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Where Minds Pollinate

Class 11
Mathematics

Chapter 7
Exercise 7.3
Notes

National Book Foundation

Exercise # 7.3

1. Expand the followings upto four terms and also find values of x for which series is convergent.

i) $(1-\sqrt{x})^{-3}$

Sol:-

$$=(1-\sqrt{x})^{-3}$$

The binomial series is:-

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\dots$$

for $x=-\sqrt{x}$, $n=-3$

$$\therefore (1-\sqrt{3})^{-3} = 1+(-3)(-\sqrt{x})+\frac{(-3)(-3-1)}{2!}(-\sqrt{x})^2+\frac{(-3)(-3-1)(-3-2)}{3!}(-\sqrt{x})^3$$

$$(1-\sqrt{3})^{-3} = 1+3\sqrt{x}+6x+10x^{3/2}$$

Binomial series converges when $|x|<1$

So, $|-\sqrt{x}|<1 \Rightarrow \sqrt{x}<1 \Rightarrow 0<x<1$

ii) $\left[3+\frac{2}{x}\right]^{-\frac{1}{3}}$

Sol:-

$$=\left[3+\frac{2}{x}\right]^{-\frac{1}{3}}$$

$$=\left[(3)\left(1+\frac{2}{3x}\right)\right]^{-\frac{1}{3}}$$

$$= (3)^{-1/3}\left[1+\frac{2}{3x}\right]^{-1/3}$$

Apply Binomial series; i.e.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

for $n = -1/3$, $x = \frac{2}{3x}$

$$\Rightarrow \left(1+\frac{2}{3x}\right)^{-\frac{1}{3}} = 1 + \left(\frac{-1}{3}\right)\left(\frac{2}{3x}\right) + \frac{(-1/3)(-1/3-1)}{2!}\left(\frac{2}{3x}\right)^2 + \frac{(-1/3)(-1/3-1)(-1/3-2)}{3!}\left(\frac{2}{3x}\right)^3$$

$$\left(1+\frac{2}{3x}\right)^{-\frac{1}{3}} = 1 - \frac{2}{9x} + \frac{8}{81x^2} - \frac{112}{2187x^3}$$

Multiply b/s by $(3)^{-1/3}$

$$(3)^{-1/3}\left[1+\frac{2}{3x}\right]^{-\frac{1}{3}} = (1)(3)^{-\frac{1}{3}} - \frac{2(3)^{-1/3}}{9x} + \frac{8(3)^{-1/3}}{81x^2} - \frac{112(3)^{-1/3}}{2187x^3}$$

$$\left[3+\frac{2}{x}\right]^{-\frac{1}{3}} = (3)^{-1/3} - \frac{2}{(3)^{2+1/3}x} + \frac{8}{(3)^{4+1/3}x^2} - \frac{112}{(3)^{7+1/3}x^3}$$

$$\left[3+\frac{2}{x}\right]^{-\frac{1}{3}} = (3)^{-1/3} - \frac{2}{(3)^{7/3}x} + \frac{8}{(3)^{13/3}x^2} - \frac{112}{(3)^{22/3}x^3}$$

Binomial series converges when $|x| < 1$

So, $\left|\frac{2}{3x}\right| < 1 \Rightarrow -\frac{3}{2} < x < \frac{3}{2}$

iii) $\left[\frac{5}{2}-\frac{3}{x^2}\right]^{\frac{1}{2}}$

Sol:-

$$=\left[\frac{5}{2}-\frac{3}{x^2}\right]^{\frac{1}{2}}$$

$$=\left[\left(\frac{5}{2}\right)\left(1-\frac{3(2)}{5x^2}\right)\right]^{\frac{1}{2}}$$

$$=\left[\frac{5}{2}\right]^{\frac{1}{2}}\left[1-\frac{6}{5x^2}\right]^{\frac{1}{2}}$$

Apply binomial series.

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3$$

for $x=\frac{-6}{5x^2}$, $n=\frac{1}{2}$

$$\therefore \left[1-\frac{6}{5x^2}\right]^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)\left(\frac{-6}{5x^2}\right)+\frac{(1/2)(1/2-1)}{2!}\left(\frac{-6}{5x^2}\right)^2+\frac{(1/2)(1/2-1)(1/2-2)}{3!}\left(\frac{-6}{5x^2}\right)$$

$$\left[1-\frac{6}{5x^2}\right]^{\frac{1}{2}} = 1-\frac{3}{5x^2}-\frac{9}{50x^4}-\frac{27}{250x^6}$$

