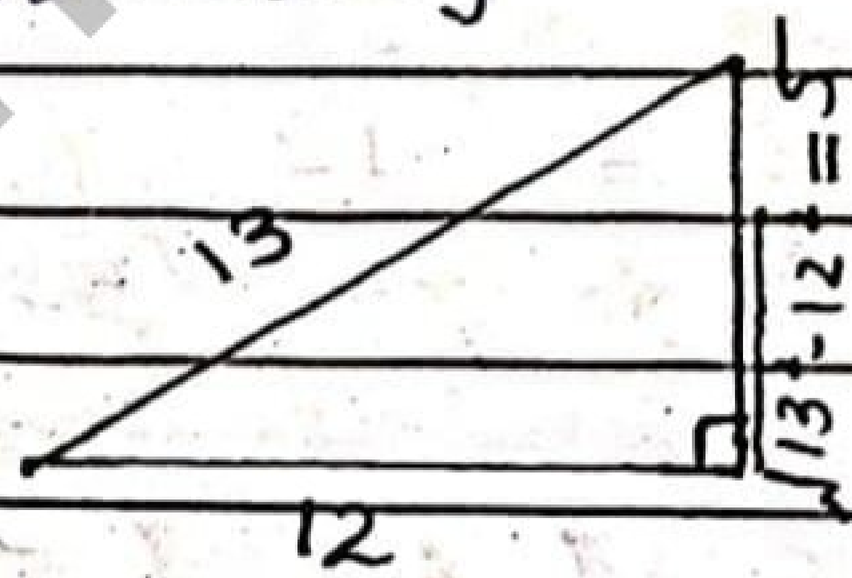
Sol:-

For $\sin\alpha = \frac{3}{5}$ {Quad. I}



$\therefore \cos\alpha = \frac{4}{5}$

Now, for $\cos\beta = \frac{12}{13}$ {Quad. III}



$\therefore \sin\beta = -\frac{5}{13}$

For, $\csc(\alpha+\beta) = \frac{1}{\sin(\alpha+\beta)}$

$$\csc(\alpha+\beta) = \frac{1}{\sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

$$= \frac{1}{(3/5)(12/13) + (4/5)(-5/13)}$$

$$\csc(\alpha+\beta) = \frac{65}{16}$$

Now, $\sec(\alpha+\beta) = \dfrac{1}{\cos(\alpha+\beta)}$

$$\sec(\alpha+\beta) = \frac{1}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$= \frac{1}{(4/5)(12/13) - (3/5)(-5/13)}$$

$$\sec(\alpha+\beta) = \frac{65}{63}$$

Now, $\cot(\alpha+\beta) = \dfrac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}$

$$\cot(\alpha+\beta) = \frac{1/\sec(\alpha+\beta)}{1/\csc(\alpha+\beta)}$$

$$\cot(\alpha+\beta) = \frac{\csc(\alpha+\beta)}{\sec(\alpha+\beta)} \Rightarrow \frac{65/16}{65/63}$$

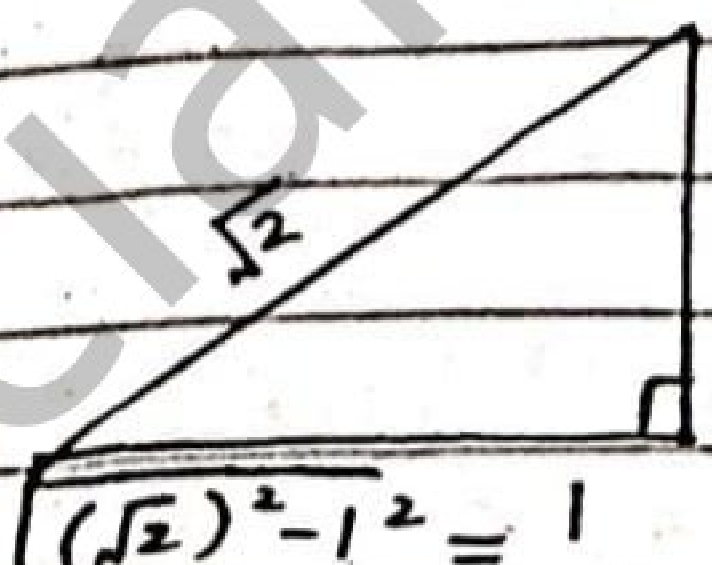
$$\cot(\alpha+\beta) = \frac{\cancel{65}}{16} \times \frac{63}{\cancel{65}} \Rightarrow \frac{63}{16}$$

