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Where Minds Pollinate

Class 11
Mathematics

Chapter 6
Exercise 6.1
Notes

National Book Foundation

Unit # 06

Permutation And Combination.

Exercise # 6.1

1. Evaluate the following.

i) $10!$

Sol:-

$$10! = 10.9.8.7.6.5.4.3.2.1$$
$$10! = 3,628,800$$

ii) $\dfrac{12!}{7!\,3!\,2!}$

Sol:-

$$\frac{12!}{7!\,3!\,2!} = \frac{12.11.10.9.8.\cancel{7!}}{\cancel{7!}\,3!\,2!}$$

$$\frac{12!}{7!\,3!\,2!} = \frac{95040}{(3.2.1)(2.1)}$$

$$\frac{12!}{7!\,3!\,2!} = \frac{95040}{12}$$

$$\frac{12!}{7!\,3!\,2!} = 7,920$$

iii) $\dfrac{4!-2!}{3!+5!}$

Sol:-

$$\frac{4!-2!}{3!+5!} = \frac{(4.3.2.1)-(2.1)}{(3.2.1)+(5.4.3.2.1)}$$

$$= \frac{24-2}{6+120}$$

$$= \frac{22}{126}$$

$$\frac{4!-2!}{3!+5!} = \frac{11}{63}$$

iv) $\dfrac{(n-1)!}{(n-2)!}$

Sol:-

$$\frac{(n-1)!}{(n-2)!} = \frac{(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$$

$$\frac{(n-1)!}{(n-2)!} = n-1$$

v) $\frac{8!}{(6!)^2}$

Sol:-

$$\frac{8!}{(6!)^2} = \frac{8 \cdot 7 \cdot \cancel{6!}}{(6!)^{\cancel{2}}}$$

$$\frac{8!}{(6!)^2} = \frac{56}{6!} \Rightarrow \frac{56}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$\frac{8!}{(6!)^2} = \frac{56}{720} \Rightarrow \frac{7}{90}$$

2.) Write the following in factorial form.

i) $14 \cdot 13 \cdot 12 \cdot 11$

Sol:-

$= 14 \cdot 13 \cdot 12 \cdot 11$

$$= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{10!}$$

$$= \frac{14!}{10!}$$

ii) $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9$

Sol:-

$= 1.3.5.7.9$

$= \frac{(1.3.5.7.9)(2.4.6.8)}{(2.4.6.8)}$

$= \frac{9.8.7.6.5.4.3.2.1}{2^4(1.2.3.4)}$

$= \frac{9!}{16.4!}$

iii) $n(n^2-1)$

Sol:-

$= n(n^2-1)$

$= n(n+1)(n-1)$

$= (n+1)(n)(n-1)$

$= \frac{(n+1)(n)(n-1)(n-2)!}{(n-2)!}$

$= \frac{(n+1)!}{(n-2)!}$

iv) $\dfrac{(n-3)(n-2)(n-1)}{n(n-4)}$

Sol:-

$$= \frac{(n-3)(n-2)(n-1)}{n(n-4)}$$

$$= \frac{(n-3)(n-2)(n-1)n!}{n(n-4)n!}$$

$$= \frac{(n-3)!}{n(n-4)n!}$$

3. Prove the following

i) $\dfrac{1}{5!} + \dfrac{3}{6!} + \dfrac{1}{7!} = \dfrac{4}{315}$

Proof:-

$$\text{L.H.S.} = \frac{1}{5!} + \frac{3}{6!} + \frac{1}{7!}$$

$$= \frac{1}{5.4.3.2.1} + \frac{3}{6.5.4.3.2.1} + \frac{1}{7.6.5.4.3.2.1}$$

$$= \frac{1}{120} + \frac{3}{720} + \frac{1}{5040}$$

$$= \frac{1}{120} + \frac{1}{240} + \frac{1}{5040}$$

$= \frac{70(1) + 120(1) + 2520(1)}{8900}$

$= \frac{2710}{8900}$

$= \frac{4}{315} \Rightarrow$ R.H.S

ii) $\frac{(n-1)!}{(n-3)!} = n^2 - 3n + 2$

Proof:-

L.H.S $\Rightarrow \frac{(n-1)!}{(n-3)!}$

$= \frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$

$= (n-1)(n-2)$

$= n^2 - 2n - n + 2$

$= n^2 - 3n + 2$

$=$ R.H.S.

