Studybeesofficial

Where Minds Pollinate

Class 11
Mathematics

Chapter 6
Exercise 6.2
Notes

National Book Foundation

Ex 6.2

Permutation:- An ordering arrangements of 'n' objects

Repeated — non-repeated.

Example:- 1, 2, 3

Number of arrangements	e.g A B C
1, 2, 3	$\underset{3}{\text{H}}\ \underset{2}{\text{H}}\ \underset{1}{\text{H}}$
1, 3, 2	$= 3 \cdot 2 \cdot 1 = 6$
2, 3, 1	$= 3!$
2, 1, 3	
3, 1, 2	^nP_r — n → Total no, r → selected no
3, 2, 1	

= 6 (ways)

Q1 (i) $^nP_r = \dfrac{n!}{(n-r)!}$

1^{st} object = n choices

2^{nd} object = $(n-1)$ choices

3^{rd} object = $(n-2)$ choices

⋮ ⋮ ⋮

r^{th} objects = $n-(r-1)$ choices

$$^nP_r = n(n-1)(n-2)(n-3)\cdots\cdots(n-r+1)$$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)!}{(n-r)!}$$

$$^nP_r = \frac{n!}{(n-r)!}$$

Proved.

(ii) $^nP_n = {}^nP_{n-1}$

L.H.S $= \dfrac{n!}{(n-n)!}$

$= \dfrac{n!}{0!} = \dfrac{n!}{1} = n!$

R.H.S $= \dfrac{n!}{[n-(n-1)]!} = \dfrac{n!}{(\cancel{n}-\cancel{n}+1)!} = \dfrac{n!}{1} = n!$

L.H.S = R.H.S

(iii) $^nP_r = n.{}^{n-1}P_{r-1}$

Proof R.H.S

$\therefore {}^nP_r = \dfrac{n!}{(n-r)!}$

$\dfrac{n.(n-1)!}{[n-1-(r-1)]!}$

$\therefore$ $^{(n)}P_{(r)}$ -ve upper with lower

$= \dfrac{n!}{n-\cancel{1}-r+\cancel{1}} \Rightarrow \dfrac{n!}{n-r} = {}^nP_r$ L.H.S

(iv) $^nP_r = {}^{n-1}P_r + r.{}^{n-1}P_{r-1}$

Proof $= \dfrac{(n-1)!}{(n-1-r)!} + \dfrac{r.(n-1)!}{(n-\cancel{1}-r+\cancel{1})!}$

$= \dfrac{(n-1)!}{(n-r-1)!} + \dfrac{r(n-1)!}{(n-r)!}$

$= \dfrac{(n-1)!}{(n-r-1)!} + \dfrac{r(n-1)!}{(n-r)(n-r-1)!}$

$= \dfrac{(n-1)!}{(n-r-1)}\left[\dfrac{1}{1} + \dfrac{r}{n-r}\right]$

$$= \frac{(n-1)!}{(n-r-1)!}\left[\frac{n-r+r}{n-r}\right]$$

$$= \frac{n(n-1)!}{(n-r)(n-r-1)} = \frac{n!}{(n-r)!} = {}^nP_r = L.H.S.$$

(v) ${}^nP_n = 2.{}^nP_{n-2}$.