Q9. Given α and β are obtuse angle with $\sin\alpha = \dfrac{1}{\sqrt{2}}$ and $\cos\beta = -\dfrac{3}{5}$, find:

i) $\sin(\alpha \pm \beta)$ ii) $\cos(\alpha \pm \beta)$ iii) $\tan(\alpha \pm \beta)$

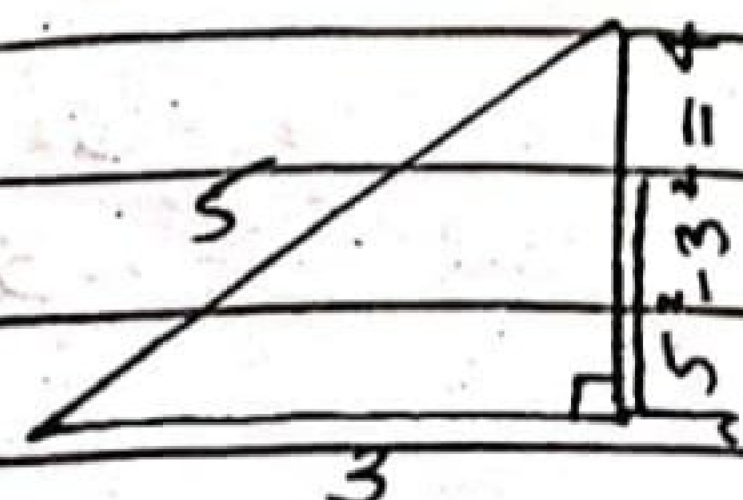
Sol:-

For $\sin\alpha = \frac{1}{\sqrt{2}}$ {Quad. II}



$\therefore \cos\alpha = -\frac{1}{\sqrt{2}}$

Now for $\cos\beta = \frac{-3}{5}$ {Quad. II}



$\therefore \sin\beta = \frac{4}{5}$

Now, $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{-3}{5}\right) \mp \left(\frac{1}{\sqrt{2}}\right)\left(\frac{4}{5}\right)$$

$$\cos(\alpha \pm \beta) = \frac{3}{5\sqrt{2}} \mp \frac{4}{5\sqrt{2}}$$

$\therefore \cos(\alpha+\beta) = -\frac{\sqrt{2}}{10}$, $\cos(\alpha-\beta) = \frac{7\sqrt{2}}{10}$

also, $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-3}{5}\right) \pm \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{4}{5}\right)$$

$$\sin(\alpha \pm \beta) = \frac{-3}{5\sqrt{2}} \pm \frac{-4}{5\sqrt{2}}$$

$$\sin(\alpha+\beta) = \frac{-7\sqrt{2}}{10}, \quad \sin(\alpha-\beta) = \frac{\sqrt{2}}{10}$$

Now, $$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$\tan(\alpha+\beta) = \frac{-7\sqrt{2}}{-\sqrt{2}} \Rightarrow 7$$

lastly, $$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$

$$\tan(\alpha-\beta) = \frac{\sqrt{2}}{7\sqrt{2}} \Rightarrow \frac{1}{7}$$

Q10. verify:-

i) $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

Sol:-

L.H.S $\Rightarrow \sin\left(\frac{\pi}{2} - \alpha\right) \Rightarrow \sin(90° - \alpha)$

$= \sin(90°)\cos\alpha - \cos 90° \sin\alpha$

$= \cos\alpha \Rightarrow$ R.H.S

ii) $\cos(\pi - \alpha) = -\cos\alpha$

Sol:-

L.H.S $\Rightarrow \cos(\pi - \alpha) \Rightarrow \cos(180° - \alpha)$

$= \cos(180°)\cos\alpha + \sin(180°)\sin\alpha$

$= -\cos\alpha$

$=$ R.H.S

iii) $\cos\left(\alpha + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

Sol:-

L.H.S $\Rightarrow \cos\left(\alpha + \frac{\pi}{4}\right) \Rightarrow \cos(\alpha + 45°)$

