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Where Minds Pollinate

Class 11
Mathematics

Chapter 4
Exercise 4.8
Notes

National Book Foundation

Exercise # 4.8

Using method of difference, find the sum of the following series.

1. $3+7+13+21+\dots$ to n-terms

Sol:-

$$\underset{T_1}{3}+\underset{T_2}{7}+\underset{T_3}{13}+\underset{T_4}{21}+\dots \text{ to } \underset{T_n}{n\text{th term}}$$

So,
$$T_2 - T_1 = 7 - 3 = 4$$
$$T_3 - T_2 = 13 - 7 = 6$$
$$T_4 - T_3 = 21 - 13 = 8$$
$$\vdots$$
$$T_n - T_{n-1} =$$

Adding all theses, we get

ii $(T_2 - T_1) + (T_3 - T_2) + (T_4 - T_3) + \dots + (T_n - T_{n-1}) = 4 + 6 + 8 + \dots (n-1)$ terms

$\cancel{T_2} - T_1 + \cancel{T_3} - \cancel{T_2} + \cancel{T_4} - \cancel{T_3} + \dots + \cancel{T_n} - T_{n-1} = 4 + 6 + 8 + \dots + (n-1)$ terms

$\therefore T_{n-1} - T_1 = 4 + 6 + 8 + \dots (n-1)$ terms

It is an arithmetic series.

$$\therefore S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_{n-1} = \frac{n-1}{2}[2(4) + (n-1-1)(2)]$$

$$= \frac{n-1}{2}[8 + 2n - 4]$$

$$= \frac{n-1}{\cancel{2}}(n+2)\cancel{2}$$

$$S_{n-1} = (n-1)(n+2)$$

iv $$\therefore T_{n-1} - T_1 = (n-1)(n+2)$$

$$T_{n-1} = n^2 + n - 2 + T_1$$

$$= n^2 + n - 2 + 3$$

$$T_{n-1} = n^2 + n + 1$$

$$\therefore S_n = n^2 + n + 1$$

2. $1 + 4 + 10 + 22 + \dots$ to n-terms

Sol:-

$S_n = 1 + 4 + 10 + 22 + \dots + T_n \quad -(i)$

OR $S_n = 1 + 4 + 10 + 22 + \dots + T_{n-1} + T_n \quad -(ii)$

$T_2 - T_1 = 4 - 1 = 3$

$T_3 - T_2 = 10 - 4 = 6$

$T_4 - T_3 = 22 - 10 = 12$

$\vdots$

$T_n - T_{n-1} = ?$

Add all these we get

$(T_2 - T_1) + (T_3 - T_2) + (T_4 - T_3) + \dots + (T_n - T_{n-1}) = 3 + 6 + 12 + \dots + (n-1)$ terms

$T_2 - T_1 + \cancel{T_3} - \cancel{T_2} + T_4 - \cancel{T_3} + \dots + T_n - T_{n-1} = 3 + 6 + 12 + \dots$ $(n-1)$ terms

$-T_1 + T_n = 3 + 6 + 12 + \dots (n-1)$ terms

It is a geometric ~~p~~ series.

$$\therefore S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_{n-1} = \frac{3(1 - 2^{n-1})}{1 - 2}$$

$$S_{n-1} = \frac{3 - 3 \cdot 2^{n-1}}{-1}$$

$$S_{n-1} = 3 \cdot 2^{n-1} - 3$$

$$\therefore -T_1 + T_n = 3 \cdot 2^{n-1} - 3$$

$$T_n = 3 \cdot 2^{n-1} - 3 + T_1$$

$$T_n = 3 \cdot 2^{n-1} - 3 + 1$$

$$T_n = 3 \cdot 2^{n-1} - 2$$

$$\therefore S_n = \sum_{k=1}^{n} T_k$$

$$S_n = \sum_{k=1}^{n} (3 \cdot 2^{n-1} - 2)$$

$$S_n = 3\sum_{k=1}^{n} 2^{n-1} - \sum_{k=1}^{n} 2$$

$$= 3\left[2^0 + 2^1 + 2^2 + \ldots 2^{n-1}\right] - 2n$$

$$= 3\left[\frac{1(1-2^n)}{1-2}\right] - 2n$$

$$= 3\left[\frac{1-2^n}{-1}\right] - 2n$$

$$S_n = 3(2^n - 1) - 2n$$

$$S_n = 3\cdot 2^n - 3 - 2n$$

5. $1+4+13+40+121+\ldots$ to n-terms.

