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*Where Minds Pollinate*

## Class 11
## Mathematics

# Chapter 7
# Exercise 7.2
# Notes

National Book Foundation

## Exercise # 7.2

1. Expand the following with the help of Pascal's triangle.

i) $\left(2\sqrt{x} + \frac{1}{\sqrt{x}}\right)^5$

**Sol:-**

$$= \left[2\sqrt{x} + \frac{1}{\sqrt{x}}\right]^2$$

By using binomial theorme for variables

$$(a+b)^5 = a^5b^0 + a^{5-1}b^1 + a^{5-2}b^2 + a^{5-3}b^3 + a^{5-4}b^4 + a^0b^5$$

$$(a+b)^5 = a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 + b^5$$

For constants from pascals triangle we adjust them in $(a+b)^5$

$$\therefore (a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

here, $a = 2\sqrt{x}$ , $b = \frac{1}{\sqrt{x}}$

$$= (2\sqrt{x})^5 + 5(2\sqrt{x})^4\left(\frac{1}{\sqrt{x}}\right) + 10(2\sqrt{x})^3\left(\frac{1}{\sqrt{x}}\right)^2 + 10(2\sqrt{x})^2\left(\frac{1}{\sqrt{x}}\right)^3 + 5(2\sqrt{x})\left(\frac{1}{\sqrt{x}}\right)^4 + \left(\frac{1}{\sqrt{x}}\right)^5$$

$$= 32x^{5/2} + 80x^{3/2} + 80\sqrt{x} + 40\left(\frac{1}{\sqrt{x}}\right) + 10\left(\frac{1}{x^{3/2}}\right) + \frac{1}{x^{5/2}}$$

$$\left[\frac{3}{x} + \frac{y}{2}\right]^6$$

**Sol:**

$$= \left[\frac{3}{x} + \frac{y}{2}\right]^6$$

By using Binomial theorme for variables. and then adjust coefficients from pascal's triangle for $n = 6$.

$$(a+b)^6 = a^6b^0 + 6a^{6-1}b^1 + 15a^{6-2}b^2 + 20a^{6-3}b^3 + 15a^{6-4}b^4 + 6a^{6-5}b^5 + a^{6-6}b^6$$

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

here, $a = \frac{3}{x}$, $b = \frac{y}{2}$

$$= \left(\frac{3}{x}\right)^6 + 6\left(\frac{3}{x}\right)^5\left(\frac{y}{2}\right) + 15\left(\frac{3}{x}\right)^4\left(\frac{y}{2}\right)^2 + 20\left(\frac{3}{x}\right)^3\left(\frac{y}{2}\right)^3 + 15\left(\frac{3}{x}\right)^2\left(\frac{y}{2}\right)^4 + 6\left(\frac{3}{x}\right)\left(\frac{y}{2}\right)^5 + \left(\frac{y}{2}\right)^6$$

$$= \frac{729}{x^6} + \frac{729y}{x^5} + \frac{1215y^2}{4x^4} + \frac{135y^3}{2x^3} + \frac{135y^4}{16x^2} + \frac{9y^5}{16x} + \frac{y^6}{64}$$

iii) $(2-x^{3/2})^7$

Sol:

$$=(2-x^{3/2})^7$$

By using Binomial theoreme and pascal triangle for constants.

$$(a+b)^7 = a^7b^0 + 7a^6b^1 + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$$

here, $a=2$, $b=-x^{3/2}$

$$= (2)^7 + 7(2)^6(-x^{3/2})^1 + 21(2)^5(-x^{3/2})^2 + 35(2)^4(-x^{3/2})^3 + 35(2)^3(-x^{3/2})^4 + 21(2)^2(-x^{3/2})^5 + 7(2)(-x^{3/2})^6 + (-x^{3/2})^7$$

$$= 128 - 448x^{3/2} + 672x^3 - 560x^{9/2} + 280x^6 - 84x^{15/2} + 14x^9 - x^{21/2}$$

iv) $\left[\frac{x^2}{y^2} - \sqrt{\frac{y}{x}}\right]^5$

Sol:

$$= \left[\frac{x^2}{y^2} - \sqrt{\frac{y}{x}}\right]^5$$

$$\because (a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

here, $a = \frac{x^2}{y^2}$, $b = -\sqrt{\frac{y}{x}}$

$$= \left[\frac{x^2}{y^2}\right]^5 + 5\left[\frac{x^2}{y^2}\right]^4\left[\sqrt{\frac{y}{x}}\right] + 10\left[\frac{x^2}{y^2}\right]^3\left[\sqrt{\frac{y}{x}}\right]^2 + 10\left[\frac{x^2}{y^2}\right]^3\left[\sqrt{\frac{y}{x}}\right]^3$$

$$+ 5\left[\frac{x^2}{y^2}\right]\left[\sqrt{\frac{y}{x}}\right]^4 + \left[\sqrt{\frac{y}{x}}\right]^5$$

$$= \frac{x^{10}}{y^{10}} + \frac{5x^8\sqrt{y}}{y^8\sqrt{x}} + \frac{10x^6 y}{y^6 x} - \frac{10x^4 y^{3/2}}{y^4 x^{3/2}} + \frac{5x^2 y^2}{y^2 x^2} - \frac{y^{5/2}}{x^{5/2}}$$

$$= \frac{x^{10}}{y^{10}} - \frac{5x^{15/2}}{y^{15/2}} + \frac{10x^5}{y^5} - \frac{10x^{5/2}}{y^{5/2}} + 5 - \frac{y^{5/2}}{x^{5/2}}$$

Expand the following by using Binomial theorem.

$$\left[\frac{2x}{3} - \frac{3}{2x}\right]^5$$

**Sol:**

$$= \left[\frac{2x}{3} - \frac{3}{2x}\right]^5$$

The general term for expansion will be:

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

here, $a = \frac{2x}{3}$, $b = \frac{-3}{2x}$, $n = 5$

$$\therefore T_{0+1} = \binom{5}{0}\left(\frac{2x}{3}\right)^{5-0}\left(\frac{-3}{2x}\right)^0$$

$$T_1 = \frac{32x^5}{243}$$

$\Rightarrow T_{1+1} = \binom{5}{1}\left(\frac{2x}{3}\right)^{5-1}\left(\frac{-3}{2x}\right)^{1}$

$T_2 = 5\left(\frac{16x^4}{81}\right)\left(\frac{-3}{2x}\right)$

$T_2 = \frac{-40x^3}{27}$

$\Rightarrow T_{2+1} = \binom{5}{2}\left(\frac{2x}{3}\right)^{5-2}\left(\frac{-3}{2x}\right)^{2}$

$T_3 = 10\left(\frac{8x^3}{27}\right)\left(\frac{9}{4x^2}\right)$

$T_3 = \frac{20x}{3}$

$\Rightarrow T_{3+1} = \binom{5}{3}\left(\frac{2x}{3}\right)^{5-3}\left(\frac{-3}{2x}\right)^{3}$

$T_4 = 10\left(\frac{4x^2}{9}\right)\left(\frac{-27}{8x^3}\right)$

$T_4 = \frac{-15}{x}$

$\Rightarrow T_{4+1} = \binom{5}{4}\left(\frac{2x}{3}\right)^{5-4}\left(\frac{-3}{2x}\right)^{4}$

$T_5 = 5\left(\frac{2x}{3}\right)\left(\frac{81}{16x^4}\right)$

$T_5 = \frac{135}{8x^3}$

$\Rightarrow T_{5+1} = \binom{5}{5}\left(\frac{2x}{3}\right)^{5-5}\left(\frac{-3}{2x}\right)^{5}$

$T_6 = \frac{-243}{32x^5}$

$\therefore \left(\frac{2x}{3} - \frac{3}{2x}\right)^5 = \frac{32x^5}{243} - \frac{40x^3}{27} + \frac{20x}{3} - \frac{15}{x} + \frac{135}{8x^3} - \frac{243}{32x^5}$

ii) $(-x+y^{-1})^6$

**Sol:**

$$=(-x+y^{-1})^6$$

As, $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}(a)^{n-2}b^2 + \dots + \binom{n}{n}b^n$

for, $n=6,\ a=-x,\ b=y^{-1}$

$$:= \binom{6}{0}(-x)^6 + \binom{6}{1}(-x)^5(y^{-1}) + \binom{6}{2}(-x)^4(y^{-1})^2 + \binom{6}{3}(-x)^3(y^{-1})^3$$

$$+ \binom{6}{4}(-x)^2(y^{-1})^4 + \binom{6}{5}(-x)(y^{-1})^5 + \binom{6}{6}(y^{-1})^6$$

$$= x^6 - \frac{6x^5}{y} + \frac{15x^4}{y^2} - \frac{20x^3}{y^3} + \frac{15x^2}{y^4} - \frac{6x}{y^5} + \frac{1}{y^6}$$

iii) $(3u-1)^7$

**Sol:**

$$=(3u-1)^7$$

As, $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n}b^n$

here, $a=3u,\ b=-1,\ n=7$

$$= \binom{7}{0}(3u)^7 + \binom{7}{1}(3u)^6(-1) + \binom{7}{2}(3u)^5(-1)^2 + \binom{7}{3}(3u)^4(-1)^3$$

$$+ \binom{7}{4}(3u)^3(-1)^4 + \binom{7}{5}(3u)^2(-1)^5 + \binom{7}{6}(3u)(-1)^6 + \binom{7}{7}(-1)^7$$

$$= 2187u^7 - 5103u^6 + 5103u^5 - 2835u^4 + 945u^3 - 189u^2 + 21u - 1$$

iv) $(a\sqrt{2} + b\sqrt{3})^5$

**Sol:**

$$= (a\sqrt{2} + b\sqrt{3})^5$$

$$\because (a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n}b^n$$

$\because$ $a = a\sqrt{2}$, $b = b\sqrt{3}$, $n = 5$

$$\therefore = \binom{5}{0}(a\sqrt{2})^5 + \binom{5}{1}(a\sqrt{2})^4(b\sqrt{3}) + \binom{5}{2}(a\sqrt{2})^3(b\sqrt{3})^2$$

$$+ \binom{5}{3}(a\sqrt{2})^2(b\sqrt{3})^3 + \binom{5}{4}(a\sqrt{2})(b\sqrt{3})^4 + \binom{5}{5}(b\sqrt{3})^5$$

$$= a^5(2)^{5/2} + 20\sqrt{3}a^4b + 30(2)^{3/2}a^3b^2 + 20(3)^{3/2}a^2b^3$$

$$+ 45\sqrt{2}ab^4 + b^5(3)^{5/2}$$

v) $(1 + 2x - y)^4$

**Sol:**

$$= (1 + 2x - y)^4$$

$$\because (a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n}b^n$$

here, $n = 4$, $a = 1 + 2x = z$, $b = -y$

$$= \binom{4}{0}(z)^4 + \binom{4}{1}(z)^3(-y) + \binom{4}{2}(z)^2(-y)^2 + \binom{4}{3}(z)(-y)^3$$