$= \cos\alpha \cos 45° - \sin\alpha \sin 45°$

$= \cos\alpha\left(\frac{1}{\sqrt{2}}\right) - \sin\alpha\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha) \Rightarrow$ R.H.S

iv) $\sin\left(\beta + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos\beta + \sin\beta)$

Sol:-

L.H.S $\Rightarrow \sin\left(\beta + \frac{\pi}{4}\right) \Rightarrow \sin(\beta + 45°)$

$= \sin\beta \cos 45° + \cos\beta \sin 45°$

$= \sin\beta\left(\frac{\sqrt{2}}{2}\right) + \cos\beta\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{2}(\cos\beta + \sin\beta) \Rightarrow$ R.H.S

v) $\tan\left(\gamma - \frac{\pi}{4}\right) = \frac{\tan\gamma - 1}{\tan\gamma + 1}$

Sol:-

L.H.S $\Rightarrow \tan\left(\gamma - \frac{\pi}{4}\right) \Rightarrow \tan(\gamma - 45°)$

$= \frac{\tan\gamma - \tan 45°}{1 + \tan\gamma \tan 45°} \Rightarrow \frac{\tan\gamma - 1}{1 + \tan\gamma(1)}$

$= \frac{\tan\gamma - 1}{\tan\gamma + 1} \Rightarrow$ R.H.S

vi) $\tan\left(\gamma + \frac{\pi}{4}\right) = \frac{1 + \tan\gamma}{1 - \tan\gamma} = \frac{\cos\gamma + \sin\gamma}{\cos\gamma - \sin\gamma}$

Sol:-

L.H.S $\Rightarrow \tan\left(\gamma + \frac{\pi}{4}\right) \Rightarrow \tan(\gamma + 45°)$

$= \frac{\tan\gamma + \tan 45°}{1 - \tan\gamma \tan 45°} \Rightarrow \frac{\tan\gamma + 1}{1 - \tan\gamma(1)}$

$$= \frac{1+\tan\gamma}{1-\tan\gamma}$$

Now, $\frac{1+\sin\gamma/\cos\gamma}{1-\sin\gamma/\cos\gamma} \Rightarrow \dfrac{\frac{\sin\gamma+\cos\gamma}{\cancel{\cos\gamma}}}{\frac{\cos\gamma-\sin\gamma}{\cancel{\cos\gamma}}}$

$$= \frac{\cos\gamma+\sin\gamma}{\cos\gamma-\sin\gamma}$$

Q11. Show that;

i) $\frac{\sin(180°+\lambda)\cos(270°+\lambda)}{\sin(180°-\lambda)\cos(270°-\lambda)} = 1$

Sol:-

L.H.S $\Rightarrow \frac{\sin(180°+\lambda)\cos(270°+\lambda)}{\sin(180°-\lambda)\cos(270°-\lambda)}$

$$= \frac{(-\cos\lambda)(+\cos\lambda)}{\cancel{(+\cos\lambda)(-\cos\lambda)}}$$

$$= \frac{\cancel{-\cos^2\lambda}}{\cancel{-\cos^2\lambda}} \Rightarrow 1 \Rightarrow \text{R.H.S}$$

ii) $$\frac{\sin(90^\circ+\alpha)-\cos(360^\circ-\alpha)+\cos\alpha}{\sin(180^\circ-\alpha)+\sin(270^\circ-\alpha)+\cos(90^\circ+\alpha)} = -1$$

Sol:-

$$L.H.S \Rightarrow \frac{\sin(90^\circ+\alpha)-\cos(360^\circ-\alpha)+\cos\alpha}{\sin(180^\circ-\alpha)+\sin(270^\circ-\alpha)+\cos(90^\circ+\alpha)}$$

$$= \frac{+\cos\alpha - \cancel{\cos\alpha} + \cancel{\cos\alpha}}{+\cancel{\sin\alpha} - \cos\alpha - \cancel{\sin\alpha}}$$

$$= \frac{\cos\alpha}{-\cos\alpha} \Rightarrow -1$$

$= R.H.S$

iii) $$\tan\alpha + \tan\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$

Sol:-

$L.H.S \Rightarrow \tan\alpha + \tan\beta$

$$= \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} \Rightarrow \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} \Rightarrow R.H.S$$

iv) $\sin(\alpha+\beta)\sin(\alpha-\beta) = \cos^2\beta - \cos^2\alpha = \sin^2\alpha - \sin^2\beta$

