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Where Minds Pollinate

Class 11
Mathematics

Chapter 7
Exercise 7.4

Notes

National Book Foundation

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$$\left(\frac{2.01}{2}\right)^{1/3} = \frac{2 + 2(2.01)}{2.01 + 4} = \frac{2 + 4.02}{6.01}$$

$$= \frac{6.02}{6.01} = 1.0016 \text{ or } \frac{602}{601} \text{ Ans}$$

Exercise 7.4

Topic: Cyclicity of Number

Cyclicity of a number is a technique of Prediction of the unit digit of any number having any exponent (where exponent is quite larger).

Cyclicity of a number is a number where the exponent repeats the unit number same as the number which we are solving the exponent.

Cyclicity 2

$$\left. \begin{array}{l} 2^1 = 2 \\ 2^2 = 4 \\ 2^3 = 8 \\ 2^4 = 16 \\ 2^5 = 32 \end{array} \right\} 1$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256$$

cyclicity of 2 is $\boxed{4}$

$$2^9 = 512$$

$$2^{10} = 1024$$

$$2^{11} = 2048$$

$$2^{12} = 4096$$

$$3^1 = 3$$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 243$$

$$3^6 = 729$$

$$3^7 = 2187$$

$$3^8 = 6561$$

cyclicity of 3 is $\boxed{4}$

$$3^9 = 19683$$

$$3^{10} = 59049$$

$$3^{11} = 177147$$

$$3^{12} = 531441$$

Cyclicity of 1: 1

$$1^1 = 1$$

$$1^2 = 1$$

$$1^3 = 1$$

$$1^4 = 1$$

Cyclicity of 4: 2

$$4^1 = 4$$

$$4^2 = 16$$

$$4^3 = 64$$

$$4^4 = 256$$

unit digit same
as 1

Cyclicity of 5: 1

$$5^1 = 5$$

$$5^2 = 25$$

$$5^3 = 125$$

$$5^4 = 625$$

Cyclicity of 8: 4

$$8^1 = 8$$

$$8^2 = 64$$

$$8^3 = 512$$

$$8^4 = 4096$$

$$8^5 = 32768$$

Cyclicity of 9: 2

$$9^1 = 9$$

$$9^2 = 81$$

$$9^3 = 729$$

$$9^4 = 6561$$

Cyclicity of 6: 4

$$6^1 = 6$$

$$6^2 = 36$$

$$6^3 = 216$$

$$6^4 = 1296$$

Cyclicity of 7: 4

$$7^1 = 7$$

$$7^2 = 49$$

$$7^3 = 343$$

$$7^4 = 2401$$

$$7^5 = 16807$$

Digit	0	1	2	3	4
Cyclicity	1	1	4	4	2
	5	6	7	8	9

Cyclicity of Table of Digits 0-9.

Digit	0	1	2	3	4	5	6	7	8	9
Cyclicity	1	1	4	4	2	1	1	4	4	2

Example:- Find unit place digit of $(64)^{20}$
 Sol: we have to find the unit place digit of $(64)^{20}$

- i) unit digit of 64 is 4
- ii) Cyclicity of 4 is 2
- iii) Dividing 20 by 2

$$\begin{array}{r} 10 \\ 2 \overline{) 20} \\ \underline{20} \\ 0 \end{array}$$

Remainder is 0

it means unit place digit is 6, of $(64)^{20}$

As remainder is '0' the unit digit will be the last digit of where cycle end

Procedure of finding the unit place digit:

To find the unit place digit of $(ab)^n$, n is large.

Step 1

First allocate the unit digit of given number $\therefore b$

Step 2: Allocate the cyclicity of unit digit $\therefore$ cyclicity of b is p

Step 3: Divide the index "n" by "p" and find the remainder. If remainder is "0" then consider the unit digit where cycle ends.