Multiply b/s $(5/2)^{1/2}$

$$\left[\frac{5}{2}\right]^{\frac{1}{2}}\left[1-\frac{6}{5x^2}\right]^{\frac{1}{2}} = 1\left(\frac{5}{2}\right)^{1/2}-\frac{3(5/2)^{1/2}}{5x^2}-\frac{9(5/2)^{1/2}}{50x^4}-\frac{27(5/2)^{1/2}}{250x^6}$$

$$\left[\frac{5}{2}-\frac{3}{x^2}\right]^{\frac{1}{2}} = \left(\frac{5}{2}\right)^{\frac{1}{2}}-\frac{3}{\sqrt{2}\sqrt{5}\,x^2}-\frac{3^2}{2^{3/2}5^{3/2}}-\frac{3^3}{2^{3/2}5^{5/2}x^6}$$

for converging series i.e

$\left| \frac{-6}{5x^2} \right| < 1 \Rightarrow \frac{6}{5x^2} < 1$

$\frac{6}{5} < |x^2|$

(v) $\frac{3+x}{3-x}$

Sol:-

$= \frac{3+x}{3-x} \Rightarrow \frac{3+x}{(3-x)^1}$

$= (3+x)(3-x)^{-1} \Rightarrow 3\left(1+\frac{x}{3}\right)(3)^{-1}\left(1-\frac{x}{3}\right)^{-1}$

$= 3^{1-1}\left(1+\frac{x}{3}\right)\left(1-\frac{x}{3}\right)^{-1} \Rightarrow \left(1+\frac{x}{3}\right)\left(1-\frac{x}{3}\right)^{-1}$

Apply binomial series on $\left(1-\frac{x}{3}\right)^{-1}$

$\left(1-\frac{x}{3}\right)^{-1} = 1 + (-1)\left(-\frac{x}{3}\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{-x}{3}\right)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}\left(\frac{-x}{3}\right)^3$

$\left(1-\frac{x}{3}\right)^{-1} = 1 + \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3}$

Multiply b/s by $\left(1+\frac{x}{3}\right)$

$\frac{3+x}{3-x} = \left[1+\frac{x}{3}\right] + \frac{x}{3}\left[1+\frac{x}{3}\right] + \frac{x^2}{3^2}\left[1+\frac{x}{3}\right] + \frac{x^3}{3^3}\left[1+\frac{x}{3}\right]$

$\frac{3+x}{3-x} = 1 + \frac{x}{3} + \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^3}{3^3} + \frac{x^4}{3^4}$

$\frac{3+x}{3-x} = 1 + \frac{2x}{3} + \frac{2x^2}{3^2} + \frac{2x^3}{3^3} + \frac{x^4}{3^4}$

for converging series

$$\left|-\frac{x}{3}\right| < 1 \Rightarrow -3 < x < 3$$

iv) $\dfrac{1-2x}{\sqrt{3+\frac{x}{2}}}$

SOL:-

$$= \frac{1-2x}{\sqrt{3+\frac{x}{2}}} \Rightarrow \frac{1-2x}{\left[3+\frac{x}{2}\right]^{\frac{1}{2}}}$$

$$= (1-2x)\left[3+\frac{x}{2}\right]^{-\frac{1}{2}} \Rightarrow (1-2x)(3)^{-\frac{1}{2}}\left[1+\frac{x}{6}\right]^{-\frac{1}{2}}$$

Apply binomial series on $\left[1+\frac{x}{6}\right]^{-\frac{1}{2}}$

$$\left[1+\frac{x}{6}\right]^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\right)\left(\frac{x}{6}\right)+\frac{(-1/2)(-1/2-1)}{2!}\left(\frac{x}{6}\right)^2+\frac{(-1/2)(-1/2-1)(-1/2-2)}{3!}\left(\frac{x}{6}\right)^3$$

$$\left[1+\frac{x}{6}\right]^{-\frac{1}{2}} = 1-\frac{x}{12}+\frac{x^2}{96}-\frac{5x^3}{3456}$$

Multiply b/s $\dfrac{1-2x}{\sqrt{3}}$

$$\frac{1-2x}{\sqrt{3+\frac{x}{2}}} = \frac{1-2x}{\sqrt{3}}-\frac{(1-2x)x}{\sqrt{3}.2^2.3}+\frac{(1-2x)x^2}{\sqrt{3}.2^5.3}-\frac{5(1-2x)x^3}{\sqrt{3}.2^7.3^3}$$

$$\frac{1-2x}{\sqrt{3+\frac{x}{2}}} = \frac{1-2x}{\sqrt{3}}-\frac{(1-2x)x}{3^{3/2}.2^2}+\frac{(1-2x)x^2}{3^{3/2}.2^5}-\frac{5(1-2x)x^3}{3^{7/2}.2^7}$$