Proof

$$L.H.S = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \boxed{n!}$$

$$R.H.S = \frac{2.n!}{[n-(n-2)]!}$$

$$= \frac{2.n!}{(n-n+2)!}$$

$$= \frac{2.n!}{2!}$$

$$= \frac{\cancel{2}.n!}{\cancel{2}.1} \Rightarrow n! \quad L.H.S = R.H.S$$

Q2 (i) ${}^nP_4 = 20\,{}^nP_2$

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Soln

$$\frac{\cancel{n!}}{(n-4)!} = 20.\frac{\cancel{n!}}{(n-2)!}$$

$$\frac{1}{\cancel{(n-4)!}} = 20.\frac{1}{(n-2)(n-3)\cancel{(n-4)!}}$$

$$(n-2)(n-3) = 20.$$

$n^2-3n-2n+6\cancel{0}-20=0$

$n^2-5n-14=0$

$n^2-7n+2n-14=0$

$n(n-7)+2(n-7)=0$

$(n+2)(n-7)=0$

$n=-2$; $\boxed{n=7}$

not possible Ans

ii) ${}^{2n}P_3 = 100\,{}^{n}P_2$

Sol: $\frac{(2n)!}{(2n-3)!} = 100\cdot\frac{n!}{(n-2)!}$

$\Rightarrow \frac{2n(2n-1)(2n-2)\cancel{(2n-3)!}}{\cancel{(2n-3)!}}$

$= \frac{\cancel{100}^{50}\cdot n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$

$\Rightarrow (2n-1)\cancel{2(n-1)} = \cancel{50}^{25}\cancel{(n-1)}$

$2n-1=25$

$2n=26$

$\boxed{n=13}$ Answer

iii) $16\,{}^{n}P_3 = 13\,{}^{n+1}P_3$

Sol: $16\frac{n!}{(n-3)!} = \frac{13(n+1)!}{(n+1-3)!}$

$\frac{16n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = \frac{13(n+1)!}{(n-2)!}$

$\Rightarrow 16\cancel{n}\cancel{(n-1)}(n-2) = \frac{13(n+1)\cancel{(n)}\cancel{(n-1)}(n-2)!}{(n-2)!}$

$16(n-2)=13(n+1)$

$16n - 32 = 13n + 13$

$16n - 13n = 13 + 32$

$3n = 45$

$\boxed{n = 15}$ Answer

(vi) $^nP_5 = 20\,^nP_3$

$$\frac{\cancel{n!}}{(n-5)!} = 20 \cdot \frac{\cancel{n!}}{(n-3)!}$$

$$\frac{1}{\cancel{(n-5)!}} = 20 \cdot \frac{1}{(n-3)(n-4)\cancel{(n-5)!}}$$

$(n-3)(n-4) = 20$

$n^2 - 4n - 3n + 12 - 20 = 0$

$n^2 - 7n - 8 = 0$

$n^2 - 8n + n - 8 = 0$

$n(n-8) + 1(n-8) = 0$

$(n+1)(n-8) = 0$

$\boxed{n = -1}$ $\boxed{n = 8}$

not possible

(v) $30\,^nP_6 = \,^{n+2}P_7$

Sol:

$$30 \cdot \frac{n!}{(n-6)!} = \frac{(n+2)!}{(n+2-7)!}$$

$$\frac{30 \cdot \cancel{n!}}{\cancel{(n-6)!}} = \frac{(n+2)(n+1)\cancel{n!}}{(n-5)\cancel{(n-6)!}}$$

$30(n-5) = n^2 + n + 2n + 2n$

$n^2+3n+2-30n+150=0$

$n^2-27n+152=0$

$n^2-8n-19n+152=0$

$(n-19)(n-8)=0$

$\boxed{n=19}$ $\boxed{n=8}$

Answer.

(vi) ${}^{n}P_5 : {}^{n-1}P_4 = 6:1$.

$$\frac{\frac{n!}{(n-5)!}}{\frac{n-1}{(n-1-4)!}} = \frac{6}{1}$$

$$\frac{n!}{\cancel{(n-5)!}} \times \frac{\cancel{(n-5)!}}{(n-1)!} = 6$$

$$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = 6$$

$\boxed{n=6}$ Answer

(vii) ${}^{n}P_4 : {}^{n-1}P_3 = 9:1$.