$$+ \binom{4}{4}(z)^0(-y)^4$$

$$= z^4 - 4z^3y + 6z^2y^2 - 4zy^3 + y^4$$

as, $z = 1 + 2x$

$$\Rightarrow (1+2x)^4 - 4(1+2x)^3 y + 6(1+2x)^2 y^2 - 4(1+2x) y^3 + y^4$$

$$= (1+2x)^4 - 4(1+2x)^3 y^2 + 6(1+4x^2+4x) y^2 - 4(y^3+2xy) + y^4$$

$$= (1+2x)^4 - 4(1+2x)^3 y^2 + 6y^2 + 24x^2y^2 + 24xy^2 - 4y^3 - 8xy + y^4$$

Apply binomial theorem on first two terms

$$(1+2x)^4 = \binom{4}{0}(1)^4 + \binom{4}{1}(1)^3(2x) + \binom{4}{2}(1)^2(2x)^2 + \binom{4}{3}(1)^1(2x)^3 + \binom{4}{4}($$

$$(1+2x)^4 = 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

and $(1+2x)^3 = 1 + 8x + 24x^2 + 32x^3$

$$(1+8x+24x^2+32x^3+16x^4) - 4y(1+8x+2x^2+32x^3)$$
$$+ 30y^2 + 24x^2y^2 - 4y^3 - 8xy + y^4$$

$$= 1+8x+24x^2+32x^3+16x^4 - 4y - 32xy - 8x^2y - 128x^3y$$
$$+ 30y^2 + 24x^2y^2 - 4y^3 - 8xy + y^4$$

$$= 16x^4 + 32x^3 + 24x^2 + 8x + y^4 - 4y^3 + 30y^2 - 4y$$
$$- 40xy - 8x^2y - 128x^3y + 24x^2y^2 + 1$$

10. The sum of coefficients of first three terms in the expansion of $(a - 3/a^2)^n$ is 559. Find the term involving $a^3$.

Sol:

$$= \left[a - \frac{3}{a^2}\right]^n$$

The $r^{th}$ term of binomial theorem is

$$\Rightarrow \quad T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

for $r = 0$ and $a = a$, $b = -\frac{3}{a^2}$

$$T_{0+1} = \binom{n}{0} (a)^{n-0} \left(-\frac{3}{a^2}\right)^0$$

$T_1 = 1(a)^n(1) \Rightarrow T_1 = a^n$

for $r = 1$ i.e.

$T_{1+1} = \binom{n}{1}(a)^{n-1}\left(\frac{-3}{a^2}\right)^1$

$T_2 = n(a)^{n-1}\left(\frac{-3}{a^2}\right)$

$T_2 = 3n\,a^{n-1-2} \Rightarrow T_2 = -3n\,a^{n-3}$

for $r = 2$, we have,

$T_{2+1} = \binom{n}{2}\cancel{(a)^{n-2}}\left(\frac{-3}{a^2}\right)^2$

$T_3 = \frac{n!}{(n-2)!\,2!}\cancel{(a)^{n-2}}\left(\frac{9}{a^4}\right)$

$T_3 = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2}(a)^{n-2-4}(9)$

$T_3 = \frac{9}{2}\cancel{n(n-1)(a)^{n-6}}$

$T_3 = \frac{9n^2 - 9n}{2}(a)^{n-6}$

Now, the sum of coefficients of $T_1$ and $T_2$ and $T_3$ is 559

$\therefore T_{1_c} + T_{2_c} + T_{3_c} = 559$

$\Rightarrow 1 - 3n + \frac{9n^2 - 9n}{2} = 559$

$2 - 6n + 9n^2 - 9n = 2(559)$

$9n^2 - 15n + 2 - 1118 = 0$

$T_1 = 1(a)^n(1) \Rightarrow T_1 = a^n$

for $r = 1$ i.e.

$T_{1+1} = \binom{n}{1}(a)^{n-1}\left(\frac{-3}{a^2}\right)^1$

$T_2 = n(a)^{n-1}\left(\frac{-3}{a^2}\right)$

$T_2 = 3n\,a^{n-1-2} \Rightarrow T_2 = -3n\,a^{n-3}$

for $r = 2$, we have,

$T_{2+1} = \binom{n}{2}(a)^{n-2}\left(\frac{-3}{a^2}\right)^2$

$T_3 = \frac{n!}{(n-2)!\,2!}(a)^{n-2}\left(\frac{9}{a^4}\right)$

$T_3 = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2}(a)^{n-2-4}(9)$

$T_3 = \frac{9}{2}n(n-1)(a)^{n-6}$

$T_3 = \frac{9n^2 - 9n}{2}(a)^{n-6}$

Now, the sum of coefficients of $T_1$ and $T_2$ and $T_3$ is 559

$\therefore T_{1c} + T_{2c} + T_{3c} = 559$

$\Rightarrow a^{n} - 3n a^{n} \frac{9n^2 - 9n}{2} = 559$

$2 - 6n + 9n^2 - 9n = 2(559)$

$9n^2 - 15n + 2 - 1118 = 0$

$9n^2 - 15n - 1116 = 0$

on solving for $n$ we get

$n = 12$ , $n = -\frac{31}{3}$ (neglect it)

$\therefore \quad n = 12$

So, $= \left[a - \frac{3}{a^2}\right]^{12}$

Now term involving $a^3$ will be

$T_{r+1} = \binom{n}{r} a^{n-r} b^r$

$T_{r+1} = \binom{12}{r} (a)^{12-r} \left(\frac{-3}{a^2}\right)^r$

$= \binom{12}{r} a^{12-r} \left[(-3)^r \cdot a^{-2r}\right]$

$T_{r+1} = \binom{12}{r} a^{12-r-2r} (-3)^r$

$T_{r+1} = \binom{12}{r} a^{12-3r} (-3)^r$

On putting $r = 3$ for $a^3$ (to get)

$T_{3+1} = \binom{12}{3} a^{12-3(3)} (-3)^3$

$T_4 = 220(-27) a^3$

$T_4 = -5940 a^3$

Hence, $T_4$ involves $a^3$.