for converging series

$\left|\frac{x}{6}\right| < 1 \Rightarrow -6 < x < 6$

vi) $\frac{\sqrt{x+1}}{\sqrt{1-x}}$

Sol:-

$$= \frac{\sqrt{x+1}}{\sqrt{1-x}} \Rightarrow \left[\frac{1+x}{1-x}\right]^{\frac{1}{2}}$$

$$= (1+x)^{1/2}(1-x)^{-1/2}$$

Consider $(1+x)^{1/2}$

$$(1+x)^{1/2} = 1 + (1/2)(x) + \frac{(1/2)(1/2-1)}{2!}(x)^2 + \frac{(1/2)(1/2-1)(1/2-2)}{3!}(x)^3$$

$$(1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} \quad -(i)$$

Now consider, $(1-x)^{-1/2}$

$$(1-x)^{-1/2} = 1 + (-1/2)(-x) + \frac{(-1/2)(-1/2-1)}{2!}(-x)^2 + \frac{(-1/2)(-1/2-1)(-1/2-2)}{3!}(-x)^3$$

$$(1-x)^{-1/2} = 1 + \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{16} \quad -(ii)$$

Multiply (i) and (ii)

$$(1+x)^{1/2}(1-x)^{-1/2} = \left[1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}\right]\left[1 + \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{16}\right]$$

$$\frac{\sqrt{x+1}}{\sqrt{1-x}} = 1 + \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{x}{2} + \frac{x^2}{4} + \frac{3x^3}{16} + \frac{5x^4}{32}$$

$$- \frac{x^2}{8} - \cancel{\frac{x^3}{16}} - \frac{3x^4}{64} - \frac{5x^5}{128} + \cancel{\frac{x^3}{16}} + \frac{x^4}{32} + \frac{3x^5}{128} + \frac{5x^6}{256}$$

$$\frac{\sqrt{x+1}}{\sqrt{1-x}} = 1 + x + \frac{x^2}{2} - \frac{x^3}{8} + \frac{9x^4}{64} - \frac{x^5}{64} + \frac{5x^6}{256}$$

4. If x is so small that its square and higher powers may be neglected then show

i) $$\frac{\sqrt{2+x}\,(1-x)^{3/2}}{3+x} \approx \frac{\sqrt{2}}{3}\left[1 - \frac{19}{12}x\right]$$

Proof:

$$\text{L.H.S} \Rightarrow \frac{\sqrt{2+x}\,(1-x)^{3/2}}{3+x}$$

$$= (2+x)^{1/2}(1-x)^{3/2}(3+x)^{-1}$$

$$= \left[(2)(1+x/2)\right]^{1/2}(1-x)^{3/2}\left[(3)(1+x/3)\right]^{-1}$$

$$= (2)^{1/2}(1+x/2)^{1/2}(1-x)^{3/2}(3)^{-1}(1+x/3)^{-1}$$

$$= \frac{\sqrt{2}}{3}(1+x/2)^{1/2}(1-x)^{3/2}(1+x/3)^{-1}$$

Now apply binomial series for 1^{st} two terms

$$\approx \frac{\sqrt{2}}{3}\left[\left(1+(1/2)(x/2)\right)\left(1+(3/2)(-x)\right)+\left(1+(x/3)(-1)\right)\right]$$

$$\approx \frac{\sqrt{2}}{3}\left[\left(1+\frac{x}{4}\right)\left(1-\frac{3x}{2}\right)\left(1-\frac{x}{3}\right)\right]$$

$$\approx \frac{\sqrt{2}}{3}\left[\left(1-\frac{3x}{2}+\frac{x}{4}-\frac{3x^2}{8}\right)\left(1-\frac{x}{3}\right)\right]$$

neglect $-\frac{3x^2}{8}$

$$\approx \frac{\sqrt{2}}{3}\left[\left(1-\frac{5x}{4}\right)\left(1-\frac{x}{3}\right)\right]$$

$$\approx \frac{\sqrt{2}}{3}\left[1-\frac{x}{3}-\frac{5x}{4}+\frac{5x^2}{12}\right]$$

neglect $5x^2/12$

$$\approx \frac{\sqrt{2}}{3}\left[1-\frac{19}{12}x\right]$$

$\approx$ R.H.S

ii) $$\frac{\left(1+\frac{2}{3}x\right)^{-5}+\sqrt{4+2x}}{(4+x)^{3/2}} \approx \frac{1}{8}\left[3-\frac{95}{24}x\right]$$