Sol

$$\frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-3)!}} = \frac{9}{1}.$$

$$\frac{n!}{\cancel{(n-4)!}} \times \frac{\cancel{(n-4)!}}{(n-1)!} = 9$$

$$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = 9$$

$\boxed{n=9}$

viii $\quad {}^{n-1}P_3 : {}^{n+1}P_3 = 5 : 12.$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-2)!}{(n+1)!} = \frac{5}{12}$$

$$\Rightarrow \frac{(n-1)(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}} \times \frac{\cancel{(n-2)!}}{(n+1)(n)\cancel{(n-1)}\cancel{(n-2)}} = \frac{5}{12}$$

$$\frac{(n-2)(n-3)}{n^2+n} = \frac{5}{12}$$

$$12[n^2-3n-2n+6] = 5n^2+5n$$

$$12[n^2-5n+6] = 5n^2+5n$$

$$12n^2-60n+72-5n^2-5n=0$$

$$7n^2-65n+72=0$$

$$a=7,\ b=-65,\ c=72$$

$$n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$n = \frac{65 \pm \sqrt{(65)^2-4(7)(72)}}{2(7)}$$

$$n = \frac{65 \pm \sqrt{4225-2016}}{14}$$

$$n = \frac{65 \pm \sqrt{2209}}{14}$$

$$n = \frac{65 \pm 47}{14}$$

$$n = \frac{65+47}{14}, \quad n = \frac{65-47}{14}$$

$n = \frac{112}{14}$; $n = \frac{18}{14}$

$\boxed{n = 8}$, $n = \frac{9}{7} \approx 1.2$

Ans so it $\notin N$

(ix) ${}^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22:7$

Sol: $\frac{(2n-1)!}{(2n-1-n)!} : \frac{(2n+1)!}{(2n+1-n+1)!} = \frac{22}{7}$

$\frac{(2n-1)!}{(n-1)!} : \frac{(2n+1)!}{(n+2)!} = \frac{22}{7}$

$\frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)!}{(2n+1)!} = \frac{22}{7}$

$\frac{\cancel{(2n-1)!}}{\cancel{(n-1)!}} \times \frac{(n+2)(n+1)(\cancel{n})\cancel{(n-1)!}}{(2n+1)(2\cancel{n})\cancel{(2n-1)!}} = \frac{22}{7}$

$\frac{(n+2)(n+1)}{2(2n+1)} = \frac{22}{7}$

$7[n^2+n+2n+2] = 44[2n+1)$

$7n^2+21n+14-88n-44 = 0$

$7n^2-67n-30 = 0$

$a = 7$, $b = -67$, $c = -30$

$n = \frac{67 \pm \sqrt{4489 - 4(7)(-30)}}{2(7)}$

$n = \frac{67 \pm \sqrt{5329}}{14}$

$n = \frac{67 \pm 73}{14}$

$n = \frac{67+73}{14}$; $n = \frac{67-73}{14}$

$n = \frac{140}{14}$; $n = \frac{-6}{14}$

$\boxed{n = 10}$ Ans

not consider $\notin N$.

Q3) (i) $^6P_{r-1} = {}^5P_4$

Sol:-

$\frac{6!}{(6-r+1)!} = \frac{5!}{(5-4)!}$

$\frac{6!}{(7-r)!} = \frac{5!}{1!}$

$\frac{6 \cdot \cancel{5!}}{(7-r)!} = \frac{\cancel{5!}}{1!}$

$6 = (7-r)!$

$(3)(2)(1) = (7-r)!$

$3! = (7-r)!$

$3 = 7-r$

$\boxed{r = 4}$ Answer.

(ii) $^{10}P_r = 2 \cdot {}^9P_r$

$\frac{10!}{(10-r)!} = \frac{2 \cdot 9!}{(9-r)!}$

$\frac{10 \cdot 9!}{(10-r)\cancel{(9-r)}} = \frac{2 \times 9!}{\cancel{(9-r)!}}$

$$\frac{5}{10-r} = 1$$

$$r = 10-5$$

$$\boxed{r=5} A.$$

(iii) $${}^{15}P_r = 210$$

$$\frac{15!}{(15-r)!} = 15 \times 14$$

$$\frac{15!}{(15-r)!} = \frac{15 \times 14 \times 13!}{13!}$$

$$\frac{15!}{(15-r)!} = \frac{15!}{13!}$$

$$13! = (15-r)!$$

$$13 = 15-r$$

$$\boxed{r=2}$$

(iv) $${}^{10}P_r = 3\ {}^{10}P_{r-1}$$

$$\frac{10!}{(10-r)!} = \frac{3 \times 10!}{(10-r+1)!}$$

$$\frac{1}{(10-r)!} = \frac{3}{(11-r)!}$$

$$\frac{1}{\cancel{(10-r)!}} = \frac{3}{(11-r)\cancel{(10-r)!}}$$

$$11-r = 3$$

$$\boxed{r=8}$$

(v) $4 \cdot {}^6P_r = {}^6P_{r+1}$

Sol:

$$4 \cdot \frac{\cancel{6!}}{(6-r)!} = \frac{\cancel{6!}}{(6-r-1)!}$$

$$\frac{4}{(6-r)!} = \frac{1}{(5-r)!}$$

$$\frac{4}{(6-r)\cancel{(5-r)!}} = \frac{1}{\cancel{(5-r)!}}$$

$$4 = 6 - r$$

$$\boxed{r = 2}$$

(vi) $2 \cdot {}^6P_{r-1} = {}^5P_r$

Sol:

$$2 \cdot \frac{6!}{(6-r+1)!} = \frac{5!}{(5-r)!}$$

$$\frac{2 \cdot 6 \cdot \cancel{5!}}{(7-r)!} = \frac{\cancel{5!}}{(5-r)!}$$

$$\frac{12}{(7-r)(6-r)\cancel{(5-r)!}} = \frac{1}{\cancel{(5-r)!}}$$

$$12 = (7-r)(6-r)$$

$$12 = 42 - 7r - 6r + r^2$$

$$r^2 - 13r + 42 - 12 = 0$$

$$r^2 - 13r + 30 = 0$$

$$r^2 - 10r - 3r + 30 = 0$$

$$(r-10)(r-3) = 0$$

$r = 10$, $\boxed{r = 3}$

not consider because $r \not> 1$.

solution A

(viii) $^{54}P_{r+3} : {}^{56}P_{r+6} = 1 : 30800$

Sol:

$$\frac{54!}{(54-r-3)!} : \frac{56!}{(56-r-6)!} = \frac{1}{30800}$$

$$\frac{54!}{(51-r)!} : \frac{56.55.54!}{(50-r)!} = \frac{1}{30800}$$

$$\frac{1}{(51-r)\cancel{(50-r)!}} \times \frac{\cancel{(50-r)!}}{56.55} = \frac{1}{30800}$$

$$\frac{1}{51-r} \times \frac{1}{3080} = \frac{1}{30800}$$

$$\frac{1}{51-r} = \frac{308\cancel{0}}{3080\cancel{0}}$$

$$\frac{1}{51-r} = \frac{1}{10}$$

$10 = 51 - r$

$\boxed{r = 41}$ Answer

Q4) Solution:- Case 1 __ __ 2

Total no = ^nP_r — Total choices left $= \frac{n!}{(n-r)!}$

$= {}^5P_2 = \frac{5!}{(5-2)!} = \frac{5.4.3!}{3!} = \boxed{20}$

Case 2: __ , __ , 4

$^5P_2 = 20$

Method 2

or $^6P_3 = 120$

Case 3: __ , __ , 6

$^5P_2 = 20$

fixed no gets cut from $n \leq 8$

Total digits = $20+20+20 = \boxed{60}$ Answer

Q5: Sol:- $0, 1, 2 \ldots\ldots 9 \implies \boxed{n = 10}$

$\underline{5}, \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}$

$\Rightarrow {}^9P_6 = \frac{9!}{(9-6)!} = 60480$ Answer

Q6 Sol:-

$\underline{6}, \underline{6}, \underline{6}, \underline{6}$

$= 6 \times 6 \times 6 \times 6$ if repitition not allowed

$= \boxed{1296}$ A then ${}^6P_4 = 360$

Q7 Sol:- Repeated Permutation:-

$= \frac{n!}{(n_1)!(n_2)!}$ → Total no.s → Repeated numbers

$\underline{0}, \underset{Even}{\underline{\quad}}, \underline{0}, \underset{Even}{\underline{\quad}}, \underline{0}, \underset{Even}{\underline{\quad}}, \underline{0}$