Sol:-

$$\underset{T_1}{1} + \underset{T_2}{4} + \underset{T_3}{13} + \underset{T_4}{40} + \underset{T_5}{121} + \ldots \text{ to } \underset{T_n}{n\text{-terms}}$$

$$T_2 - T_1 = 4 - 1 = 3$$
$$T_3 - T_2 = 13 - 4 = 9$$
$$T_4 - T_3 = 40 - 13 = 27$$
$$T_5 - T_4 = 121 - 40 = 81$$
$$\vdots$$
$$T_n - T_{n-1}$$

Add all these, we get

$$(T_2 - T_1) + (T_3 - T_2) + (T_4 - T_3) + (T_5 - T_4) + \ldots + (T_n - T_{n-1}) = 3 + 9 + 27 + 81 + \ldots \text{ to } (n-1) \text{ terms}$$

$$\cancel{T_2} - T_1 + \cancel{T_3} - \cancel{T_2} + \cancel{T_4} - \cancel{T_3} + \cancel{T_5} - \cancel{T_4} + \ldots + T_n - \cancel{T_{n-1}} = 3 + 9 + 27 + 81 + \ldots \text{ to } (n-1) \text{ terms}$$

$\therefore \quad -T_1 + T_n = 3 + 9 + 27 + 81 + \dots$ to $(n-1)$ terms

It is a geometric series

$$\therefore \quad S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{n-1} = \frac{3(1-3^{n-1})}{1-3}$$

$$= \frac{3-3\cdot 3^{n-1}}{-2}$$

$$S_{n-1} = \frac{(3\cdot 3^{n-1}-3)}{2}$$

$$\therefore \quad -T_1 + T_n = (3\cdot 3^{n-1} - 3) \div 2$$

$$T_n = \frac{3\cdot 3^{n-1}-3}{2} + 1$$

$$T_n = \frac{3\cdot 3^{n-1}-3+2}{2}$$

$$T_n = \frac{3\cdot 3^{n-1}-1}{2}$$

Replace "n" with "k"

$$T_k = \frac{3\cdot 3^{k-1}-1}{2}$$

As $S_n = \sum_{k=1}^{n} T_k$

$$S_n = \sum_{k=1}^{n} \left(\frac{3\cdot 3^{k-1}-1}{2}\right)$$

$$= \frac{1}{2}\sum_{k=1}^{n} 3\cdot 3^{k-1} - \frac{1}{2}\sum_{k=1}^{n} 1$$

$$S_n = \frac{3}{2}\sum_{k=1}^{n} 3^{k-1} - \frac{n}{2}$$

$$S_n = \frac{3}{2}\left[3^0 + 3^1 + 3^2 + \dots + 3^{n-1}\right) - \frac{n}{2}$$

$$S_n = \frac{3}{2}\left[\frac{1(3^n - 1)}{3-1}\right] - \frac{n}{2}$$

$$S_n = \frac{3}{2}\left[\frac{3^n - 1}{2}\right] - \frac{n}{2}$$

$$S_n = \frac{3 \cdot 3^n - 3}{4} - \frac{n}{2}$$

$$= \frac{3 \cdot 3^n - 3 - 2n}{4}$$

$$S_n = \frac{3 \cdot 3^n - 2n - 3}{4}$$

4. $1 + 2 + 4 + 7 + 11 + 16 + \ldots\ n$ - terms

Sol:-

$$\underset{T_1}{1} + \underset{T_2}{2} + \underset{T_3}{4} + \underset{T_4}{7} + \underset{T_5}{11} + \underset{T_6}{16} + \ldots \text{ to } \underset{T_n}{n} \text{ terms}$$

$T_2 - T_1 = 2 - 1 = 1$

$T_3 - T_2 = 4 - 2 = 2$

$T_4 - T_3 = 7 - 4 = 3$

$T_5 - T_4 = 11 - 7 = 4$

$\vdots$

$T_n - T_{n-1}$

Now add a all these, we get,

$(T_2 - T_1) + (T_3 - T_2) + (T_4 - T_3) + \ldots + (T_n - T_{n-1}) = 1 + 2 + 3 + 4 + \ldots (n-1)$ terms

$\cancel{T_2} - T_1 + \cancel{T_3} - \cancel{T_2} + \cancel{T_4} - \cancel{T_3} + \ldots + T_n - \cancel{T_{n-1}} = 1 + 2 + 3 + 4 + \ldots (n-1)$ terms