4. Show that

i) $\frac{(2n)!}{n!} = 2^n (1.3.5 \ldots (2n-1))$

Proof:-

L.H.S. $\Rightarrow \dfrac{(2n)!}{n!}$

$$= \frac{(2n)(2n-1)(2n-2)(2n-3)\ldots 5.4.3.2.1}{n!}$$

$$= \frac{[2n(2n-2)\ldots 4.2][(2n-1)(2n-3)\ldots 5.3.1]}{n(n-1)(n-2)\ldots 3.2.1}$$

$$= \frac{2^n\cancel{[n(n-1)(n-2)\ldots 2.1]}[(2n-1)(2n-3)\ldots 5.3.1]}{\cancel{n(n-1)(n-2)\ldots 2.1}}$$

$$= 2^n[(2n-1)(2n-3)\ldots 5.3.1]$$

$$= 2^n[1.3.5\ldots(2n-1)]$$

= R.H.S.

ii) $\dfrac{(2n-1)!}{n!} = 2^{n-1}(1.3.5\ldots(2n-1))$

Proof:-

L.H.S. $\Rightarrow \dfrac{(2n-1)!}{n!}$

$$= \frac{(2n-1)(2n-2)(2n-3)(2n-4)\ldots 5.4.3.2.1}{n!}$$

$$= \frac{[(2n-2)(2n-4)\ldots 4.2][(2n-1)(2n-3)\ldots 5.3.1]}{n!}$$

$$= \frac{2^{n-1}[(n-1)(n-2)\ldots 2.1][(2n-1)(2n-3)\ldots 5.3.1]}{n!}$$

$$= \frac{2^{n-1}\cancel{(n!)}[(2n-1)(2n-3)\ldots 5.3.1]}{\cancel{n!}}$$

$$= 2^{n-1}[1.3.5\ldots(2n-1)]$$

$= R.H.S$

5. Find the value of n in the following.

i) $$\frac{n}{(n-4)!} = \frac{3.3!}{(n-3)!}$$

Sol:-

$$\frac{n}{(n-4)!} = \frac{3.3!}{(n-3)!}$$

$$n(n-3)! = 3.3!(n-4)!$$

$$n(n-3)\cancel{(n-4)!} = 3.3!\cancel{(n-4)!}$$

$$n(n-3) = 3[3.2.1]$$

$$n^2-3n = 3(6)$$

$$n^2-3n = 18$$

$$n^2-3n-18=0$$

$$n^2+3n-6n-18=0$$

$$n(n+3)-6(n+3)=0$$

$$(n+3)(n-6)=0$$

$\therefore \quad n+3=0 \quad$ OR $\quad n-6=0$

$n=-3 \quad$ OR $\quad n=6$ (n cannot be −ve for factorial)

$\therefore \quad n = 8$

ii) $\dfrac{n!}{(n-4)!} : \dfrac{(n-1)!}{(n-3)!} = 36:2$

Sol:-

$$\frac{n!}{(n-4)!} : \frac{(n-1)!}{(n-3)!} = 36:2$$

$$\frac{n!}{(n-4)!} \times \frac{(n-3)!}{(n-1)!} = \frac{36}{2}$$

$$\frac{n\cancel{(n-1)!}}{\cancel{(n-4)!}} \cdot \frac{(n-3)\cancel{(n-4)!}}{\cancel{(n-1)!}} = 18$$

$n(n-3) = 18$

$n^2 - 3n = 18$

$n^2 - 3n - 18 = 0$

$n^2 + 3n - 6n - 18 = 0$

$n(n+3) - 6(n+3) = 0$

$(n+3)(n-6) = 0$

$\therefore \quad n+3 = 0 \quad$ OR $\quad n-6=0$

$n = -3 \quad$ OR $\quad n = 6$

factorial is not defined for −ve no.