Q. Prove that sum of all bionomial coefficients in the expansion of $(a+b)^n$ is $2^n$; hence or otherwise prove that sum of odd coefficients is $2^{n-1}$.

**Proof:**

Binomial expansion for $(a+b)^n$ is:-

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n \quad \text{—(i)}$$

Now to get $2^n$ put $a=b=1$

$$\therefore (1+1)^n = \binom{n}{0}(1)^n + \binom{n}{1}(1)^{n-1}(1) + \dots + \binom{n}{n-1}(1)(1)^{n-1} + \binom{n}{n}(1)^n$$

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n-1} + \binom{n}{n}$$

Hence proved. —(i)

$\therefore$ Now, put $a = 1$ and $b = -1$

$$(1-1)^n = \binom{n}{0}(1)^n + \binom{n}{1}(1)^{n-1}(-1) + \binom{n}{2}(1)^{n-2}(-1)^2 + \dots + \binom{n}{n-1}(1)(-1)^{n-1} + \binom{n}{n}(1)(-1)$$

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots + \binom{n}{n-1} - \binom{n}{n}$$

$$0 = \left[\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\dots+\binom{n}{n-1}\right] - \left[\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\dots+\binom{n}{n}\right]$$

OR,

$$\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\dots+\binom{n}{n-1} = \binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\dots+\binom{n}{n} \quad -(ii)$$

Put eq.(ii) in (i)

$$\therefore 2^n = \binom{n}{1}+\binom{n}{1}+\binom{n}{3}+\binom{n}{3}+\dots+\binom{n}{n-1}\binom{n}{n-1}$$

$$2^n = 2\binom{n}{1}+2\binom{n}{3}+2\binom{n}{5}+\dots 2\binom{n}{n-1}$$

$$\frac{2^n}{2} = \binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\dots+\binom{n}{n-1}$$

$$2^{n-1} = \binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\dots+\binom{n}{n-1}$$

Hence, proved.

8. Find the $r^{th}$ term from end in the expansion of $(a+b)^n$ where $0 \le r \le n$.

Sol:-

$$= (a+b)^n \qquad 0 \le r \le n$$

The $r^{th}$ term in expansion will be.

$$T_{k+1} = \binom{n}{k} a^{n-k} b^k$$

Here, $a = b$, $b = a$ and $p+1 = r \Rightarrow p = r-1$

$$T_r = \binom{n}{r-1} (b)^{n-(r-1)} (a)^{r-1}$$

$$T_r = \binom{n}{r-1} (b)^{n-r+1} (a)^{r-1}$$

which is required term.

Find the term independent of $x$ in the expansion of the following.

$$\left[2x^2 - \frac{1}{x}\right]^{12}$$

**Sol:**

$$= \left[2x^2 - \frac{1}{x}\right]^{12}$$

The $(r+1)$th term for the expansion will be:-

$$T_{r+1} = \binom{n}{r} (a)^{n-r} (b)^r$$

Here, $a = 2x^2$, $b = -\frac{1}{x}$, $n = 12$

$$\therefore T_{r+1} = \binom{12}{r} (2x^2)^{12-r} \left(-\frac{1}{x}\right)^r$$

$$= \binom{12}{r} (2)^{12-r} \cdot (x)^{2(12-r)} \cdot (-1)^r (x)^{-1(r)}$$

$$T_{r+1} = \binom{12}{r}(2)^{12-r} \cdot (-1)^r \cdot (x)^{2(12-r)-r} \quad \text{--- (i)}$$

for term having no $x$.

$$\Rightarrow \quad 2(12-r) - r = 0$$
$$24 - 2r - r = 0$$
$$24 = 3r$$
$$r = \frac{24}{3}$$
$$r = 8 \rightarrow \text{Put in (i)}$$

$$\therefore \quad T_{8+1} = \binom{12}{8}(2)^{12-8}(-1)^8 \cdot (x)^0$$

$$T_9 = 495(16)$$

$$T_9 = 7920$$

ii) $\left[\sqrt{x} + \dfrac{1}{3x^2}\right]^{10}$

**Sol:**

$$= \left[\sqrt{x} + \frac{1}{3x^2}\right]^{10}$$

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

Here, $a = \sqrt{x}$, $b = \dfrac{1}{3x^2}$, $n = 10$

$$\therefore \quad T_{r+1} = \binom{10}{r}(\sqrt{x})^{10-r}\left(\frac{1}{3x^2}\right)^r$$

$$T_{r+1} = \binom{10}{r} (x^{1/2})^{10-r} (1)^r \left(\frac{1}{3}\right)^r (x^{-2})^r$$

$$= \binom{10}{r} (x)^{5-1/2r} \left(\frac{1}{3}\right)^r (x)^{-2r}$$

$$T_{r+1} = \binom{10}{r} \left(\frac{1}{3^r}\right) (x)^{5-1/2r-2r} \quad -(i)$$

for term involving no $x$

$$\Rightarrow \quad 5 - \frac{r}{2} - 2r = 0$$

$$5 - \frac{5}{2} r = 0$$

$$\cancel{5} = \frac{\cancel{5} r}{2}$$

$$2(1) = r$$

$$r = 2 \quad \text{— Put in (i)}$$

$$T_{2+1} = \binom{10}{2} \left(\frac{1}{3^2}\right) (x)^0$$

$$T_3 = 45 \left(\frac{1}{9}\right)$$

$$T_3 = 5$$

6. Find the specified term in the following expansions.

i) Term involving $b^6$ in the expansion of $\left[\frac{a^2}{2} + 2b^2\right]^{10}$