Proof:-

L.H.S $\Rightarrow$ $$\frac{\left(1+\frac{2}{3}x\right)^{-5}+\sqrt{4+2x}}{(4+x)^{3/2}}$$

$$= \frac{(1+2x/3)^{-5}+\left[(4)(1+x/2)\right]^{1/2}}{\left[(4)(1+x/4)\right]^{3/2}}$$

$$= \frac{(1+2x/3)^{-5}+(4)^{1/2}(1+x/2)^{1/2}}{(4)^{3/2}(1+x/4)^{3/2}}$$

$$= \left[(1+2x/3)^{-5}+2(1+x/2)^{1/2}\right]\left[(4)^{-3/2}(1+x/4)^{-3/2}\right]$$

Now apply Binomial series. for first two terms.

$$\approx \left\{\left[1+(-5)(2x/3)\right]+2\left[1+(1/2)(x/2)\right]\right\}\left\{(4)^{-3/2}\left[1+(-3/2)(x/4)\right]\right\}$$

$$\approx \left[1-\frac{10x}{3}+2+\frac{x}{2}\right]\left[(4)^{-3/2}\left(1-\frac{3x}{8}\right)\right]$$

$$\approx \left[3-\frac{17x}{6}\right]\left[(4)^{-3/2}\left(1-\frac{3x}{8}\right)\right]$$

$$\approx (4)^{-3/2}\left[3-\frac{9x}{8}-\frac{17x}{6}+\frac{17}{16}x^2\right]$$

neglect $\frac{17}{16}x^2$

$$\approx \left[\frac{1}{4^{3/2}}\right]\left[3-\frac{95}{24}x\right]$$

$$\approx \frac{1}{8}\left(3-\frac{95}{24}x\right)$$

$\approx$ R.H.S

iii) $$\frac{\sqrt{9-x}+\left(1+\frac{3}{4}x\right)^{-5}}{2+x} \approx 2-\frac{71}{24}x$$

Proof:-

L.H.S $\Rightarrow$ $$\frac{\sqrt{9-x}+(1+3x/4)^{-5}}{2+x}$$

$$= \frac{(9-x)^{1/2}+(1+3x/4)^{-5}}{2(1+x/2)}$$

$$= \frac{(9)^{1/2}(1 - x/9)^{1/2} + (1 + 3x/4)^{-5}}{2(1 + x/2)}$$

$$= \frac{3(1 - x/9)^{1/2} + (1 + 3x/4)^{-5}}{2(1 + x/2)}$$

$$= \frac{\left[3(1 - x/9)^{1/2} + (1 + 3x/4)^{-5}\right]\left[(1 + x/2)\right]^{-1}}{2}$$

Apply binomial series for first two terms

$$\approx \frac{\left\{3\left[1 + (1/2)(-x/9)\right] + \left[1 + (-5)(3x/4)\right]\right\}\left\{1 + (-1)(x/2)\right\}}{2}$$

$$\approx \frac{\left[3 + - x/6 + 1 - 15x/4\right]\left[1 - x/2\right]}{2}$$

$$\approx \frac{\left[4 - \frac{47}{12}x\right]\left[1 - x/2\right]}{2}$$

$$\approx \left[4 - 2x - \frac{47}{12}x + \frac{47x^2}{24}\right]\frac{1}{2}$$

$$\approx \left(4 - \frac{71}{12}x\right)\left(\frac{1}{2}\right) \qquad \text{neglect } \frac{47}{24}x^2$$

$$\approx 2 - \frac{71}{24}x$$

$\approx$ R.H.S.

5. If x is so small that its cube and higher powers can be neglected then show:-

$$\frac{(1+x)^{3/2}-(1+x^2)^3}{\sqrt{1-x}} \approx \frac{3x}{2}-\frac{15x^2}{8}$$

PROOF:-

$$L.H.S \Rightarrow \frac{(1+x)^{3/2}-(1+x^2)^3}{\sqrt{1-x}}$$

$$= \frac{(1+x)^{3/2}-(1+x^2)^3}{(1-x)^{+1/2}}$$

$$= \left[(1+x)^{3/2}-(1+x^2)^3\right]\left[(1-x)^{-1/2}\right]$$

Now apply binomial ~~series~~ upto x^2 term involving.