Example: Find unit digit of $(8)^{51}$

- 1) Unit place digit of 8 is 8
- 2) Cyclicity of 8 is 4
- 3) Dividing 51 by 4

$$\begin{array}{r} 12 \\ 4 \overline{) 51} \\ \underline{4} \\ 11 \end{array}$$

$$\begin{array}{r} 11 \\ 8 \\ 3 \end{array}$$

$\rightarrow (8)^3 \rightarrow 512 \rightarrow$ unit digit of $(8)^{51}$

As Remainder is 3, so unit place

Example Find digit of $(17)^{109}$

- i) unit place digit of 17 is 7
- ii) cyclicity of 7 is 4
- iii) dividing 109 by 4

$$\begin{array}{r} 27 \\ 4 \overline{) 109} \\ \underline{8} \\ 29 \\ \underline{28} \\ 1 \end{array}$$

As remainder is 1
so the unit digit
of $(17)^{109}$ is

7

Qno 1 (i) $(27)^{304}$

(i) Unit digit of $27 \equiv 7$

(ii) Cyclicity of $7 \equiv 4$

(iii) Divide 304 by 4

$$\begin{array}{r} 76 \\ 4 \overline{) 304} \\ \underline{28} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Remainder

As Remainder is "0" which shows the cycle is end for '7' so $7^4 =$

$2401 \Rightarrow$ which has unit digit (1)

Hence $(27)^{304}$ has unit digit $\boxed{1}$

(ii) 108^{33}

(i) Unit place digit of 84 is 4

(ii) Cyclicity of 4 is 2

(iii) divide 203 by 2

$$\begin{array}{r} 101 \\ 2 \overline{) 203} \\ \underline{20} \\ 3 \\ \underline{2} \\ 1 \end{array}$$

As Remainder is $\boxed{1}$

so unit digit is $\boxed{4}$

As period of 8 is 4 at unit place.

$$= (100 + 8)^{33}$$

$$= (100 + 8)^{8 \times 4 + 1} = \text{unit place digit is } 8^1 = 8$$

iv) $(503)^{43}$

① Unit place digit is **3** of 503

② Cyclicity of **3** is **4**

③ Divide 43 by 4

$$\begin{array}{r} 10 \\ 4 \overline{) 43} \\ \underline{40} \\ 3 \end{array}$$

As Remainder is 3

so $(3)^3 = 27$ so

unit digit is 7 of $(503)^{43}$

Concept Remainder

$$\frac{10}{2} = 2 \overline{) 10}$$

$$\begin{array}{r} 10 \\ 2 \overline{) 10} \\ \underline{10} \\ 0 \end{array} \text{--- (Remainder)}$$

$$\frac{11}{2} = 2 \overline{) 11}$$

$$\begin{array}{r} 11 \\ 2 \overline{) 11} \\ \underline{10} \\ 1 \end{array}$$

when a number is divisible by 2 we have remainders 0, 1

When $\frac{19}{8} = 2 \overline{) 19}$

$$\begin{array}{r} 4 \\ 8 \overline{) 19} \\ \underline{20} \end{array}$$

$$\begin{array}{r} 3 \\ 8 \overline{) 19} \\ \underline{16} \\ 3 \end{array}$$

Two possibilities -1

! -ve remainder + divisor = +ve Remainder

Tricks to Find Remainder

+ve Remainder

$$\frac{19}{8} = 4$$

-ve remainder +

divisor = +ve remainder

-ve Remainder

$$\frac{19}{8} = -1$$

Addition and Product Rule of Remainder.

$$\bullet \frac{43+38}{2} \Rightarrow R(43)+R(38) = 1+0 = \boxed{1}$$

$$\bullet \frac{89+120+52+79}{11} \Rightarrow 1+(-1)+(-3)+2 = -1$$

-ve Remainder + divisor = +ve remainder
 $(-1) + 11 = 10$ ✓

Product rule:

$$\bullet \frac{37 \times 43}{7} = 2 \times 1 = 2 \quad \checkmark$$

$$\bullet \frac{(51)^{25}}{13} = \left(\frac{52-1}{13}\right)^{25} = (-1)^{25} = -1$$

$-1 + 13 = \boxed{12}$ ✓

$$\bullet \frac{(49)^{78}}{10} = \left(\frac{50-1}{10}\right)^{78} = (-1)^{78} = \boxed{1}$$

① no 2:

② $(94)^{205}$ is divided by 31

$$(94)^{205} = (93+1)^{205} = \binom{205}{0}(93)^{205}(1) + \binom{205}{1}(93)^{204}(1) + \dots + \binom{205}{205}(93)^0(1)^{205}$$

$$\Rightarrow \frac{(94)^{205}}{31} = \frac{\binom{205}{0}(93)^{205}}{31} + \frac{\binom{205}{1}(93)^{204}}{31} + \dots + \frac{(1)^{205}}{31}$$