For Even:- 2, 2, 4

$= \frac{3!}{2!1!} = \frac{3 \cdot \cancel{2} \cdot 1}{\cancel{2} \cdot 1} = \boxed{3}$

For odd arrangements:- 1, 1, 3, 3

$\frac{4!}{2!2!} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}2!} = \frac{12}{2} = \boxed{6}$

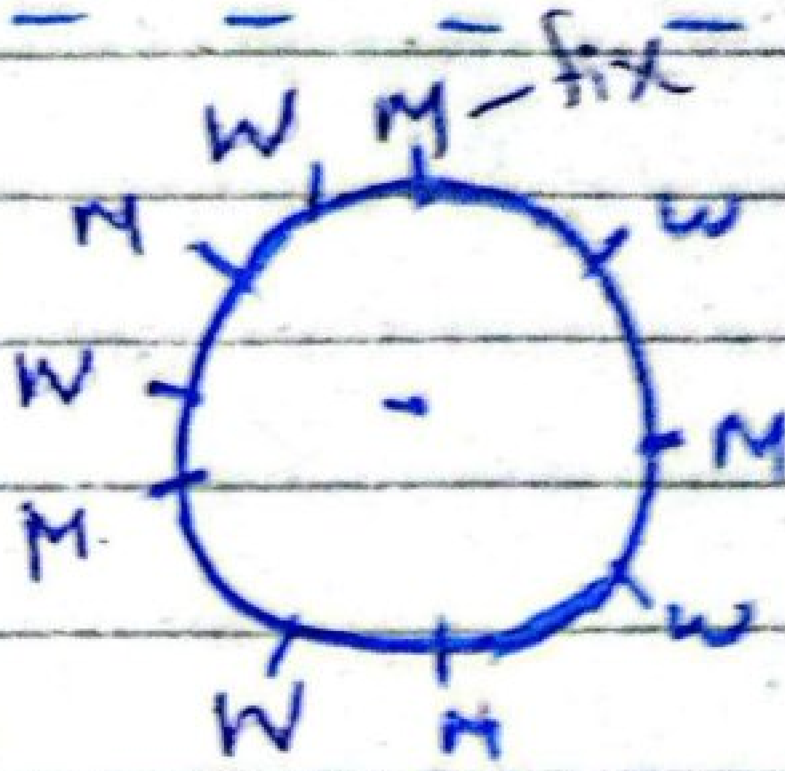
in Highway $\therefore \frac{}{2}$

now total arrangement

$= 3 \times 6 = \boxed{18}$

Q8:- for men.

$= {}^4P_4 = \frac{4!}{0!} = 4\cdot3\cdot2\cdot1! = \boxed{24}$



for women

$\therefore {}^5P_5 = \boxed{120}$

Total $= 24 \times 120$

ways $= 2880$ Ans

Q9 Sol Repeated Permutation

$= \frac{9!}{2!\cdot3!\cdot4!} = \frac{9\cdot8\cdot7\cdot6\cdot5\cdot4!}{2\cdot6\cdot4!} = \boxed{1260}$ Ans

Q10) Sol:- (a) Total no of words $= 6$

Total no of choices $= 6$ non repeated

${}^6P_6 = 6! = \boxed{720}$ Answer

(b) __ , __ , __ , __ , __ , F

${}^5P_5 = \boxed{120}$ Ans

Q11) Solution:- Repeated Permutation.

Total words $= 9$.

$S = 2$ $E = 1$

$T = 2$ $M = 1$

$A = 2$ $N = 1$

Repeated permutation

$= \frac{n!}{(n_1)!\,(n_2)!\,\ldots}$

$= \frac{9!}{2!\cdot 2!\cdot 2!} \Rightarrow \frac{362880}{8} = \boxed{45360}$ Answer.

IMP

Q12:- ___ , ___ , ___ , ___ , ___
odd , even , odd , even , odd

X wrong way

Vowel letters = O, E , Consonent letters = V, W, L

~~r>n so can't solve with this method.~~

Contrary Method

· let vowel words occupy even place .

vowel no of arrangements even $= {}^2P_2 = \boxed{2}$

no consonents words $= {}^3P_3 = 3! = \boxed{6}$.