$$\approx \left\{\left[1+(3/2)(x)+\frac{(3/2)(3/2-1)}{2!}(x)^2\right]-\left[1+(3)(x^2)\right]\right\}$$

$$\left\{1+(-1/2)(-x)+\frac{(-1/2)(-1/2-1)}{2!}(-x)^2\right\}$$

$$\approx \left[1+\frac{3x}{2}+\frac{3x^2}{8}-(1+3x^2)\right]\left[1+\frac{x}{2}+\frac{3}{8}x^2\right]$$

$$\approx \left[\cancel{1}+\frac{3x}{2}+\frac{3x^2}{8}-\cancel{1}-3x^2\right]\left[1+\frac{x}{2}+\frac{3}{8}x^2\right]$$

$$\approx \left[\frac{3x}{2}-\frac{21x^2}{8}\right]\left[1+\frac{x}{2}+\frac{3}{8}x^2\right]$$

$$\approx \frac{3x}{2} + \frac{3x^2}{4} + \frac{9x^3}{16} - \frac{21x^2}{8} - \frac{21x^3}{16} - \frac{63x^4}{64}$$

~~neglect~~ $\frac{9x^3}{16}, -\frac{21x^3}{16}$ and $-\frac{63x^4}{64}$

$$\approx \frac{3x}{2} + \frac{3x^2}{4} - \frac{21x^2}{8}$$

$$\approx \frac{3x}{2} - \frac{15x^2}{8}$$

$\approx$ R.H.S

ii) $$\frac{(4+x)^{-2} + (1-2x)^{-5}}{(1+2x)^2} \approx \frac{17}{16} + \frac{183}{32}x + \frac{8419}{256}x^2$$

PROOF:-

L.H.S $\Rightarrow \dfrac{(4+x)^{-2} + (1-2x)^{-5}}{(1+2x)^2}$

$$= \left[(4)^{-2}(1+x/4)^{-2} + (1-2x)^{-5}\right]\left[(1+2x)^{-2}\right]$$

$$= \left[(1/16)(1+x/4)^{-2} + (1-2x)^{-5}\right]\left[(1+2x)^{-2}\right]$$

Apply Binomial series for first 3-terms

$$\approx \left[(1/16)\left[1+(-2)(x/4)+\frac{(-2)(-2-1)}{2!}(x/4)^2\right] + \left[\cancel{1+(-5)(-2x)}\right.\right.$$

$$\left.\left. + \frac{(-5)(-5-1)}{2!}(-2x)^2\right]\right]\left[1+(-2)(2x)+\frac{(-2)(-2-1)}{2!}\cancel{(2x)^2}\right.$$

$$\approx \left[\frac{1}{16} - \frac{x}{32} + \frac{3x^2}{256} + 1 + 10x + 160x^2\right]\left[1 - 4x + 12x^2\right]$$

$$\approx \left[\frac{17}{16} + \frac{319x}{32} + \frac{15363x^2}{256}\right]\left[1 - 4x + 12x^2\right]$$

$$\approx \frac{17}{16} - \frac{17}{4}x + \frac{51}{4}x^2 + \frac{319x}{32} - \frac{319}{8}x^2 + \frac{957}{8}x^3$$

$$+ \frac{15363x^2}{256} - \frac{15363}{64}x^3 + \frac{46089}{64}x^4$$

neglect term having x^3 and x^4

$$\therefore \approx \frac{17}{16} - \frac{17}{4}x + \frac{319}{32}x + \frac{51}{4}x^2 - \frac{319}{8}x^2 + \frac{15363}{256}x^2$$

$$\approx \frac{17}{16} + \frac{183}{32}x + \frac{8419}{256}x^2$$

$\approx$ R.H.S.

3. Find the term involving x^{14} in the product of $(1+x^2)(2+\sqrt{3}\,x^3)^{-\frac{1}{2}}$.

Sol:-

$$= (1+x^2)(2+\sqrt{3}x^3)^{-1/2}$$

$$= (1+x^2)(2)^{-1/2}\left(1+ \sqrt{3}x^3/2\right)^{-1/2}$$

$$= \frac{1}{\sqrt{2}}(1+x^2)\left(1+\frac{\sqrt{3}x^3}{2}\right)^{-1/2}$$

The general term of binomial series is

$$T_{r+1} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} x^r$$

for $\left(1 + \frac{\sqrt{3}x^3}{2}\right)^{-1/2}$

$$T_{r+1} = \frac{(-1/2)(-1/2-1)(-1/2+2)\cdots(-1/2+r+1)}{r!} \left(\frac{\sqrt{3}}{2}x^3\right)^r$$

$$T_{r+1} = \frac{(-1/2)(-1/2-1)(-1/2-2)\cdots(-1/2-r+1)(\sqrt{3})^r}{(2)^r r!} (x^{3r})$$