As on R.H.S Every Term is divisible by 31 except last term which is $(1)^{205} = 1$ so remainder is 1

OR by Product.

$$\text{Alternative} = \frac{(94)^{204}}{31} = \frac{94 \times 94 \times 94 \times \dots \times 94}{31} \quad (204 \text{ times})$$

$$= (1)(1)(1) \dots (1) \quad (204 \text{ times})$$

= 1 Answer

(8)²⁰⁵ is ÷ by 48 (b)

$$\frac{(8)^{205}}{48} = \frac{(2^3)^{205}}{48} = \frac{2^{615}}{48} = \frac{2^4 \cdot 2^{611}}{48} = \frac{16 \cdot 2^{611}}{48}$$

$$= \frac{2^{611}}{3} \quad \text{--- (i)}$$

$$\frac{2 \cdot 2^{610}}{3} = \frac{2(2^2)^{305}}{3} = \frac{2(4)^{305}}{3} = \frac{2(3+1)^{305}}{3}$$

$$= \frac{2}{3} \left[\binom{305}{0} (3)^{305} (1) + \binom{305}{1} (3)^{304} (1) + \dots + \binom{305}{305} (3)^0 (1)^{305} \right]$$

3

= 2 (Every term is divisible by 3 except last)

$$(1)^{305} = 1$$

2(1) = 2 Remainder 2

By product

$$\frac{2^{611}}{3} = \frac{2 \times 2 \times \dots \times 2}{3} \quad 611 \text{ times}$$

$$= (-1)(-1)(-1) \dots 611 \text{ times}$$

$$= (-1)^{611} = -1 + 3 = \boxed{2} \text{ Ans}$$

Example :-

$$\frac{(80)^{7218}}{9}$$

$$= \frac{(80)(80)(80) \dots \dots \dots (80) \text{ 7218 times}}{9}$$

$$= \frac{(-1)(-1)(-1) \dots \dots \dots (-1) \text{ 7218 times}}{9}$$

$$= (-1)^{7218} = \boxed{1} \text{ Answer}$$

$$\frac{2^{510}}{31} \text{ is } \frac{K}{31}$$

$$\text{for } \frac{(2)^{510}}{31} = \frac{(2^5)^{102}}{31} = \frac{(32)^{102}}{31}$$

$$= \frac{(32)(32)(32) \dots \dots \dots (32) \text{ 102 times}}{31}$$

$$= \frac{2^{510}}{31} = \frac{(1)(1)(1) \dots \dots \dots (1) \text{ 102 times}}{31}$$

$$= (1)^{102} = \boxed{1} \checkmark$$

$$\boxed{K=1} \checkmark$$

By binomial

$$\frac{(32)^{102}}{31} = \frac{(31+1)^{102}}{31} = \binom{102}{0}(31)^{102}(1)^0 + \dots \dots \dots$$

By Binomial $\frac{2510}{31} = \frac{(25)^{102}}{31} = \frac{(32)^{102}}{31}$

$$\frac{(32)^{102}}{31} = \frac{(31+1)^{102}}{31}$$

$$= \frac{\binom{102}{0}(31)^{102}(1)^0 + \dots + \binom{102}{102}(31)^0(1)^{102}}{31}$$

$\frac{(32)^{102}}{31}$ Every term is divisible by 31 except last $\frac{1}{31} = \frac{K}{31} \Rightarrow K=1$

04
(a)

158

Sol

$$(10+5)^8 = \binom{8}{0}(10)^8(5)^0 + \binom{8}{1}(10)^7(5)^1 + \binom{8}{2}(10)^6(5)^2 + \binom{8}{3}(10)^5(5)^3 + \binom{8}{4}(10)^4(5)^4 + \binom{8}{5}(10)^3(5)^5 + \binom{8}{6}(10)^2(5)^6 + \binom{8}{7}(10)^1(5)^7 + \binom{8}{8}(10)^0(5)^8$$

$$= 10^2 \left[\binom{8}{0}(10)^6 + \binom{8}{1}10^5(5) + \dots + \binom{8}{6}(5)^6 \right] + \binom{8}{7}(10)(5)^7 + \binom{8}{8}(5)^8$$

$$= 100(\underline{m}) + \binom{8}{7}10(5)^7 + \binom{8}{8}5^8$$

$$= 100m + 8(10)(78125) + 390625$$

* (last two digits are always zero) + 6640685

= So Resultant have last two digits as

Unit digit of $(15)^8$ is 5

last two digits of $(15)^8$ is 25

(b)

