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Where Minds Pollinate

Class 11

Mathematics

Chapter 4

Review Exercise 4

Notes

National Book Foundation

Review Exercise 4

Q2) let the numbers are

$a-3d,\ a-d,\ a+d,\ a+3d$ in A.P

Given Condition

$a-3d+a-d+a+d+a+3d=24$.

$4a=24 \Rightarrow \boxed{a=6}$

2nd Condition:-

$(a-3d)(a+3d)(a-d)(a+d)=945$

$(a^2-9d^2)(a^2-d^2)=945$

let $d^2=x$

$[(6)^2-9x][6^2-x]=945$

$(36-9x)(36-x)=945$

$9(4-x)(36-x)=945$

$144-4x-36x+x^2=\frac{945}{9}$

$x^2-40x+144=105$

$x^2-40x+144-105=0$

$x^2-40x+39=0$

$x^2-x-39x+39=0$

$x(x-1)-39(x-1)=0$

$(x-39)(x-1)=0$

$x=39$; $x=1$

replace x with d^2

$d^2=39$, $d^2=1$

$d=\pm\sqrt{39}$, $d=1$

So neglect

$a-3d=6-3(1)=3$

$a-d=6-1=5$

$a+d=6+1=7$

$a+3d=6+3(1)=9$

Hence 3, 5, 7, 9

$a-3d$, $a-2d$, $a-d$, a, $a+d$, $a+2d$, $a+3d$
~~for odd~~ for even no

(3) $a-3d, a-d, a+d, a+3d$ in A.P

Sum; first Condition:-

$$a-\cancel{3d}+a-\cancel{d}+a+\cancel{d}+a+\cancel{3d}=6$$

$$4a=6 \qquad \boxed{a=3/2}$$

2nd Condition:-

$$(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=14$$

$$a^2+9d^2-\cancel{6ad}+a^2+d^2-\cancel{2ad}+a^2+d^2+\cancel{2ad}+a^2+9d^2+\cancel{6ad}=14$$

$$4a^2+20d^2=14 \Rightarrow 4\left(\frac{3}{2}\right)^2+20d^2=14$$

$$4\left(\frac{9}{4}\right)+20d^2=14$$

$$20d^2=14-9$$

$$d^2=\frac{5}{20} \Rightarrow \boxed{d=\frac{1}{2}}$$

$$a-3d=\frac{3}{2}-3\left(\frac{1}{2}\right)=0$$

$$a-d=\frac{3}{2}-\frac{1}{2}=\frac{3-1}{2}=\frac{2}{2}=1$$

$$a+d=\frac{3}{2}+\frac{1}{2}=\frac{3+1}{2}=\frac{4}{2}=2$$

$$a+3d=\frac{3}{2}+3\left(\frac{1}{2}\right)=\frac{3+3}{2}=\frac{6}{2}=3$$

Hence 0,1,2,3

Answer.

for odd no GP skip middle terms

④ $2, A_1, A_2, 6 \ldots A_{20}, 86$ in A.P

$a_1 = 2, \quad a_{22} = 86$

$a_{22} = a_1 + (22-1)d$

$86 = 2 + 21d \Rightarrow \boxed{d = 4}$

So $A_1 = a_2 = a_1 + d \Rightarrow A_1 = 2 + 4 = \boxed{6}$

$A_2 = a_3 = a_1 + 2d = 2 + 2(4) = \boxed{10}$

similarly

$\boxed{A_3 = 14}, \boxed{A_4} = 18 \ldots\ldots A_{20} = 82$

⑤ $S_n = 3 + 33 + 333 + \ldots n$ term — use method of difference

~~multiply & divide by 3~~

$S = 3[1 + 11 + 111 + \ldots n \text{ terms}]$ — $\times$ & divide by 9 always.

$S = \frac{3}{9}[9 + 99 + 999 + \ldots n \text{ terms}]$

$S = \frac{3}{9}((10-1) + (100-1) + (1000-1) + \ldots n \text{ terms})$

$S = 3/9 (10 + 10^2 + 10^3 + \ldots n) - (1 + 1 + 1 + \ldots n)$

$\Rightarrow S_n = \frac{1}{3}\left(\frac{10(10^n - 1)}{10-1} - n\right)$

Its G.P i.e
$a = 10$
$r = 10$
$S_n = \frac{a(r^n - 1)}{r-1}$

$S_n = \frac{1}{3}\left(\frac{10^{n+1} - 10}{9} - n\right)$

$S_n = \frac{1}{27}(10^{n+1} - 10 - 9n)$

3 terms

$\frac{a}{r^2}, \left(\frac{a}{r}, a, ar\right), ar^2$

⑥ Sol let $\frac{a_1}{r}, a_1, a_1 r$ are in G.P

1st Condition: $\left(\frac{a_1}{r}\right)(a_1)(a_1 r) = 216$

$a_1^3 = 6^3 \Rightarrow \boxed{a_1 = 6}$

2nd Condition

$\frac{a_1}{r} + a_1 + a_1 r = 19$

$\frac{6}{r} + 6 + 6r = 19$

$\frac{6}{r} + 6r = 19 - 6$

$\frac{6 + 6r^2}{r} = 13 \Rightarrow 6r^2 + 6 = 13r$

$6r^2 - 13r + 6 = 0$

36

$6r^2 - 9r - 4r + 6 = 0$

$3r(2r - 3) - 2(2r - 3) = 0$

$(3r - 2)(2r - 3) = 0$

$3r - 2 = 0 \qquad 2r - 3 = 0$

$\boxed{r = 2/3} \qquad \boxed{r = 3/2}$

$\frac{a_1}{r} = \frac{6}{2/3} = \boxed{9} \qquad \frac{a_1}{r} = \frac{6}{3/2} = \boxed{4}$