Now multiply general term with

$\frac{1}{\sqrt{2}}(1+x^2)$

$$= \left[\frac{1}{\sqrt{2}}(1+x^2)\right]\left[\frac{(-1/2)(-1/2-1)(-1/2-2)\cdots(-1/2-r+1)(\sqrt{3})^r}{2^r r!} x^{3r}\right]$$

$$= \left[\frac{(-1/2)(-1/2-1)\cdots(-1/2-r+1)(\sqrt{3})^r}{2^r\sqrt{2}\, r!} x^{3r}\right] + \left[\frac{(-1/2)(-1/2-1)\cdots(-1/2-r+1)(\sqrt{3})^r x^{3r+2}}{2^r\sqrt{2}\, r!}\right]$$

To get integer value of r

use $3r+2 = 14$

$r = 4$

Put $r=4$ in 2^{nd} term we get

$$= \frac{315\sqrt{2}}{4096} x^{14}$$

8. Find sum of the following series.

i) $1 - \frac{3}{7} + 4.\left(\frac{3}{7}\right)^2 - 8\left(\frac{3}{7}\right)^3 + \dots$

Sol:-

$$= 1 - \frac{3}{7} + 4.\left(\frac{3}{7}\right)^2 - 8\left(\frac{3}{7}\right)^3 + \dots$$

As binomial series is given as:-

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \dots$$

On comparing with given series

$nx = -\frac{3}{7}$, squaring on b/s

$$n^2x^2 = \frac{9}{49} \quad -(i)$$

and, $\frac{n(n-1)}{2!}x^2 = 4\left(\frac{3}{7}\right)^2$

$$\frac{n^2x^2 - nx^2}{2} = \frac{36}{49}$$

OR $\frac{9}{49} - nx^2 = \frac{36}{49} \times 2$

$$-nx^2 = \frac{72}{49} - \frac{9}{49}$$

$$nx^2 = \frac{9}{7} \quad -(ii)$$

Divide eq.(i) and (ii)

$$\frac{n^2x^2}{nx^2} = \frac{9/49}{-9/7}$$

$$n = -\frac{9}{49} \times \frac{7}{9} \Rightarrow n = -\frac{1}{7}$$

As, $nx = -\frac{3}{7} \Rightarrow x = \frac{+3}{-7} \times \frac{1}{n}$

$$x = -\frac{3}{\cancel{7}} \times -\cancel{7} \Rightarrow x = +3$$

$$\therefore 1 - \frac{3}{7} + 4.\left(\frac{3}{7}\right)^2 - 8\left(\frac{3}{7}\right)^3 + \dots = (1+3)^{-1/7}$$
$$= \frac{1}{\sqrt[7]{4}}$$

ii) $1 - \frac{1}{15} + \frac{4}{2!\,15^2} - \frac{4.7}{3!\,15^3} + \dots$

Sol:-

$$= 1 - \frac{1}{15} + \frac{4}{2!\,15^2} - \frac{4.7}{3!\,15^3} + \dots$$

As binomial series is given as,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

On comparing with original series:-

$$nx = -\frac{1}{15} \text{ -(i)} \Rightarrow n^2x^2 = \frac{1}{225}$$

and $\frac{n(n-1)}{\cancel{2!}}x^2 = \frac{4}{\cancel{2!}15^2}$

$$n^2x^2 - nx^2 = \frac{4}{225}$$

$$\frac{1}{225} - nx^2 \frac{4}{225}$$

$$-nx^2 = \frac{4}{225} - \frac{1}{225}$$

$$nx^2 = -\frac{1}{75} \quad \text{— (ii)}$$

Divide (i) by (ii)

$$\frac{n^2 x^2}{n x^2} = \frac{+1/225}{-1/75}$$

$$n = -\frac{1}{3}$$

eq. (i) $\Rightarrow$ $nx = \frac{-1}{15} \Rightarrow x = \frac{1}{5}$

$$\therefore \quad = 1 - \frac{1}{15} + \frac{4}{2!\,15^2} + \frac{4.7}{3!\,15^3} + \dots$$

$$= \left(1 + \frac{1}{5}\right)^{-1/3}$$

$$= \left(\frac{6}{5}\right)^{-1/3}$$

$$= \left(\frac{5}{6}\right)^{1/3}$$

$$= \sqrt[3]{\frac{5}{6}}$$

(ii) $1 - \frac{2.4}{5} + \frac{3.4^2}{5^2} + \frac{4.4^3}{5^3} + \dots$

Sol:-

$$= 1 - \frac{24}{5} + \frac{3.4^2}{5^2} + \frac{4.4^3}{5^3} + \dots$$

As binomial series is given as

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$$

on composing we get

$$nx = -\frac{2.4}{5} \text{ —(iv)} \Rightarrow \quad n^2x^2 = \frac{64}{25} \text{ —(i)}$$