Sol:-

L.H.S $\Rightarrow \sin(\alpha+\beta)\sin(\alpha-\beta)$

$= [\sin\alpha\cos\beta + \cos\alpha\sin\beta][\sin\alpha\cos\beta - \cos\alpha\sin\beta]$

$= [\sin\alpha\cos\beta]^2 - [\cos\alpha\sin\beta]^2$ —(i)

use $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\therefore (1-\cos^2\alpha)\cos^2\beta - [(1-\cos^2\beta)\cos^2\alpha]$

$= \cos^2\beta - \cos^2\alpha\cos^2\beta - (\cos^2\alpha - \cos^2\alpha\cos^2\beta)$

$= \cos^2\beta - \cancel{\cos^2\alpha\cos^2\beta} - \cos^2\alpha + \cancel{\cos^2\alpha\cos^2\beta}$

$= \cos^2\beta - \cos^2\alpha$

Now, use $\cos^2\theta = 1 - \sin^2\theta$ in (i)

$= \sin^2\alpha(1-\sin^2\beta) - [(1-\sin^2\alpha)\sin^2\beta]$

$= \sin^2\alpha - \sin^2\alpha\sin^2\beta - (\sin^2\beta - \sin^2\alpha\sin^2\beta)$

$= \sin^2\alpha - \cancel{\sin^2\alpha\sin^2\beta} - \sin^2\beta + \cancel{\sin^2\alpha\sin^2\beta}$

$= \sin^2\alpha - \sin^2\beta$

v) $\dfrac{\tan(x+y)}{\cot(x-y)} = \dfrac{\tan^2 x - \tan^2 y}{1-\tan^2 x \tan^2 y}$

Sol:-

L.H.S $\Rightarrow \dfrac{\tan(x+y)}{\cot(x-y)} \Rightarrow \dfrac{\tan(x+y)}{1/\tan(x-y)}$

$= \dfrac{\tan x + \tan y / 1 - \tan x \tan y}{1 + \tan x \tan y / \tan x - \tan y}$

$= \dfrac{\tan x + \tan y}{1 - \tan x \tan y} \cdot \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

$= \dfrac{(\tan x + \tan y)(\tan x - \tan y)}{(1 - \tan x \tan y)(1 + \tan x \tan y)}$

$= \dfrac{\tan^2 x - \tan^2 y}{1 - \tan^2 x \tan^2 y} \Rightarrow$ R.H.S

vi) $\dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \dfrac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$

Sol:-

L.H.S $\Rightarrow \dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$

$$= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

"$\cos\alpha\cos\beta$" → Divide it in nominater and denominater

$$= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta / \cos\alpha\cos\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta / \cos\alpha\cos\beta}$$

$$= \frac{\cancel{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}} - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}{\cancel{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}} + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{1 - \tan\alpha\tan\beta}{1 + \tan\alpha\tan\beta}$$

vi) $\cot(\alpha - \beta) = \dfrac{\cot\alpha\cot\beta + 1}{\cot\beta - \cot\alpha}$

Sol:-

L.H.S $\Rightarrow \cot(\alpha - \beta) \Rightarrow \dfrac{1}{\tan(\alpha - \beta)}$

$$= \frac{1 + \tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$$

$$= \frac{1 + (1/\cot\alpha)(1/\cot\beta)}{1/\cot\alpha \quad 1/\cot\beta}$$

$$= \frac{\frac{\cot\alpha\cot\beta + 1}{\cot\alpha\cot\beta}}{\frac{\cot\beta - \cot\alpha}{\cot\alpha\cot\beta}}$$

$$= \frac{\cot\alpha\cot\beta + 1}{\cot\beta - \cot\alpha}$$

vii) $\frac{\cos 4\theta}{\csc\theta} + \frac{\sin 4\theta}{\sec\theta} = \sin 5\theta$

Sol:-

L.H.S $\Rightarrow \frac{\cos 4\theta}{\csc\theta} + \frac{\sin 4\theta}{\sec\theta}$