37⁷

Sol: $(30+7)^7$

$$(37)^7 = (30+7)^7 = \binom{7}{0}(30)^7(7)^0 + \binom{7}{1}(30)^6(7)^1$$

$$+ \binom{7}{2}(30)^5(7)^2 + \binom{7}{3}(30)^4(7)^3 + \binom{7}{4}(30)^3(7)^4$$

$$+ \binom{7}{5}(30)^2(7)^5 + \binom{7}{6}(30)^1(7)^6 + \binom{7}{7}(30)^0(7)^7$$

= Taking $(30)^2$ common from First 6 terms

$$= (30)^2 \left[\text{---} \right] + \binom{7}{6}(30)^1(7)^6 + (7)^7$$

$$= 900 \left(\text{---} \right) + 25529833$$

= last two digits 33 ✓

Q4 (c)

for

$$(29)^{10} = (20+9)^{10} = \binom{10}{0}(20)^{10}(9)^0 + \binom{10}{1}(20)^9(9)^1 + \binom{10}{2}(20)^8(9)^2 + \binom{10}{3}(20)^7(9)^3 + \binom{10}{4}(20)^6(9)^4 + \binom{10}{5}(20)^5(9)^5 + \binom{10}{6}(20)^4(9)^6 + \binom{10}{7}(20)^3(9)^7 + \binom{10}{8}(20)^2(9)^8 + \binom{10}{9}(20)^1(9)^9 + \binom{10}{10}(20)^0(9)^{10}$$

$$= (20)^2 \left[\frac{1}{(20)^8} \right] + \binom{10}{9}(20)^1(9)^9 + (9)^{10}$$

$$\cancel{10(20)^9} + 809$$

(Having last digits 00) + 80970882201

= 01 Answer

Q5 or (102)⁵⁰

$$(102)^{50} = (100+2)^{50} = \binom{50}{0}(100)^{50}(2)^0 +$$

$$\binom{50}{1}(100)^{49}(2)^1 + \dots + \binom{50}{50}(100)^0(2)^{50}$$

$$(102)^{50} = (100)^{50} + 50 \cdot (100)^{49} \cdot 2 + \dots + 2^{50} \quad \text{--- (i)}$$

$$(98)^{50} = (100-2)^{50} = \binom{50}{0} (100)^{50} (-2)^0 + \binom{50}{1} (100)^{49} (-2)^1 + \dots + (-2)^{50} \quad \text{--- (ii)}$$

Subtracting (i) and (ii)

$$(102)^{50} - (98)^{50} = 50 \cdot (100)^{49} \cdot 2 + 50(100)^{49}(2) + \dots + 100(100)^{49} + 100(100)^{49}$$

$$(102)^{50} - (98)^{50} = 2(100)^{50} + \dots > (100)^{50}$$

$$(102)^5 - (98)^5 > (100)^5$$

$$(102)^{50} > (100)^{50} + (98)^{50} \quad \text{Proved}$$

$$47^{30} + 50^{30} > 53^{30}$$

$$(53)^{30} = (50+3)^{30} = \binom{30}{0} (50)^{30} (3)^0 + \binom{30}{1} (50)^{29} (3)^1 + \binom{30}{2} (50)^{28} (3)^2 + \dots + \binom{30}{30} (50)^0 (3)^{30}$$

$$(53)^{30} = (50)^3 + (30)(50)^{29}(3) + (435)(50)^{28}(3)^2 + \dots + (3)^{30} \quad \text{--- (i)}$$

$$(47)^{30} = (50-3)^{30} = \binom{30}{0} (50)^{30} (-3)^0 + \binom{30}{1} (50)^{29} (-3)^1 + \binom{30}{2} (50)^{28} (-3)^2 + \dots + \binom{30}{30} (50)^0 (-3)^{30}$$

$$(47)^{30} = (50)^{30} - (30)(50)^{29}(3) + (435)(50)^{28}(3)^2 + \dots + (3)^{30} \quad \text{--- (ii)}$$