$\boxed{a_1 = 6} \qquad \boxed{a_1 = 6}$

$a_1 r = 6\left(\frac{2}{3}\right) \qquad \boxed{a_1 r = 9}$

$\boxed{r = 9}$

Q7 Sol: $2, G_1, G_2, G_3, G_4, 486$ in G.P

$a_1 = 2, \quad a_6 = 486$

$a_1 r^{6-1} = 486$

$2r^5 = 486 \Rightarrow r^5 = 243$

$r^5 = 3^5 \quad \boxed{r=3}$

$G_1 = a_2 = a_1 r = 2(3) = \boxed{6}$

$G_2 = a_3 = a_2 r = 6(3) = \boxed{18}$

$G_3 = a_4 = a_3 r = 18(3) = \boxed{54}$

$G_4 = a_5 = a_4 r = 54(3) = \boxed{162}$ Ans

Q8) Sol:- $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n} = \dfrac{2ab}{a+b}$

$H.M = \dfrac{2ab}{a+b}$

$(a+b)(a^{n+1}+b^{n+1}) = 2ab(a^n+b^n)$

$a^{n+2} + ab^{n+1} + ba^{n+1} + b^{n+2} = 2a^{n+1}b + 2ab^{n+1}$

$a^{n+2} + b^{n+2} = -ab^{n+1} - ba^{n+1} + 2a^{n+1}b + 2ab^{n+1}$

$a^{n+2} + b^{n+2} = ab^{n+1} + a^{n+1}b$

$a^{n+1+1} - a^{n+1}b = ab^{n+1} - b^{n+1+1}$

$a^{n+1}(\cancel{a-b}) = b^{n+1}(\cancel{a-b})$

$\dfrac{a^{n+1}}{b^{n+1}} = 1$

then $\left(\dfrac{a}{b}\right)^{n+1} = \left(\dfrac{a}{b}\right)^0$

$n+1=0 \qquad \boxed{p^0 = 1}$ key point

$\boxed{n=-1}$ Answer

Q9) $H_3 = \frac{6}{7}$ in H.P and $a_3 = \frac{7}{6}$ in A.P

$H_{14} = \frac{1}{3}$ in H.P & $a_{14} = \frac{3}{1}$ in A.P

In A.P { $a_n = a_1 + (n-1)d$

$a_3 = a_1 + (3-1)d$

$\frac{7}{6} = a_1 + 2d$ — (1)

$a_{14} = a_1 + (14-1)d$

$3 = a_1 + 13d$ — (2)

Solve (1) & (2)

$$\begin{aligned} a_1 + 2d &= 7/6 \\ \pm a_1 \pm 13d &= \pm 3 \\ \hline -11d &= \frac{7}{6} - 3 \end{aligned}$$

$-11d = -\frac{11}{6}$

$\boxed{d = \frac{1}{6}}$ put in (2)

$3 = a_1 + 13\left(\frac{1}{6}\right)$

$3 - \frac{13}{6} = a_1$

$\boxed{a_1 = \frac{5}{6}}$

$a_n = a_1 + (n-1)d$

$a_n = \frac{5}{6} + (n-1)\frac{1}{6}$

$a_n = \frac{1}{6}\left[5 + n - 1\right]$

$a_n = \frac{4+n}{6}$ in A.P

So $H_n = \frac{6}{4+n}$ in H.P A

Q10) (i) $\sum_{k=1}^{10} \frac{2k}{2k+1}$

$= \sum_{k=1}^{10} \frac{2k}{2k \cdot 2} = \sum_{k=1}^{10} \frac{1}{2} = \frac{1}{2} \sum_{k=1}^{10} (1)$

$= \frac{1}{2}(1+1+1+\dots+1)$ 10times

$= \frac{1}{2}(\cancel{10})^{5} = \boxed{5}$ Answer.

(ii) $\sum_{k=1}^{8} (-1)^{k+1} 3^k$

$= (-1)^2 \cdot 3^1 + (-1)^3(3)^2 + (-1)^4(3)^3 + (-1)^5(3)^5 + (-1)^6(3)^5 + (-1)^7(3)^6 + (-1)^8(3)^7 + (-1)^9(3)^8$

$= 3 - 3^2 + 3^3 - 3^4 + 3^5 - 3^6 + 3^7 - 3^8$

$= (3 + 3^3 + 3^5 + 3^7) - (3^2 + 3^4 + 3^6 + 3^8)$

$= 3(1 + 3^2 + 3^4 + 3^6) - 3^2(1 + 3^2 + 3^4 + 3^6) \dots$

$= (3 - 3^2)(1 + 3^2 + 3^4 + 3^6)$

$= (3-9)(1+9+81+729)$

$= -6(820) = -4920$ A.