$\therefore \quad n = 6$

6. Prove the following for $n \in N$

i) $(2n)! = 2^n (n!)[1.3.5 \ldots (2n-1)]$

Proof:-

L.H.S $\Rightarrow$ $(2n)!$

As, $\dfrac{(2n)!}{n!} = 2^n[1.3.5 \ldots (2n-1)]$

$$(2n)! = 2^n(n!)[1.3.5 \ldots (2n-1)]$$

Hence proved.

ii) $(n+1)[n!\,n + (n-1)!(2n-1) + (n-2)!(n-1)] = (n+2)!$

Proof:-

$= (n+1)[n!\,n + (n-1)!(2n-1) + (n-2)!(n-1)!]$

Multiply and divide by n of 2^{nd} and 3^{rd} term in bracket

$$= (n+1)\left[n!\,n + \frac{(n)(n-1)!(2n-1)}{n} + \frac{n(n-1)(n-2)!}{n}\right]$$

$$= (n+1)\left[\cancel{n!\,n+}\ \frac{n!(2n-1)}{n} + \frac{n!}{n}\right]$$

$$= n!(n+1)\left[n + \frac{2n-1}{n} + \frac{1}{n}\right]$$

$$= (n+1)\, n! \left[\frac{n^2 + 2n - 1 + 1}{n} \right]$$

$$= (n+1)! \left[\frac{\cancel{n}(n+2)}{\cancel{n}} \right]$$

$$= (n+1)!\,(n+2)$$

$$= (n+2)(n+1)!$$

$$= (n+2)!$$

$= R.H.S$

iii) $$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!}$$

Proof:-

$L.H.S \Rightarrow$ $$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!(n-r+1) + n!(r)}{r(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n![n - \cancel{r} + 1 + \cancel{r}]}{r!(n-r+1)!}$$

$$= \frac{n!(n+1)}{r!(n-r+1)!}$$

$$= \frac{(n+1)!}{r!(n-r+1)!} \Rightarrow R.H.S$$

iv) $\frac{n!}{r!} = n(n-1)(n-2)\ldots(r+1)$

Proof:-

R.H.S $\Rightarrow n(n-1)(n-2)\ldots(r+1)$

Multiply and divide with $r!$

$$= \frac{n(n-1)(n-2)\ldots(r+1)r!}{r!}$$

$$= \frac{n!}{r!} \Rightarrow \text{R.H.S}$$

v) $(n-r+1) \cdot \frac{n!}{(n-r+1)!} = \frac{n!}{(n-r)!}$

Proof:-

L.H.S $\Rightarrow (n-r+1) \cdot \frac{n!}{(n-r+1)!}$

$$= \cancel{(n-r+1)} \cdot \frac{n!}{\cancel{(n-r+1)}(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

= R.H.S

vii) $\dfrac{2n!}{[(n-1)!]^2} = \dfrac{n(n+1)(n+2)\dots(2n-1)2n}{(n-1)!}$

Proof:-

R.H.S $\Rightarrow \dfrac{n(n+1)(n+2)\dots(2n-1)2n}{(n-1)!}$

$= \dfrac{2n(2n-1)\dots(n+2)(n+1)(n)}{(n-1)!}$

Multiply and divide with $(n-1)!$

$= \dfrac{2n(2n-1)\dots(n+2)(n+1)(n)(n-1)!}{(n-1)!(n-1)!}$

$= \dfrac{2n!}{[(n-1)!]^2} \Rightarrow$ L.H.S

ix) $(n!)^2 \leq n!\,n^n < (2n)!$

Proof:-

Consider,	Consider
$n \leq n$	$n < n+1$
$n-1 \leq n$	$n < n+2$
$n-2 \leq n$	$\vdots$
$\vdots \quad \vdots$	$n < n+n$
$2 \leq n$	OR $n < 2n$
$1 \leq n$	
$\Rightarrow n(n-1)(n-2)\dots 2\cdot 1 \leq n^n$	$\Rightarrow n^n < 2n(2n-1)\dots(n+2)(n+1)$
$n! \leq n^n$	Multiply by $n!$. we get, $n!\,n^n < (2n)!$

Multiply both sides with $n!$

$\therefore (n!)^2 \leq n^n \cdot n! < (2n)!$

vi) $33!$ is divisible by 2^{15}

Proof:-

$33! = 33 \cdot 32 \cdot 31 \cdot 30 \ldots 16 \cdot 15 \cdot 14 \ldots 3 \cdot 2 \cdot 1$

$33! = 33 \cdot 2^5 \cdot 31 \cdot 30 \ldots 2^4 \ldots 2^3 \ldots 2^2 \ldots 2^1 \cdot 1$

$33! = 2^{5+4+3+2+1}(33 \cdot 31 \cdot 30 \ldots 3 \cdot 1)$

$33! = 2^{15}(33 \cdot 31 \cdot 30 \ldots 3 \cdot 1)$

Hence, $33!$ is divisible by 2^{15}

viii) $(n!+1)$ is not divisible by any natural no. 2 and n.