$$(53)^{30} - (47)^{30} = (80)^3 + (30)(50)^{29}(3) + (435) \\ + (435)(50)^{28}(3)^2 + \dots + (3)30 -$$

$$(50)^{30} + (30)(50)^{29}(3) - 435(50)^{28}(3)^2 + \dots - (30)30 \\ \geq 90 = (90)(50)^{29} + (90)(50)^{29} + \dots -$$

$$(53)^{30} - (47)^{30} = (180)(50)^{29} + \dots$$

$$(53)^{30} - (47)^{30} = (130 + 50)(50)^{29} + \dots$$

$$(53)^{30} - (47)^{30} = 50 \cdot 50^{29} + (50)(130)^{29} + \dots$$

$$(53)^{30} - (47)^{30} = (50)^{30} + (50)^{29}(130) + \dots$$

$$(53)^{30} - (47)^{30} = (50)^{30} + (50)^{29}(130) + \dots > (50)^{30}$$

$$(53)^{30} > (50)^{30} + (47)^{30} \quad \checkmark$$

⑥ $12^{15} + 8^{15}$ is divisible by 10.

for

$$(12)^{15} = (10 + 2)^{15} = \binom{15}{0} (10)^{15} (2)^0 + \binom{15}{1} (10)^{14} (2)^1 +$$

$$\binom{15}{2} (10)^{13} (2)^2 + \dots + \binom{15}{15} (10)^0 (2)^{15}$$

$$(12)^{15} = 10^{15} + 15(10)^{14}(2) + 105(10)^{13}(2)^2 + \dots$$

$$\dots + (2)^{15} \quad \text{--- ⑦}$$

$$(8)^{15} = (10 - 2)^{15} = \binom{15}{0} (10)^{15} (-2)^0 + \binom{15}{1} (10)^{14} (-2)^1 +$$

$$+ \binom{15}{2} (10)^{13} (-2)^2 + \dots + \binom{15}{15} (10)^0 (-2)^{15}$$

$$(8)^{15} = (10)^{15} - 15(10)^{14}(2) + 105(10)^{13}(2)^2 + \dots - (2)^{15}$$

Adding (i) & (ii)

$$(12)^{15} + (8)^{15} = 2(10)^{15} + 2(105)(10)^{13}(2)^2 + \dots + 2\binom{15}{14}(10)^1(2)^{14}$$

$$= 2(\dots) \text{ as given R.H.S is multiple}$$

of 2 so divisible by 2

Show $22^{25} + 18^{25}$ is divisible by 20.

$$(22)^{25} = (20+2)^{25} = \binom{25}{0}(20)^{25}(2)^0 + \binom{25}{1}(20)^{24}(2)^1 + \dots + \binom{25}{24}(20)^1(2)^{24} + \binom{25}{25}(20)^0(2)^{25} \quad \text{--- (i)}$$

$$(18)^{25} = (20-2)^{25} = \binom{25}{0}(20)^{25}(-2)^0 + \binom{25}{1}(20)^{24}(-2)^1 + \dots + \binom{25}{24}(20)^1(-2)^{24} + \binom{25}{25}(20)^0(-2)^{25} \quad \text{--- (ii)}$$

Adding (i) and (ii)

$$(22)^{25} + (18)^{25} = 2(20)^{25} + 2\binom{25}{2}(20)^{23}(2)^2 + \dots + 2\binom{25}{24}(20)^1(2)^{24}$$

$= 20[\dots] = \text{AS R.H.S is multiple of 20 so divisible by 20}$

Q no 8

5^{103} is divisible by 13

$$\frac{5^{103}}{13} = \frac{5 \cdot 5^{102}}{13} = \frac{5(5^2)^{51}}{13} = \frac{5(25)^{51}}{13}$$

$$\frac{5(26-1)^{51}}{13} = \frac{5}{13} \left[\binom{51}{0} (26)^{51} (-1)^0 + \dots + \binom{51}{51} (26)^0 (-1)^{51} \right]$$

$$= \frac{5}{13} \left[\binom{51}{0} (26)^{51} + \dots + \binom{51}{51} (26)^0 (-1)^{51} \right]$$

As 13 divides the entire terms except last term which is $(-1)^{51}$ so Remainder is $5(-1)^{51} = -5$