and, $\frac{n(n-1)x^2}{2!} = \frac{3.4^2}{5^2}$

$$n^2x^2 - nx^2 = \frac{96}{25} \Rightarrow -nx^2 = \frac{96}{25} - \frac{64}{25}$$

$$nx^2 = \frac{-32}{5} \text{ —(ii)}$$

Divide eq.(i) and (ii)

$$\frac{n^{\cancel{2}}\cancel{x^2}}{\cancel{n}\cancel{x^2}} = \frac{-64/225}{-32/5} \Rightarrow n = -\frac{2}{5}$$

eq.(iv) $\Rightarrow \quad nx = -\frac{2.4}{5} \Rightarrow x = 4$

$\therefore = (1+x)^n$

$= (1+4)^{-2/5}$

$= (5)^{-2/5}$

iv) $3 - \frac{3}{18} - \frac{3}{2!\,18^2} - \frac{3^2}{3!\,18^3} - \dots$

Soln

$$= 3 - \frac{3}{18} - \frac{3}{2!\,18^2} - \frac{3^2}{3!\,18^3} - \dots$$

$$= 3\left[1 - \frac{1}{18} - \frac{1}{2!\,8^2} - \frac{3}{3!\,18^3} \dots\right] \quad -(\alpha)$$

As, $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$

on comparing

$$nx = -\frac{1}{18} \Rightarrow n^2x^2 = \frac{1}{324} \quad -(i)$$

and, $\frac{n(n-1)x^2}{\cancel{2!}} = -\frac{1}{\cancel{2!}\,18^2}$

$$\cancel{n^2x^2 - nx^2} = -\frac{1}{324}$$

~~OR~~ $$-nx^2 = -\frac{1}{324} - \frac{1}{324}$$

$$nx^2 = \frac{1}{162} \quad -(ii)$$

Divide (i) by (ii)

$$\frac{n^{\cancel{2}}\cancel{x^2}}{\cancel{n}\cancel{x^2}} = \frac{1/324}{1/162}$$

$$n = \frac{1}{2}$$

$nx = -\frac{1}{18} \Rightarrow x = -\frac{1}{9}$

$\Rightarrow (1+x)^n$

$= (1-\frac{1}{9})^{1/2}$

$= (\frac{8}{9})^{1/2}$

$= \sqrt{\frac{8}{9}} \Rightarrow \frac{2\sqrt{2}}{3} \Rightarrow$ put in (x)

$= 2\sqrt{2}$

Q6. If $(\frac{1}{x})^2$ is so large and its higher power may be neglected than show that :-

$$\sqrt{x^2+25} - \sqrt{x^2+9} \approx \frac{8}{x}$$

Proof:-

$\Rightarrow \sqrt{x^2+25} - \sqrt{x^2+9} \approx \frac{8}{x}$

L.H.S $\Rightarrow \sqrt{x^2+25} - \sqrt{x^2+9}$

$= (x^2+25)^{1/2} - (x^2+9)^{1/2}$

$= (x^2)^{1/2}(1+25/x^2)^{1/2} - (x^2)^{1/2}(1+9/x^2)^{1/2}$

$= x(1+\frac{25}{x^2})^{1/2} - x(1+\frac{9}{x^2})^{1/2}$

Apply Binomial series upto $\frac{1}{x^2}$ containing term

$$\approx x\left[1+\left(\tfrac{1}{2}\right)\left(25/x^2\right)\right]-x\left[1+\left(\tfrac{1}{2}\right)\left(9/x^2\right)\right]$$

$$\approx x\left(1+\frac{25}{2x^2}\right)-x\left(1+\frac{9}{2x^2}\right)$$

$$\approx \cancel{x}+\frac{25}{2x}-\cancel{x}+\left(\frac{-9}{2x}\right)$$

$$\approx \frac{25}{2x}-\frac{9}{2x}$$

$$\approx \frac{8}{x} \Rightarrow \text{R.H.S.}$$

Q 7. Find the term involving x^n while simplifying $\frac{(2+x)^2}{(1+x)^3}$

Sol:-

$$=\frac{(2+x)^2}{(1+x)^3}$$

$$=(2+x)^2(1+x)^{-3}$$
$$=(4+4x+x^2)(1+x)^{-3}$$

As, $T_{r+1}=\frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}x^r$

$$=(4+4x+x^2)\left[\frac{(-3)(-3-1)(-3-2)\cdots(-3-r+1)}{r!}x^r\right]$$

$$=\frac{(4)(-3)(-4)(-5)\cdots(-3-r+1)}{r!}x^r+\frac{(4)(-3)(-4)(-5)\cdots(-3-r+1)}{r!}x^{r+1}+\frac{(-3)(-4)(-5)\cdots(-3-r+1)}{r!}x^{r+2}$$