Total arrangements vowel words occupy even places $= 2\times 6 = \boxed{12}$

Total arrangements : ${}^5P_5 = 5! = 120$

Now

total arrangements vowel words occupy odd places $= 120 - 12$

$= \boxed{108}$ Answer

Q13) Solution: Total arrangment = $^7P_7$ = 5040

vowels = AIE

consonants = MCHN.

Contrary method

Let vowels are together

= $\boxed{AIE}$, M, C, H, N

$^5P_5 = 5! = 120$

vowels letters arrangements = $^3P_3 = 3! = \boxed{6}$

Total way vowels are together

$= 120 \times 6 = \boxed{720}$

Now

Total ways vowels are never together

$= 5040 - 720 = \boxed{4320}$ Ans

Q14 Total words = n = 7

total choices = 3 = r

$^nP_r = {}^7P_3 = \frac{7!}{(7-3)!} = \boxed{210}$ Ans

Q15) Sol: Total arrangements.

same subjects.

$^3P_3 = 3! = 6$

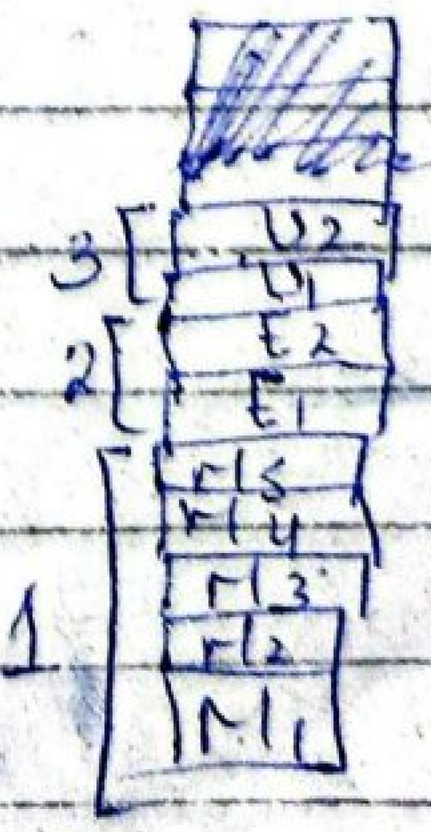
Math arrangments

$= {}^5P_5 = 5! = 120$

English arrangments

$= {}^3P_3 = 5! = 6$

123
132
213
231
312
321