$= \frac{\cos 4\theta}{1/\sin\theta} + \frac{\sin 4\theta}{1/\cos\theta} \Rightarrow$

$= \sin\theta\cos 4\theta + \cos\theta\sin 4\theta$

$= \sin(\theta + 4\theta) \Rightarrow \sin 5\theta \Rightarrow$ R.H.S

$\frac{t\gamma}{2}$

$\frac{1}{1/2}$

Q12. If $\alpha+\beta+\gamma = 180^\circ$, prove that:

i) $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha \tan\beta \tan\gamma$

Proof:-

Given, $\alpha+\beta+\gamma = 180^\circ$

OR $\alpha+\beta = 180^\circ - \gamma$

Now take "tan" on b/s

$$\therefore \tan(\alpha+\beta) = \tan(180^\circ - \gamma)$$

$$\frac{\tan\alpha + \tan\beta}{1 \tan\alpha \tan\beta} = \frac{\tan(180^\circ) - \tan\gamma}{1 + \tan(180^\circ)\tan\gamma}$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = \frac{-\tan\gamma}{1}$$

$$1(\tan\alpha + \tan\beta) = -\tan\gamma\,[1 - \tan\alpha \tan\beta]$$

$$\tan\alpha + \tan\beta = -\tan\gamma + \tan\alpha \tan\beta \tan\gamma$$

OR, $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha \tan\beta \tan\gamma$

Proved.

ii) $\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$

Proof:-

Given, $\alpha + \beta + \gamma = 180°$

$\alpha + \beta = 180° - \gamma$

Divide "2" on b/s and apply cot.

$$\therefore \cot\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \cot\left(90° - \frac{\gamma}{2}\right)$$

$$\frac{\cot(\alpha/2)\cot(\beta/2) - 1}{\cot(\alpha/2) + \cot(\beta/2)} = \frac{\cot(90)\cot(\gamma/2) + 1}{-\cot(90) + \cot(\gamma/2)}$$

$$\frac{\cot(\alpha/2)\cot(\beta/2) - 1}{\cot(\alpha/2) + \cot(\beta/2)} = \frac{1}{\cot(\gamma/2)}$$

$$1[\cot(\alpha/2) + \cot(\beta/2)] = \cot(\gamma/2)[\cot(\alpha/2)\cot(\beta/2) - 1]$$

$$\cot(\alpha/2) + \cot(\beta/2) = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2} - \cot$$

$$\therefore \cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2}$$

→ Proved.

iii) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} \quad \tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} - 1 = 0$

Proof:-

Given, $\alpha+\beta+\gamma = 180^\circ$

$\alpha+\beta = 180^\circ - \gamma$

Divide "2" on b/s and apply

$\therefore \left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \left(90^\circ - \frac{\gamma}{2}\right)$

$\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \tan\left(90^\circ - \frac{\gamma}{2}\right)$

$\frac{\tan\alpha/2 + \tan\beta/2}{1-\tan\alpha/2\,\tan\beta/2} = \cot\frac{\gamma}{2} = \frac{1}{\tan\gamma/2}$

$1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \tan\frac{\gamma}{2}\left[\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}\right]$

$1-\tan\alpha/2\,\tan\beta/2 = \tan\frac{\alpha}{2}\tan\frac{\gamma}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

$\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\alpha}{2}\tan\frac{\gamma}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} - 1 = 0$

Hence proved...

Q13. Express the following in $r\sin(\theta+\phi)$

i) $12\sin\theta - 5\cos\theta$

Sol:-

Given, $= 12\sin\theta - 5\cos\theta$ — (i)

Let $12 = r\cos\phi$ — (ii)

$-5 = r\sin\phi$ — (iii)

Now, put these value in (i)

$= (r\cos\phi)\sin\theta + (r\sin\phi)\cos\theta$

$= r\cos\phi\sin\theta + r\cos\theta\sin\phi$

$= r(\cos\phi\sin\theta + \cos\theta\sin\phi)$

$= r\sin(\theta+\phi)$

Now, we will define r and ϕ.