Sol:

$$= \left[\frac{a^2}{2} + 2b^2\right]^{10}$$

The $(r+1)^{th}$ term in expansion will be,

i.e. $T_{r+1} = \binom{n}{r}(a)^{n-r} b^r$

here, $n = 10$, $a = \frac{a^2}{2}$, $b = 2b^2$

$\therefore$ $T_{r+1} = \binom{10}{r}\left(\frac{a^2}{2}\right)^{10-r}(2b^2)^r$

$T_{r+1} = \binom{10}{r}\left(\frac{a^2}{2}\right)^{10-r}(2)^r(b)^{2r}$ — (i)

for term involving $b^6$

$\Rightarrow 2r = 6 \Rightarrow r = 3 \rightarrow$ Put in (i)

$\therefore$ $T_{3+1} = \binom{10}{3}\left(\frac{a^2}{2}\right)^{10-3} \cdot (2)^3(b)^{2(3)}$

$T_4 = (120)\left(\frac{a^{2(7)}}{2^7}\right) \cdot (8)(b)^6$

$= \frac{(120)(8)}{2^7}(a)^{14}(b)^6$

$T_4 = \frac{15}{2}a^{14}b^6$

ii) Term involving $q^8$ in the expansion of $\left[\frac{p^2}{2}+6q^2\right]^{12}$

**Sol:**

$$=\left[\frac{p^2}{2}+6q^2\right]^{12}$$

As, $T_{r+1}=\binom{n}{r}a^{n-r}b^r$

here, $a=\frac{p^2}{2}$, $b=6q^2$, $n=12$

$$\therefore T_{r+1}=\binom{12}{r}\left(\frac{p^2}{2}\right)^{12-r}(6q^2)^r$$

$$T_{r+1}=\binom{12}{r}\left(\frac{p^2}{2}\right)^{12-r}(6)^r(q)^{2r} \quad -(i)$$

for term involving $q^8$

$\Rightarrow 2r=8 \Rightarrow r=4 \rightarrow$ put in (i)

$$\therefore T_{4+1}=\binom{12}{4}\left(\frac{p^2}{2}\right)^{12-4}(6)^4(q)^{2(4)}$$

$$T_5=(495)\left(\frac{1}{2^8}\right)(p)^{2(8)}(1296)(q)^8$$

$$T_5=\frac{495(1296)}{2^8}p^{16}q^8$$

$$T_5=\frac{40095}{16}p^{16}q^8$$

iii) Term involving $x^4y^3$ in expansion $(3x^4-y)^4$

**Sol:**

$= (3x^4 - y)^4$

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

here, $a = 3x^4$, $b = -y$, $n = 4$

$\therefore T_{r+1} = \binom{4}{r} (3x^4)^{4-r} (-y)^r$

for term involving $x^4 y^3$ put $r=3$

$\Rightarrow T_{3+1} \binom{4}{3} (3x^4)^{4-3} (-y)^3$

$T_4 = 4(3)^1 (x^4)^1 (y)^3$

$T_4 = 12x^4 y^3$

iv) Term involving $y^8 x^3$ in expansion of $(y^4 - 3x)^5$

**Sol:** $= (y^4 - 3x)^5$

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

here, $a = y^4$, $b = -3x$, $n = 5$

$\therefore T_{r+1} = \binom{5}{r} (y^4)^{5-r} (-3x)^r$

$T_{r+1} = \binom{5}{r} y^{20-4r} (-3)^r (x)^r$

for term involving $y^8 x^3$ put $r = 3$

$\Rightarrow T_{3+1} = \binom{5}{3} y^{20-4(3)} (-3)^3 (x)^3$

$T_4 = 10 (y)^8 (-27)(x^3)$

$T_4 = -270 y^8 x^3$

Find the coefficients of the $8^{th}$ term in the expansion of $\left(x^2 + \frac{y}{2}\right)^{10}$

**Sol:**

$$= \left[x^2 + \frac{y}{2}\right]^{10}$$

The $(r+1)^{th}$ term for expansion will be;

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Here, $a = x^2$, $b = \frac{y}{2}$, $n = 10$, $r = 7$

$$\therefore T_{7+1} = \binom{10}{7} (x^2)^{10-7} \left(\frac{y}{2}\right)^7$$

$$T_8 = \frac{(120)(x)^6 (y)^7}{2^7}$$

$$T_8 = \frac{15}{16} x^6 y^7$$

Therefore coefficient of $8^{th}$ term is $\frac{15}{16}$

5) Find the middle term in the expansion of the following.

$$\left[3x^2 - \frac{1}{2x}\right]^{10}$$

**Sol:**

$$= \left[3x^2 - \frac{1}{2x}\right]^{10}$$

The middle term will be $\frac{10+2}{2} = 6$

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

for, $r = 5$, $a = 3x^2$, $b = -\frac{1}{2x}$, $n = 10$

$$\Rightarrow T_{5+1} = \binom{10}{5}(3x^2)^{10-5}\left(\frac{-1}{2x}\right)^5$$

$$T_6 = 252\,(243x^{10})\left(\frac{-1}{32x^5}\right)$$

$T_6 = \frac{-15309x^5}{8}$ is middle term

ii) $\left[2x^2 - \frac{1}{5x}\right]^{11}$

**Sol:**

$$= \left[2x^2 - \frac{1}{5x}\right]^{11}$$

Since, $n = 11$ is odd so, it has two middle terms i.e. $6^{th}$ and $7^{th}$.