Q11) Sol $4 + 14 + 30 + 52 + 80 + \dots$

$(1\times4) + (2\times7) + (3\times10) + (4\times13) + (5\times16) + \dots$

$\boxed{T_K = k(3K+1)}$

Apply sum on b/s

$\sum_{k=1}^{n} T_K = 3\sum_{k=1}^{n} K^2 + \sum_{k=1}^{n} K$

$S_n = 3\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{n(n+1)}{2}$

Taking common $\frac{n(n+1)}{2}$

$S_n = \frac{n(n+1)}{2}\left[2n+1+1\right]^2$

$= \frac{n(n+1)}{2}(2n+2)$

$= S_n = \frac{n(n+1)}{2} \cdot 2(n+1)$

$S_n = n(n+1)^2$ Ans

Q12) $1+4+10+21+39+\ldots n$

$T_n = 1+(4-1)+(10-4)+(21-10)+(39-21)+ \ldots$ n term.

$T_n = 1+[3+6+11+18 \ldots (n-1)\text{term}]$ — (1)

now consider

$3+6+11+18+ \ldots (n-1)$ term

now apply difference

$T_{n-1} = 3+(6-3)+(11-6)+(18-11)+ \ldots (n-1)$ term

$T_{n-1} = 3+[3+5+7+ \ldots (n-2)\text{term}]$ — (2)

now it is A.P

$a_1 = 3, d = 2, n = n-2.$

Sum of n-terms in A.P

$S_n = \frac{n}{2}[2a_1+(n-1)d]$

from (2)

$T_{n-1} = 3+\left[\frac{n-2}{2}[2(3)+(n-2-1)2]\right]$

$T_{n-1} = 3+\left[\frac{n-2}{2} \cdot 2[3+n-3]\right]$

$T_{n-1} = 3+[(n-2)n]$

$T_{n-1} = n^2 - 2n + 3$

$$\boxed{T_{n-1} = n^2 - 2n + 3}$$

now apply sum for eq (2)

$$\sum_{n=2}^{n} T_{n-1} = \sum_{k=2}^{n} k^2 - 2\sum_{k=2}^{n} k + 3\sum_{k=2}^{n} 1$$

$$S_{n-1} = \left[\frac{n(n+1)(2n+1)}{6} - 1\right] - 2\left[\frac{n(n+1)}{2} - 1\right] + 3(n-1)$$

$$S_{n-1} = \frac{n(n+1)(2n+1)}{6} - 2\left[\frac{n(n+1)}{2}\right] + 3n - 3 + 2 - 1$$

$$S_{n-1} = \frac{n}{6}\left[2n^2 + 3n + 1 - 6n - 6 + 18\right] - 2$$

$$S_{n-1} = \frac{n}{6}\left[2n^2 - 3n + 13\right] - 2 \quad \text{put in (1)}$$

$$T_n = 1 + \frac{n}{6}\left(2n^2 - 3n + 13\right) - 2$$

$$T_n = \frac{n}{6}\left(2n^2 - 3n + 13\right) - 1$$

$$T_n = \frac{2n^3 - 3n^2 + 13n - 6}{6}$$

Now

$$T_n = \frac{2n^3 - 3n^2 + 13n - 6}{6}$$

Apply Σ on b/s

$$\sum_{k=1}^{n} T_k = \frac{1}{6}\left[2\sum_{k=1}^{n} k^3 - 3\sum_{k=1}^{n} k^2 + 13\sum_{n=1}^{n} k - 6\sum_{k=1}^{n} 1\right.$$

$$S_n = \frac{1}{6}\left[2\left(\frac{n(n+1)}{2}\right)^2 - 3\left[\frac{n(n+1)(2n+1)}{6}\right] + 13\left(\frac{n(n+1)}{2}\right)\right] - 6n$$

$$S_n = \frac{1}{6}\left[\left\{\frac{2(n(n+1))^2}{4}\right\} - \frac{n(n+1)(2n+1)}{2} + 13\left\{\frac{n(n+1)}{2}\right\}\right] - 6n$$

$$S_n = \frac{1}{6}\left[\frac{n(n+1)}{2}\left[n(n+1) - (2n+1) + 13\right]\right] - 6n$$

$$S_n = \frac{n(n+1)}{12}\left[n^2 + n - 2n - 1 + 13\right] - \frac{6n}{6}$$

$$S_n = \frac{n(n+1)}{12}\left[n^2 - n + 12\right] - n\left(\frac{12}{12}\right)$$

$$S_n = \frac{n}{12}\left[n^3 - n^2 + 12n + n^2 - n + 12 - 12\right]$$

$$S_n = \frac{n}{12}\left[n^3 + 11n\right]$$

$$S_n = \frac{n^4 + 11n^2}{12}$$