square eq. (ii) and (iii) and add them

$144 + 25 = r^2\cos^2\phi + r^2\sin^2\phi$

$169 = r^2(\cos^2\phi + \sin^2\phi)$

$r^2 = 169 \Rightarrow \sqrt{r^2} = \sqrt{169}$

$r = 13$

ii Now divide eq(iii) by (ii)

$$-\frac{5}{12} = \frac{r\sin\phi}{r\cos\phi} \Rightarrow \frac{\sin\phi}{\cos\phi} = -\frac{5}{12}$$

OR $\phi = \tan^{-1}\left[-\frac{5}{12}\right]$

ii) $3\sin\theta + 4\cos\theta$

Sol:-

$= 3\sin\theta + 4\cos\theta$

Let, $3 = r\cos\phi$ —(i)

$4 = r\sin\phi$ —(iii)

$\therefore = (r\cos\phi)\sin\theta + (r\sin\phi)\cos\theta$

$= r\cos\phi\sin\theta + r\cos\theta\sin\phi$

$= r(\cos\phi\sin\theta + \cos\theta + \sin\phi)$

$= r\sin(\theta+\phi)$

Now, square on b/s in (i),(ii) and add them

$9+16 = r^2\cos^2\phi + r^2\sin^2\phi$

$25 = r^2(\cos^2\phi + \sin^2\phi)$

$r^2 = 25$

$r = 5$

Now divide eq(ii) by (iii)

$$\frac{4}{3} = \frac{r\sin\phi}{r\cos\phi} \Rightarrow \tan\phi = \frac{4}{3}$$

$$\phi = \tan^{-1}\left[\frac{4}{3}\right]$$

iii) $\sin\theta - \cos\theta$

Sol:-

$= \sin\theta - \cos\theta$

let $1 = r\sin\phi$

$-1 = r\cos\phi$.

$\therefore = +(r\sin\phi)\cos\theta + (r\cos\phi)\sin\theta$

$= r\cos\phi\sin\theta + r\sin\phi\cos\theta$

$= r(\cos\phi\sin\theta + \cos\theta\sin\phi)$

$= r\sin(\theta+\phi)$

Square on b/s in (i) and (ii) and add them

$1 + 1 = r^2(\sin^2\phi + \cos^2\phi)$

$r^2 = 2$

$r = \sqrt{2}$

Divide eq (ii) by (i)

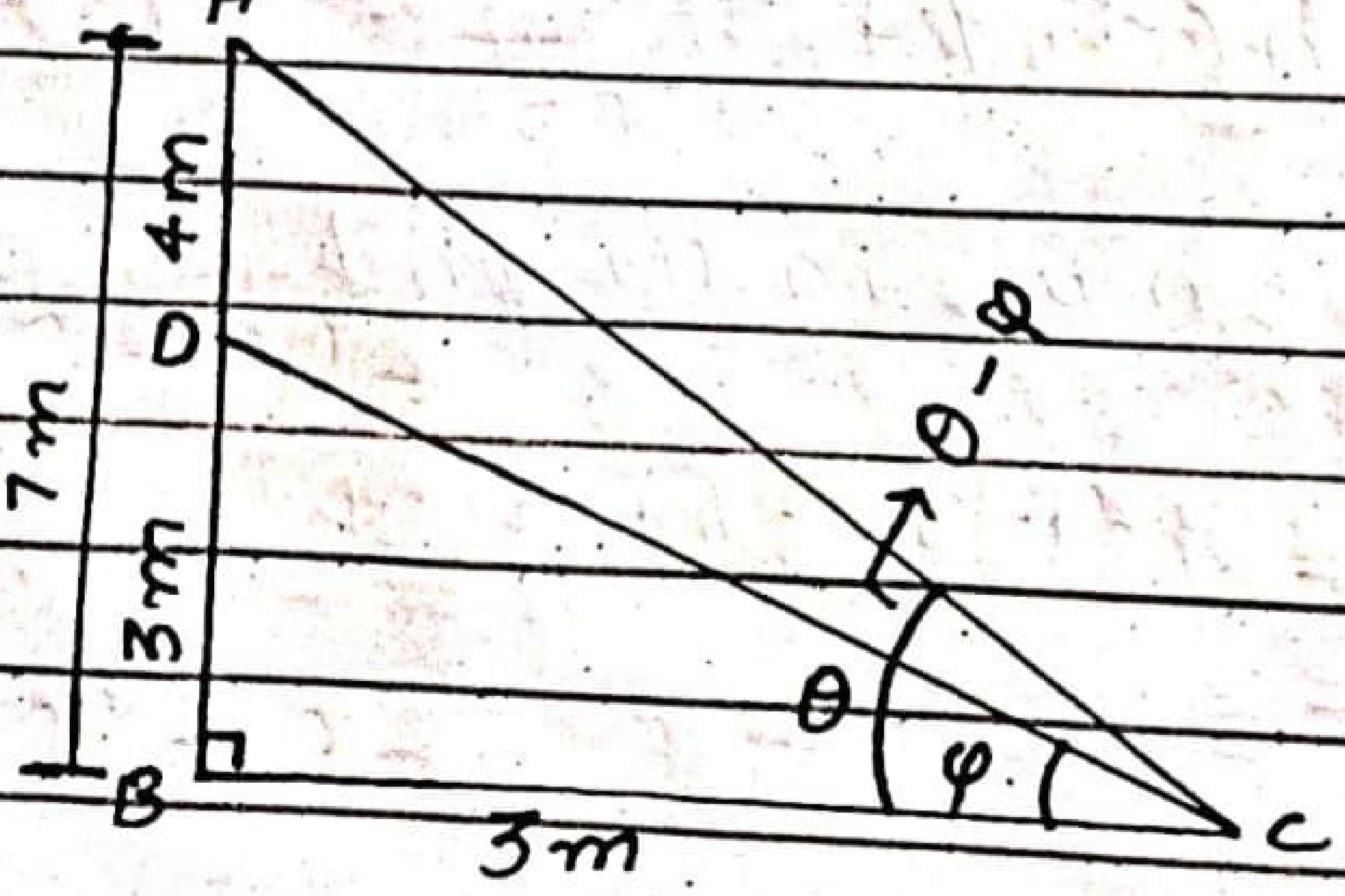
$$\frac{1}{1} = \frac{\cancel{r}\sin\phi}{\cancel{r}\cos\phi} \Rightarrow \tan\phi = -\frac{1}{1}$$