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

for, $r = 5$ and $6$, $n = 11$, $a = 2x^2$, $b = -\frac{1}{5x}$

$T_{5+1} = \binom{11}{5}(2x^2)^{11-5}\left(\frac{-1}{5x}\right)^5$

$T_6 = (462)(64x^{12})\left(\frac{-1}{3125x^5}\right)$

$T_6 = -9.46176x^7$

$T_{6+1} = \binom{11}{6}(2x^2)^{11-6}\left(\frac{-1}{5x}\right)^6$

$T_7 = (462)(32x^{10})\left(\frac{1}{15625x^6}\right)$

$T_7 = 0.946176x^4$

iii) $\left[\frac{a}{\sqrt{x}} + \sqrt{x}\right]^8$

**Sol:**

$$= \left[\frac{a}{\sqrt{x}} + \sqrt{x}\right]^8$$

The middle term in expansion will be $5^{th}$

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

$r = 4$, $a = \frac{a}{\sqrt{x}}$, $b = \sqrt{x}$, $n = 8$

$\Rightarrow T_{4+1} = \binom{8}{4}\left(\frac{a}{\sqrt{x}}\right)^{8-4}(\sqrt{x})^4$

$T_5 = (70)\left(\frac{a^4}{x^{4/2}}\right)(x^{4/2})$

$T_5 = 70a^4$

iv) $\left[a - \frac{3}{x^2}\right]^{12}$

**Sol:-**

$$= \left[a - \frac{3}{x^2}\right]^{12}$$

The middle term is $\frac{12+2}{2} = 7^{th}$ term

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

here, $a = a$, $b = \frac{3}{x^2}$, $n = 12$, $r = 6$

$\Rightarrow T_{6+1} = \binom{12}{6} (a)^{12-6} \left(\frac{3}{x^2}\right)^6$

$T_7 = (924)(a)^6 \left(\frac{729}{x^{12}}\right)$

$T_7 = \frac{673596 a^6}{x^{12}}$

12. If the coefficients of $2^{nd}$, $3^{rd}$ and $4^{th}$ terms in the expansion of $(1+x)^{2m}$ are in A.P; show that $2m^2 - 9m + 7 = 0$

**Sol:**

$= (1+x)^{2m}$

The general term for binomial theorem is:-

$T_{r+1} = \binom{n}{r} a^{n-r} b^r$

for, $r = 1, 2$ and $3$, $a = 1$, $b = x$, $n = 2m$

$T_{1+1} = \binom{2m}{1} (1)^{2m-1} (x)$

$T_2 = 2m(1)x$

$T_2 = 2mx$

also, $T_{2+1} = \binom{2m}{2} (\ )^{2m-2} (x)^2$

$T_3 = \binom{2m}{2} x^2$

$T_3 = \dfrac{(2m)!}{(2m-2)!\,2!} x^2$

$T_3 = \dfrac{(\cancel{2}m)(2m-1)\cancel{(2m-2)}}{\cancel{(2m-2)!}\,\cancel{2}} x^2$

$T_3 = m(2m-1)x^2$

and $T_{3+1} = \binom{2m}{3} \cancel{(1)^{2m-3}} (x)^3$

$T_4 = \binom{2m}{3} x^3$

$T_4 = \dfrac{(2m)!}{(2m-3)!\,3!} x^3$

$T_4 = \dfrac{(\cancel{2}m)(2m-1)(2m-2)\cancel{(2m-3)!}}{\cancel{(2m-3)!}\,3\cdot\cancel{2}} \cdot x^3$

$T_4 = \dfrac{m(2m-1)(2m-2)x^3}{3}$

Since, $T_2$, $T_3$ and $T_4$ coefficients are in arithmetic progression

$\Rightarrow T_2, T_3, T_4 \rightarrow A.P$

$\Rightarrow T_3 - T_2 = T_4 - T_3 \rightarrow$ common difference

~~$T_3 + T_3 = T_4 + T_2$~~

$2T_3 = T_4 + T_2 \longrightarrow$ coefficients

OR $2m(2m-1) = \dfrac{m(2m-1)(2m-2)}{3} + 2m$

Taking "m" common on b/s and we get.

$$2(2m-1)^{n-2} = \frac{(2m-1)(2m-2)^{n-3}}{3} + 2^{n}$$

$$3(4m^{n-2} - 2^{n}) = (4m^2 - 4m - 2m + 2)(1^3) + 3(2)$$

$$12m - 16 = 4m^2 - 4m - 2m + 2 + 6$$

$$4m^2 - 4m - 12m - 2m + 2 + 6 + 6 = 0$$

$$4m^2 - 18m + 14 = 0$$

$$2(2m^2 - 9m + 7) = 0$$

OR $2m^2 - 9m + 7 = 0$ $\{2 \neq 0\}$

11. If the coefficients of $(r-5)^{th}$ and $(2r-1)^{th}$ term in the expansion of $(1+a)^{34}$ are equal, the find the vale of r.

**Sol:**

$$= (1+a)^{34}$$

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^{r}$

Here, $a = 1$, $b = a$, $n = 34$, $R+1 = r-5$

~~find $(r-5)^{th}$ and $(2r-1)^{th}$ term.~~ $R = r-6$

for $(r-5)^{th}$ term

$$T_{r-5} = \binom{34}{r-6} (1)^{34-(r-6)} (a)^{r-6}$$

$$T_{r-5} = \binom{34}{r-6} (a)^{r-6}$$

Now for $(2r-1)^{th}$ term

$$T_{2r-1} = \binom{34}{2r-2} (1)^{34-(2r-2)} (a)^{2r-2}$$

$$T_{2r-1} = \binom{34}{2r-2} (a)^{2r-2}$$

Since, the coefficients of $(r-5)^{th}$ and ~~$(2r-1)^{th}$ term are equal.~~

$$\Rightarrow \binom{34}{r-6} = \binom{34}{2r-2}$$

$\because \quad {}^nC_r = {}^nC_{n-r}$

$\therefore \quad r-6 = 34-(2r-2)$

$r-6 = 34-2r+2$

$r+2r = 36+6$

$3r = 42$

$r = \frac{42}{3}$

$\Rightarrow \quad r = 14$

13. If coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 6: 33: 110, then find the value of $n$ and the position of terms?