Urdu arrangments = $^2P_2 = 2! = 2$

Total arrangments = $6 \times 120 \times 6 \times 2$

= $\boxed{8640}$ Answer

Q16) Sol: $\boxed{\text{Case 1}}$

__, __, __, __, __, 1

Arrangment $^5P_5 = 5! = \boxed{120}$

$\boxed{\text{Case 2}}$

__, __, __, __, __, 3

Arrangment $^5P_5 = 5! = 120$

First are independ on ev

$\boxed{\text{Case 3}}$

__, __, __, __, __, 5

Arrangment = $^5P_5 = 5! = \boxed{120}$

Total odd numbers = $120 + 120 + 120$

= $\boxed{360}$ Answer

add becaus they are independent of each other

Q17) Sol: Case 1

__, __, __, 1

Arrangments = $^4P_3 = \frac{4!}{1!} = \boxed{24}$

Case 2

__, __, __, 3

Arrangments = $^4P_3 = 24$

Case 3 __, __, __, 5

Five digits odd numbers

$\underline{0}, \_\_, \_\_, \_\_, \underline{3} \quad {}^3P_3 = 6$

$\underline{0}, \_\_, \_\_, \_\_, \underline{5} \quad {}^3P_3 = 6$ } $= 12$

Total odd no less than 10,000

$= 2 + 8 + 24 + 48 + 12 = \boxed{94}$ Answer

Q19) Sol:- No of circular permutation