$$\varphi = \tan^{-1}(-1)$$

Q14. A telophone pole is braced by two wiries that are both fastened to the ground at a point 3m from the base of the pole. The shorter wire is fastened 3m above from the base of the pole and the longer wire 7m above the ground.

Sol:-

According to the given situation, the diagram will be:-



a. What is the measure in degrees, of the angle that the shorter wire makes with the ground?

Sol:-

Let shorter angle be φ.

In ΔDBC, we have

$$\therefore \tan\varphi = \frac{3m}{3m}$$

$$\tan\varphi = 1 \Rightarrow \varphi = \tan^{-1}(1)$$

$$\varphi = 45^\circ$$

b. Let θ be the measure of the angle that the longer wire makes with the ground: Find $\sin\theta$ and $\cos\theta$.

Sol:-

In ΔABC

$$(mAC) = \sqrt{(mAB)^2 + (mBC)^2}$$

$$mAC = \sqrt{7^2 + 3^2}$$

$$mAC = \sqrt{58}$$

Now, $\sin\theta = \frac{3}{\sqrt{58}}$ — (x)

and $\cos\theta = \frac{7}{\sqrt{58}}$

c. Find the cosine of the angle between the wires where they meet at the ground.

Sol:-

Angle between wires $= \theta - \varphi$

So, $\cos(\theta - \varphi) = \cos\theta \sin\varphi + \cos\varphi \sin\theta$

Now, we have $\varphi = 45°$ from above calculations and

eq.(x) $\Rightarrow \theta = \sin^{-1}\left[\frac{3}{\sqrt{58}}\right] \Rightarrow 23.199°$

$\therefore \cos(\theta - \varphi) = \cos(45°)\sin(23.199°) + \cos(23.199°)\sin(45°)$

$\cos(\theta - \varphi) = 0.9285$

d. Find, to the nearest degree, the measure of the angle between wires.

Sol:-

From previous calculations

$$\cos(\theta - \varphi) = 0.9285$$

$$\theta - \varphi = \cos^{-1}(0.9285)$$

Let $\theta - \varphi = \alpha \rightarrow$ angle b/w wires

$$\therefore \quad \alpha = 21.7978^\circ$$

OR $\alpha \approx 22^\circ$