**Solution:**

$= (1+x)^n$

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

for, $n = n$, $r =$ consecutive, $a = 1$, $b = x$

Let $(r-1)^{th}$, $(r)^{th}$ and $(r+1)^{th}$ term be consecutive.

$\therefore \quad T_r = \binom{n}{r-1} (1)^{n-(r-1)} (x)^{r-1}$

$T_r = \binom{n}{r-1} (x)^{r-1}$

Now, $\quad T_{r+1} = \binom{n}{r} (1)^{n-r} (x)^r$

$T_{r+1} = \binom{n}{r} (x)^r$

and $\quad T_{r+2} = \binom{n}{r+1} (1)^{n-(r+1)} (x)^{r+1}$

$T_{r+2} = \binom{n}{r+1} (x)^{r+1}$

As, the coefficients of consecutive term are in ratio $6 : 33 : 110$

$\Rightarrow \quad \binom{n}{r-1} : \binom{n}{r} = 6 : 33$

$$\frac{n!}{[n-(r-1)]!\,(r-1)!} \times \frac{(n-r)!\,r!}{n!} = 6 : 33$$

$$\frac{(n-r)!\,r!}{(n-r+1)!\,(r-1)!} = 6:33$$

$$\frac{\cancel{(n-r)!}\,r.\cancel{(r-1)!}}{(n-r+1)\cancel{(n-r)!}\,\cancel{(r-1)!}} = \frac{6}{33}$$

$$\frac{r}{n-r+1} = \frac{6}{33}$$

$$33r = 6(n-r+1)$$
$$33r = 6n - 6r + 6$$
$$6n - 39n + 6 = 0$$
$$3(2n - 13r + 2) = 0$$
$$2n - 13r + 2 = 0 \quad -(i)$$

and $\binom{n}{r} : \binom{n}{r+1} = 33:110$

$$\frac{\cancel{n!}}{(n-r)!\,r!} \times \frac{(n-(r+1))!\,(r+1)!}{\cancel{n!}} = \frac{33}{110}$$

$$\frac{(n-r-1)!\,(r+1)!}{(n-r)!\,r!} = \frac{3}{10}$$

$$\frac{\cancel{(n-r-1)!}\,(r+1)\cancel{(r)!}}{(n-r)\cancel{(n-r-1)!}\,\cancel{r!}} = \frac{3}{10}$$

$$10(r+1) = 3(n-r)$$
$$10r + 10 = 3n - 3r$$
$$3n - 13r - 10 = 0 - (ii)$$

On solving (i) and (ii) we get,

$$n = 12, \quad r = 2$$

Hence, the consecutive terms are, $2^{nd}$, $3^{rd}$ and $4^{th}$.

18. Use pascal triangle to find no. of heads when six coins are tossed simultaneously?

**Sol:-**

Six coins are tossed means, $n = 6$

The pascal triangle is

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n=0$ | | | | | | | 1 | | | | | | | |
| $n=1$ | | | | | | 1 | | 1 | | | | | | |
| $n=2$ | | | | | 1 | | 2 | | 1 | | | | | |
| $n=3$ | | | | 1 | | 3 | | 3 | | 1 | | | | |
| $n=4$ | | | 1 | | 4 | | 6 | | 4 | | 1 | | | |
| $n=5$ | | 1 | | 5 | | 10 | | 10 | | 5 | | 1 | | |
| $n=6$ | 1 | | 6 | | 15 | | 20 | | 15 | | 6 | | 1 | |
| | 0-H | | 1-H | | 2-H | | 3-H | | 4-H | | 5-H | | 6-H | |

No. of heads $= 0+1+2+3+4+5+6 = 21$

So, 0 heads will appear 1 time on tossing
1 head will appear 6 times on tossing
2 head will appear 15 times on tos
3 head will appear 20 times on tossing
4 head will appear 15 times on tossing
5 head will appear 6 times on tossing
6 head will appear 1 times on tossing

If seven coins are tossed how many times 5-heads will appear?

Sol:-

7-coins are tossed means $n = 7$

The pascal triangle for $n=7$ will be

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $n=7$ | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 |
| | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| | 0-H | 1-H | 2-H | 3-H | 4-H | 5-H | 6-H | 7-H |

So, 21 results of tossing will give 5-heads.

If coin is tossed 8 times how many times 3 tails will appear?

Sol:-

8-coins tossed i.e. $n = 8$

The pascal triangle for $n=8$ is

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | $n=8$ |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | |
| 0-H | 1-H | 2-H | 3-H | 4-H | 5-H | 6-H | 7-H | 8-H | |
| 8-T | 7-T | 6-T | 5-T | 4-T | 3-T | 2-T | 1-T | 0-T | |

So, 56 results will give 3-tails.

14. Prove that $\binom{n}{0}+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+\dots+\frac{1}{n+1}\binom{n}{n}=\frac{2^{n+1}-1}{n+1}$

Solution:-

we know that,

$$(a+b)^n=\binom{n}{0}a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\dots+\binom{n}{n}b^n$$