Proof:-

$\because n! = n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1$

$\forall$ all positive natural numbers

$(n!+1)$ is not divisible by any natural no. between 2 and n because $x+y$ both are not divisible by between 2 and n.

7. Find n, if

xi) $\frac{1}{2(n-2)!} : \frac{1}{4!\,(n-4)!} = 2 \quad , \quad n \geq 4$

Sol:-

$$\frac{1}{2(n-2)!} : \frac{1}{4!(n-4)!} = 2$$

$$\frac{1}{2(n-2)!} \cdot \frac{4!(n-4)!}{1} = 2$$

$$\frac{1}{\cancel{2}(n-2)(n-3)\cancel{(n-4)!}} \cdot \frac{\overset{12}{\cancel{24}}\cancel{(n-4)!}}{1} = 2$$

$$\frac{12}{(n-2)(n-3)} = 2$$

$$\frac{12}{n^2-3n-2n+6} = 2$$

$$2n^2 - 10n + 12 = 12$$

$$n^2 - 5n + \cancel{6} = \cancel{6}$$

$$n^2 - 5n = 0$$

$$n(n-5) = 0$$

$\therefore \quad n = 0 \quad , \quad n - 5 = 0$

$\qquad\qquad\qquad\qquad n = 5$

As, $n \geq 4$

$\therefore \quad n = 5$

x) $\frac{(2n)!}{4!(2n-3)!} \cdot \frac{4!(n-4)!}{n!} = 52$

Sol:-

$$\frac{(2n)!}{\cancel{4!}(2n-3)!} \cdot \frac{\cancel{4!}(n-4)!}{n!} = 52$$

$$\frac{2n(2n-1)(2n-2)\cancel{(2n-3)!}}{\cancel{(2n-3)!}} \cdot \frac{\cancel{(n-4)!}}{n(n-1)(n-2)(n-3)(n-4)!} = 52$$

$$\frac{2\cancel{n}(2n-1)(2n-2)}{\cancel{n}(n-1)(n-2)(n-3)} = 52$$

$$\frac{2(2n-1)2\cancel{(n-1)}}{\cancel{(n-1)}(n-2)(n-3)} = 52$$

$$\frac{2 \cdot 2(2n-1)}{(n-2)(n-3)} = 52$$

$$\frac{2(4n-2)}{n^2-3n-2n+6} = 52$$

$$\cancel{2}\cdot\cancel{2}(2n-1) = \cancel{52}(n^2-5n+6)$$

$2n-1 = 13(n^2-5n+6)$

$2n-1 = 13n^2 - 65n + 78$

$13n^2 - 65n - 2n + 78 + 1 = 0$

$13n^2 - 67n + 79 = 0$

On solving we get

$n = 3.32$, $n = 1.826$

By the way this question wrong.

ix) $\frac{(n+2)!}{(2n-1)!} = \frac{(n+3)!}{(2n+1)!} \cdot \frac{72}{7}$.

Sol:-

$$\frac{(n+2)!}{(2n-1)!} = \frac{(n+3)!}{(2n+1)!} \cdot \frac{72}{7}$$

$$\frac{\cancel{(n+2)!}}{\cancel{(2n-1)!}} = \frac{(n+3)\cancel{(n+2)!}}{(2n+1)(2n)\cancel{(2n-1)!}} \cdot \frac{72}{7}$$

$$1 = \frac{n+3}{2n(2n+1)} \cdot \frac{72}{7}$$

$14n(2n+1) = 72(n+3)$

$28n^2 + 14n = 72n + 216$

$28n^2 + 14n - 72n - 216 = 0$

$28n^2 - 58n - 216 = 0$

OR $14n^2 - 29n - 108 = 0$

On solving it we get

$n = 4$, $n = \frac{-27}{14}$

A $n \in N$

$\therefore$ $n = 4$

viii) $(n+1)! = 6 \cdot (n-1)!$

Sol:-

$(n+1)! = 6 \cdot (n-1)!$

$(n+1)(n)\cancel{(n-1)!} = 6 \cdot \cancel{(n-1)!}$

$n(n+1) = 6(1)$

$n^2 + n = 6$

$n^2 + n - 6 = 0$

On solving it we get.