$= -ve \text{ Remainder} + \text{divisor} = +ve \text{ Remainder}$

$$-5 + 13 = \boxed{8} \text{ Ans}$$

Q 9

$(17)^{1717}$ is $\div$ by 9

$$\frac{(17)^{1717}}{9} = \frac{(18-1)^{1717}}{9}$$

$$\frac{(18-1)^{1717}}{9} = \frac{1}{9} \left[\binom{1717}{0} (18)^{1717} (-1)^0 + \binom{1717}{1} (18)^{1716} (-1)^1 + \dots + \binom{1717}{1717} (18)^0 (-1)^{1717} \right]$$

As we see entire values are divisible by 9 except last one

$$= (-1)^{1717} \Rightarrow \frac{1}{9} (-1)^{1717} = (-1)^{1717} = -1$$

$$= -1 + 9 = \boxed{8} \text{ } \text{By using Binomial}$$

OR

By rule

$$\frac{(-1)^{1717}}{9} = \frac{(-1)(-1)(-1) \dots (-1)^{1717 \text{ times}}}{9} \Rightarrow (-1)(-1)(-1) \dots (-1)^{1717 \text{ times}}$$

$$(-1)^{1717} = (-1) + 9 = \boxed{8} \text{ } \text{Ans}$$

$$(1.1)^{10,000} \text{ } \text{Q10} \text{ } \text{or } 1000$$

$$(1.1)^{10,000} = (1 + 0.1)^{10,000} = \binom{10,000}{0} (1)^{10,000} (0.1)^0 + \binom{10,000}{1} (1)^{9999} (0.1)^1 + \dots + (0.1)^{10,000}$$

$$(1.1)^{10,000} = 1 + (10,000)(0.1) + \dots + (0.1)^{10,000}$$

$$(1.1)^{10,000} = 1 + 1000 + \dots + (0.1)^{1000} > 1000$$

2nd term is 1000 itself so

$$(1.1)^{10,000} > 1000 \text{ } \text{Ans}$$

Q no 11. $9^{n+1} - 8n - 9$ is divisible by 64.

or second method on end

$$\begin{aligned}
 9^{n+1} - 8n - 9 &= (8+1)^{n+1} - 8n - 9 \\
 &= \binom{n+1}{0} (8)^{n+1} (1)^0 + \binom{n+1}{1} (8)^n (1)^1 + \dots + \binom{n+1}{n-1} 8^2 \\
 &\quad + \binom{n+1}{n} 8^1 (1)^n + \binom{n+1}{n+1} 8^0 (1)^{n+1} - 8n - 9 \\
 &= \left(8^{n+1} + \binom{n+1}{1} 8^n + \dots + \binom{n+1}{n-1} 8^2 + 1 \right) - 8n - 9 \\
 &= 8^{n+1} + \binom{n+1}{1} 8^n + 8n + 8 + 1 - 8n - 9 \\
 &= 8^{n+1} + \binom{n+1}{1} 8^n + 8n + 8 + 1 - 8n - 9
 \end{aligned}$$

one of the term is divisible by 64

Q 12.

$(a-b)$ is factor of $a^n - b^n$.

$$(a^n - b^n) = (a - b + b)^n - b^n = ((a-b) + b)^n - b^n$$

$$\begin{aligned}
 ((a-b) + b)^n - b^n &= \left[\binom{n}{0} (a-b)^n (b)^0 + \binom{n}{1} (a-b)^{n-1} (b)^1 \right. \\
 &\quad + \binom{n}{2} (a-b)^{n-2} (b)^2 + \dots + \binom{n}{n-1} (a-b) (b)^{n-1} \\
 &\quad \left. + \binom{n}{n} (a-b)^0 (b)^n \right] - b^n
 \end{aligned}$$

$$(a-b)^n + b^n = (a-b)^n + \binom{n}{1}(a-b)^{n-1}b + \binom{n}{2}(a-b)^{n-2}b^2 + \dots + \binom{n}{n-1}(a-b)b^{n-1} + b^n - b^n$$

$a^n - b^n$ clearly every term has $(a-b)$ as a factor and is divisible by $a-b$

Q13 $6^{n+3} - 18n - 6$ is divisible by 6

$$= 6^{n+3} - 6 \cdot 3n - 6$$

$$= 6^{n+3} \cdot 6 - 6 \cdot 3n - 6$$

$= 6(6^{n+2} - 3n - 1)$ clearly 6 is the factor so terms are divisible by 6