$$= \frac{(4)(-1)(-2)(-3)\dots(-3-r+1)}{2r!} + \frac{(4)(-1)(-2)\dots(-3+r+1)}{2r!}x^{r+1}$$

$$+ \frac{(-1)(-2)(-3)\dots(-3-r+1)}{2r!}x^{r+2}$$

$$= \frac{(4)(-1)^{r+2}(1)(2)(3)\dots(r+2+r)}{2r!}x^{r} + \frac{(4)(-1)^{r+2}(1)(2)\dots(r+2)}{2r!}x^{r+1}$$

$$+ \frac{(-1)^{r+2}(1)(2)(3)\dots(r+2)}{2r!}x^{r+2}$$

Put $r = n$, $r+1 = n$, $r+2 = n$

$r = n-1$, $n = r = 2$

$$= \frac{(4)(-1)^{n+2}(1)(2)\dots(n+2)}{2n!}x^{n} + \frac{(4)(-1)^{n+1}(1)(2)\dots(n+1)}{2(n-1)!}x^{n}$$

$$+ \frac{(-1)^{n}(1)(2)\dots(n)}{2(n-2)!}x^{n}$$

$$= \frac{(-1)^n x^n}{2}\left[\frac{4(-1)^2(1)(2)\dots(n+2)}{n!} + \frac{4(-1)(1)(2)\dots(n+1)}{(n-1)!} + \frac{1(2)(3)\dots(n)}{(n-2)!}\right]$$

$$= \frac{(-1)^n x^n}{2}\left[\frac{4(n+2)!}{n!} + \frac{-4(n+1)!}{(n-1)!} + \frac{n!}{(n-2)!}\right]$$

$$= \frac{(-1)^n x^n}{2}\left[\frac{4(n+2)(n+1)\cancel{n!}}{\cancel{n!}} - \frac{4(n+1)n\cancel{(n-1)!}}{\cancel{(n-1)!}} + \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}\right]$$

$$= \frac{(-1)^n x^n}{2}\left[4(n^2+3n+2) - 4(n^2+n) + n^2 - n\right]$$

$$= \frac{(-1)^n x^n}{2}\left[4n^2+12n+8 - 4n^2 + 4 + n^2 - n\right]$$

$$= \frac{(-1)^n x^n}{2}\left[\frac{n^2+7n+8}{2}\right]$$

Q2. Approximate the value upto four places of decimal.

ii) $(.95)^{2/7}$

Sol:-

$$= (0.95)^{2/7}$$

$$= (1-0.05)^{2/7}$$

$$= 1 + (2/7)(-0.05) + \frac{(2/7)(2/7-1)}{2!}(-0.05)^2 + \frac{(2/7)(2/7-1)(2/7-2)}{3!}(-0.05$$

$$= 1 - \frac{1}{70} - \frac{1}{3920} + \frac{1}{137200}$$

$$= 0.985$$

iii) $(1.03)^{1/3}$

Sol:-

$$= (1.03)^{1/3}$$

$= (1+0.03)^{1/3}$

$= 1 + (1/3)(0.03) + \frac{(1/3)(1/3-1)}{2!}(0.03)^2 + \frac{(1/3)(1/3-1)(1/3-2)}{3!}(0.03)^3$

$= 1 + \frac{1}{100} + \frac{-1}{10000} + \frac{1}{600000}$

$= 1.0099$

$\sqrt{\frac{5}{3}}$

Sol:-

$= \sqrt{\frac{5}{3}}$

$= \left(\frac{5}{3}\right)^{1/2} \Rightarrow \left(1 + \frac{2}{3}\right)^{1/2}$

$= 1 + (1/2)(2/3) + \frac{(1/2)(1/2-1)}{2!}\left(\frac{2}{3}\right)^2 + \frac{(1/2)(1/2-1)(1/2-2)}{3!}\left(\frac{2}{3}\right)^3$

$= 1 + \frac{1}{3} - \frac{1}{18} + \frac{1}{54}$

$= 1.2963$

$\sqrt[6]{65}$

Sol:-

$= \sqrt[6]{65} \Rightarrow (1+64)^{1/6}$

$= (64)^{1/2}\left(1 + \frac{1}{64}\right)^{1/6} = 2\left(1 + \frac{1}{64}\right)^{1/6}$

$= 2\left[1 + (1/6)(1/64) + \frac{(1/6)(1/6-1)}{2!}\left(\frac{1}{64}\right)^2 + \frac{(1/6)(1/6-1)(1/6-2)}{3!}\left(\frac{1}{64}\right)^3\right]$

$= 2.0051$