$n = 2 \quad , \quad n = -3$

As $n \in N$

$\therefore \quad n = 2$

viii) $n! = 990 \cdot (n-3)!$

Sol:-

$n! = 990 \cdot (n-3)!$

$n(n-1)(n-2)\cancel{(n-3)!} = 990 \cdot \cancel{(n-3)!}$

$n(n-1)(n-2) = 990(1)$

$n(n^2 - 3n + 2) = 990$

$n^3 - 3n^2 + 2n - 990 = 0$

Put $n = 11$

$\therefore \quad (11)^3 - 3(11)^2 + 2(11) - 990 = 0$

$990 - 990 = 0$

$0 = 0$

$\therefore \quad n = 11$

vi) $\frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$

Sol:-

$$\frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$$

$$\frac{1}{9!} + \frac{1}{10.9!} = \frac{n}{11!}$$

$$\frac{10+1}{10.9!} = \frac{n}{11!}$$

$$\frac{11}{10!} = \frac{n}{11.10!}$$

$$\frac{n}{11} = 11 \implies n = (11)^2$$

$$n = 121$$

v) $(n+2)! = 56 \cdot n!$

Sol:-

$$(n+2)! = 56 \cdot n!$$

$$(n+2)(n+1)\cancel{n!} = 56 \cdot \cancel{n!}$$

$$(n+2)(n+1) = 56(1)$$

$$n^2 + 3n + 2 = 56$$

$$n^2 + 3n + 2 - 56 = 0$$

$$n^2 + 3n - 54 = 0$$

on solving we get

$$n = 6 \quad , \quad n = -4$$

$$\therefore n = 6 \quad , \quad n \in N$$

iv) $(n+2)! = 132 \cdot n!$

Sol:-

$(n+2)! = 132 \cdot n!$

$(n+2)(n+1)\cancel{(n)!} = 132 \cdot \cancel{n!}$

$(n+2)(n+1) = 132$

$n^2 + 3n + 2 = 132$

$n^2 + 3n + 2 - 132 = 0$

$n^2 + 3n - 130 = 0$

On solving it we get

$n = 10 \quad , \quad n = -13$

~~As, $n \in N$~~

$\therefore \; n = 10$

iii) $(n+2)! = 60 \cdot (n-1)!$

Sol:-

$(n+2)! = 60 \cdot (n-1)!$

$(n+2)(n+1)(n)\cancel{(n-1)!} = 60 \cdot \cancel{(n-1)!}$

$(n+2)(n+1)n = 60$

$(n^2 + 3n + 2)n = 60$

$n^3 + 3n^2 + 2n - 60 = 0$

Put $n = 3$

$\therefore \; (3)^3 + 3(3)^2 + 2(3) - 60 = 0$

$60 - 60 = 0$

$0 = 0 \qquad \therefore n = 3$

ii) $\frac{n!}{(n-3)!} = 20 . \frac{n!}{(n-3)!}$ $\quad n \geq 5$

Sol:-

$$\frac{n!}{(n-5)!} = 20 . \frac{n!}{(n-3)!}$$

$$\frac{1}{\cancel{(n-5)!}} = \frac{20}{(n-3)(n-4)\cancel{(n-5)!}}$$

$$1 = \frac{20}{n^2 - 7n + 12}$$

$n^2 - 7n + 12 - 20 = 0$

$n^2 - 7n - 8 = 0$

on solving we get.

$n = -1, \quad n = 8$

as $n \geq 5$

$\therefore \quad n = 8$

i) $\frac{n!}{(n-2)!} = 930 \quad n \geq 2$

Sol:-

$$\frac{n!}{(n-2)!} = 930$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 930$$

$n(n-1) = 930$

$n^2 - n - 930 = 0$

on solving we get

$n = -31, \; n = 30$

as $n \geq 2$

$\therefore \; n = 30$