Put $a=1, b=1, n=n+1$

$$\therefore (2)^{n+1}=\binom{n+1}{0}(1)^{n+1}(1)^0+\binom{n+1}{1}(1)^n(1)+\binom{n+1}{2}(1)^{n-1}(1)^2$$
$$+\binom{n+1}{3}(1)^{n-2}(1)^3+\dots+\binom{n+1}{n+1}(1)^{n+1}$$

$$2^{n+1}=1+(n+1)+\frac{(n+1)!}{(n+1-2)!\,2!}+\frac{(n+1)!}{(n+1-3)\,3!}+\dots+1$$

$$2^{n+1}-1=(n+1)+\frac{(n+1)!}{(n-1)!\,2!}+\frac{(n+1)!}{(n-2)!\,3!}+\dots+1$$

$$2^{n+1}-1=(n+1)+\frac{(n+1)n\cancel{(n-1)!}}{\cancel{(n-1)!}\,2!}+\frac{(n+1)n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,3!}+\dots$$

$$2^{n+1}-1=(n+1)+\frac{n(n+1)}{2}+\frac{n(n-1)(n+1)}{6}+\dots+1$$

Divide b/s by $(n+1)$

$$\frac{2^{n+1}-1}{n+1}=\frac{\cancel{n+1}}{\cancel{n+1}}+\frac{n\cancel{(n+1)}}{2}\cdot\frac{1}{\cancel{(n+1)}}+\frac{n(n-1)\cancel{(n+1)}}{6}\cdot\frac{1}{\cancel{(n+1)}}$$
$$+\dots+\frac{1}{n+1}$$

$$\frac{2^{n+1}-1}{n+1}=1+\frac{n}{2}+\frac{n(n-1)}{6}+\dots\frac{1}{n+1}$$

$$\frac{2^{n+1}-1}{n+1} = \binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{n+1}\binom{n}{n}$$

15. Prove that $\binom{n}{0} - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \cdots + \frac{(-1)^n}{n+1}\binom{n}{n} = \frac{1}{n+1}$

**Proof :-**

we know that

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n}b^n$$

Put $a = 1$, $b = -1$ and $n = n+1$

$$(1-1)^{n+1} = \binom{n+1}{0}(1)^{n+1} + \binom{n+1}{1}(1)^n(-1) + \binom{n+1}{2}(1)^{n-1}(-1)^2$$

$$+ \binom{n+1}{3}(-1)^{n-2}(-1)^3 + \cdots + \binom{n+1}{n+1}(-1)^{n+1}$$

$$0 = 1 + (n+1)(-1) + \frac{(n+1)!}{(n+1-2)!\,2!} + \frac{(n+1)!}{(n+1-3)!\,3!}(-1)$$

$$+ \cdots + \cancel{\binom{n+1}{n+1}(-1)^n(1)^n(-1)}$$

$$0 = -1 + (n+1) - \frac{(n+1)n\cancel{(n-1)!}}{\cancel{(n-1)!}\,2} + \frac{(n+1)n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\;3.2.1} - \cdots (-1)^n$$

$$1 = (n+1) - \frac{n(n+1)}{2} + \frac{n(n-1)(n+1)}{6} - \cdots + (-1)^n$$

Divide b/s by "$(n+1)$"

$$\therefore \frac{1}{n+1} = 1 - \frac{n}{2} + \frac{n(n-1)}{3\times 2} - \cdots + \frac{(-1)^n}{n+1}$$

$$\frac{1}{n+1} = \binom{n}{0} - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \cdots + \frac{(-1)^n}{n+1}\binom{n}{n}$$

16. $\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{2^2}\binom{n}{2} + \dots + \frac{1}{2^n}\binom{n}{n} = \left(\frac{3}{2}\right)^n$

**Proof:-**

We know that

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n}b^n$$

Put $a = 1$, $b = \frac{1}{2}$, $n = n$

$$\left(1+\frac{1}{2}\right)^n = \binom{n}{0}(1)^n + \binom{n}{1}(1)^{n-1}\left(\frac{1}{2}\right) + \binom{n}{2}(1)^{n-2}\left(\frac{1}{2}\right)^2$$
$$+ \binom{n}{3}(1)^{n-3}\left(\frac{1}{2}\right)^3 + \dots + \binom{n}{n}\left(\frac{1}{2}\right)^n$$

$$\left(\frac{3}{2}\right)^n = \binom{n}{0} + \binom{n}{1}\left(\frac{1}{2}\right) + \binom{n}{2}\left(\frac{1}{2^2}\right) + \dots + \binom{n}{n}\left(\frac{1}{2^n}\right)$$

$$\left(\frac{3}{2}\right)^n = \binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{2^2}\binom{n}{2} + \dots + \frac{1}{2^n}\binom{n}{n}$$

17. $\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}$

**Proof:-**

We know that,

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n}b^n$$

Put $a=1$, $n = 2n$

$$(1+b)^n = \binom{n}{0}(1)^n + \binom{n}{1}(1)^{n-1}(b) + \dots + \binom{n}{n}b^n$$

$$(1+b)^n = \binom{n}{0} + \binom{n}{1}b + \binom{n}{2}b^2 + \dots + \binom{n}{n}b^n$$

Multiply b/s by $(1+b)^n$.

$$(1+b)^{2n} = \left[\binom{n}{0} + \binom{n}{1} b + \dots + \binom{n}{n} b^n\right] \cdot \left[\binom{n}{n} b^n + \dots + \binom{n}{1} b + \binom{n}{0}\right]$$

$$(1+b)^{2n} = \binom{n}{0}\binom{n}{n} b^n + \binom{n}{1}\binom{n}{n-1} b^{1+n-1} + \dots + \binom{n}{n}\binom{n}{0} b^n]$$

$$(1+b)^{2n} = \left[\binom{n}{0}^2 + \binom{n}{1}^2 + \dots + \binom{n}{n}^2\right] b^n$$

Now consider, $(1+b)^{2n}$, find term $b^n$

As, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

for, $n = 2n$, $a = 1$

$$T_{r+1} = \binom{2n}{r} (1)^{2n-r} (b)^r$$

Put $r = n$.

$$\therefore T_{n+1} = \binom{2n}{n} (b)^n$$

$$T_{n+1} = b^n \binom{2n}{n}$$

$$\therefore \binom{2n}{n} \cancel{b^n} = \left[\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2\right] \cancel{b^n}$$

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2$$