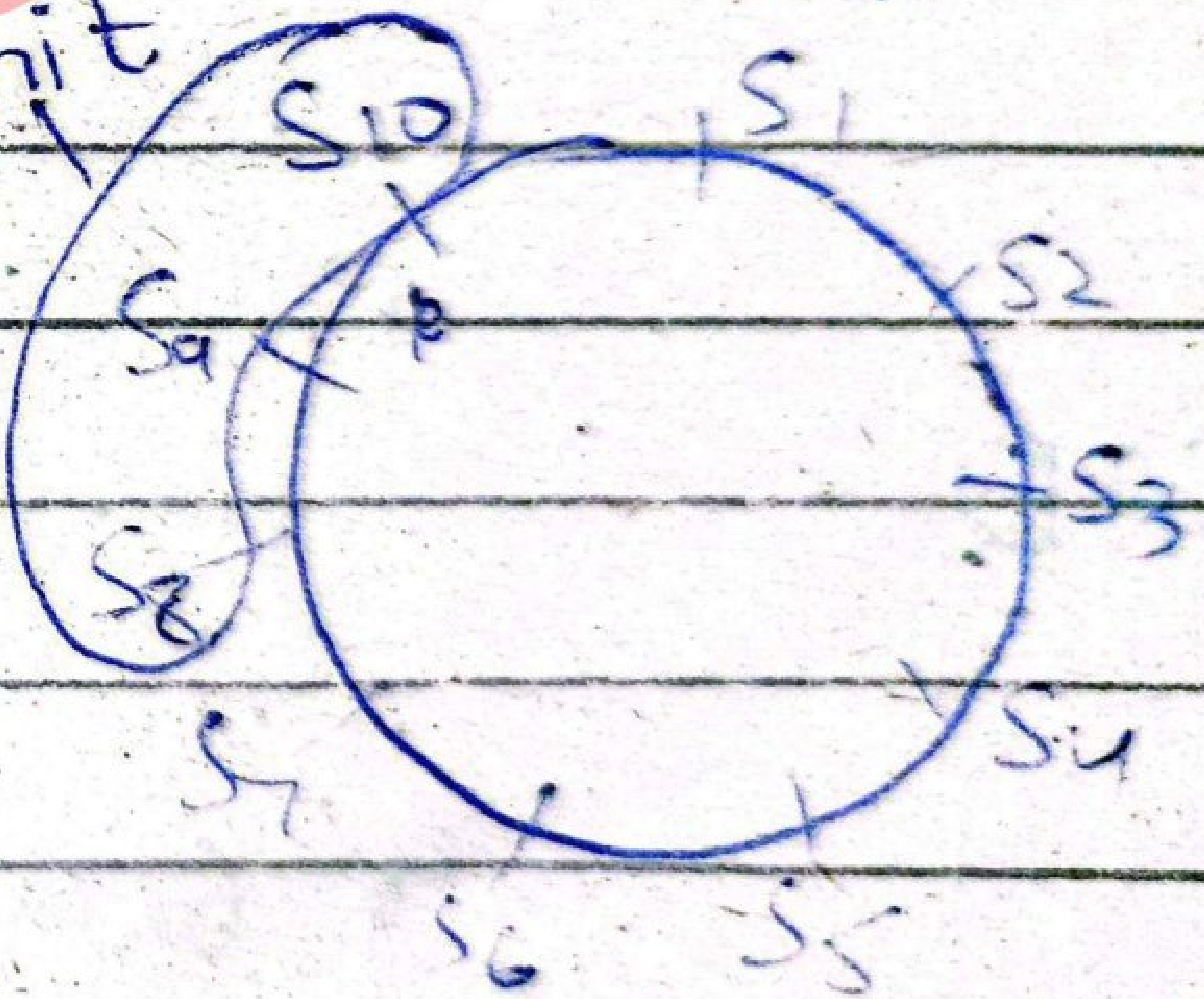
$(n-1)!$

$n = 8$

no of permutation

$= (8-1)! = 7!$

${}^3P_3 = \frac{3!}{0!} = 6$

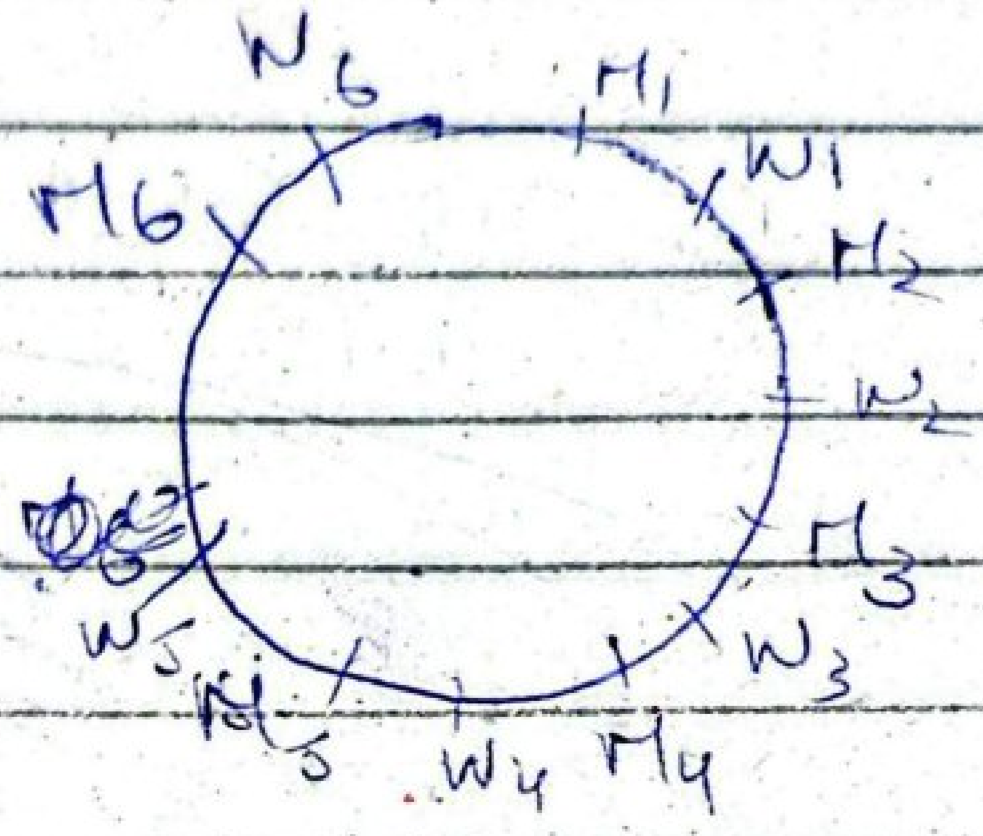


Total no of Permutation = $7! \times 6$

$= 5040 \times 6$

$= 30240$ A

Q20



• for Men arrangments

$= {}^5P_5 = 120$

• For Women arrangments

$= {}^6P_6 = 720$

Total arrangments $= 720 \times 120$

$= \boxed{86400}$ Ans

Q21) ① WHY

${}^3P_3 = 3! = 6$ (ways)

WHY
WYH
HWY
HYW
YWH
YHW Answers.

② SAD

SAD	ASD	DAS
SAD	ADS	DSA
SDA		

6ways Ans

Q21) (iii) TWO

$^3P_3 = 3! = 6$

TOW WTO OTW

TWO WOT OWT Answer

(ii) MADE

$^4P_4 = 4! = 24$

MADE AMDE DMAE EMAD

MAED AMED DMEA EMDA

MDAE ADME DAME EAMD

MDEA ADEM DAEM EADM

MEAD AEMD DEMA EDMA

MEDA AEDM DEAM EDAM

Q22:- Sol LAHORE

(1 2 3 4 5 6)

Encrypt (3 4 6 1 5 2)

= H O E L R A Answer

Q23:- Sol:- T N L U M A

Labelling (4 6 3 2 1 5)

Decrypt (1 2 3 4 5 6)

= M U L T A N