$-T_1 + T_n = 1 + 2 + 3 + 4 + \ldots (n-1)$ - terms

It is an arithmetic series

$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{n-1} = \frac{n-1}{2}\left[2(1) + (n-1-1)(1)\right]$

$S_{n-1} = \frac{n-1}{2}\left[n\right] \Rightarrow S_{n-1} = \frac{n(n-1)}{2}$

$\therefore$ Sum of $-T_1$ and T_n is $n(n-1)/2$

So. $-T_1 + T_n = \frac{n(n-1)}{2}$

$T_n = n(n-1)/2 + T_1$

$T_n = \frac{n(n-1)}{2} + 1$

$T_n = n^2 - n + 2 \,/2$

Replace n with k

$\therefore T_k = k^2 - k + 2/2$

As, $S_n = \sum_{k=1}^{n} T_k$

$S_n = \sum_{k=1}^{n}\left(\frac{k^2 - k + 2}{2}\right)$

$= \frac{1}{2}\sum_{k=1}^{n} k^2 - \frac{1}{2}\sum_{k=1}^{n} k + \frac{1}{2}\sum_{k=1}^{n} 2$

$S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right] - \frac{1}{2}\left[\frac{n(n+1)}{2}\right] + 2n\left(\frac{1}{2}\right)$

$S_n = \frac{n}{6}\left[n^2 + 5\right]$

5. $1 + 4 + 8 + 14 + 24 + 42 + 76 + \ldots$ n-term

Sol:-

$1 + 4 + 8 + 14 + 24 + \ldots$ n-terms

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad \ldots \quad T_n$

$T_2 - T_1 = 4 - 1$

$T_3 - T_2 = 8 -$

5. $3+4+6+10+18+34+66+\ldots$ n-terms

Sol:-

$= 3+4+6+10+18+\ldots$ n-terms

$\downarrow T_1 \quad \downarrow T_2 \quad \downarrow T_3 \quad \downarrow T_4 \quad \downarrow T_5 \quad \ldots \quad \downarrow T_n$

$T_2 - T_1 = 4-3 = 1$

$T_3 - T_2 = 6-4 = 2$

$T_4 - T_3 = 10-6 = 4$

$T_5 - T_4 = 18-10 = 8$

$\vdots$

$T_n - T_{n-1}$

Add all these, we get

$(T_2 - T_1) + (T_3 - T_2) + (T_4 - T_3) + \ldots + (T_n - T_{n-1}) = 1+2+4+8+\ldots$ (n-1)-te

$\cancel{T_2} - T_1 + \cancel{T_3} - \cancel{T_2} + \cancel{T_4} - \cancel{T_3} + \ldots + T_n - \cancel{T_{n-1}} = 1+2+4+8+\ldots$ (n-1)-terms

$-T_1 + T_n = 1+2+4+8+\ldots$ (n-1)-terms

It is a geometric series

$\therefore S_n = \frac{a_1(r^n - 1)}{r-1}$

$S_{n-1} = \frac{1(2^{n-1} - 1)}{2-1}$

$S_{n-1} = 2^{n-1} - 1$

So, $-T_1 + T_n = 2^{n-1} - 1$

$T_n = 2^{n-1} - 1 + 3$

$T_n = 2^{n-1} + 2$

Replace : n with k

$T_k = 2^{k-1} + 2$

As, $S_n = \sum_{k=1}^{n} T_k$

$= \sum_{k=1}^{n} (2^{k-1}+2)$

$S_n = \sum_{k=1}^{n} 2^k \cdot 2^{-1} + \sum_{k=1}^{n} 2$

$S_n = 2^{-1}\sum_{k=1}^{n} 2^k + 2n$

$= 2^{-1}\left[2^1+2^2+2^3+\dots+2^n\right]+2n$

$= \frac{1}{2}\left[\frac{2(2^n-1)}{2-1}\right]+2n$

$S_n = \frac{1}{\cancel{2}}\left[\cancel{2}(2^n-1)\right]+2n$

$S_n = 2^n + 2n - 1$

Find sum of n-terms of series.

7. $\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\dots$

Sol:-

$= \frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\dots$ n-terms

let two Harmonic progressions

$\frac{1}{1}, \frac{1}{4}, \frac{1}{7}, \dots$ n-terms	$\frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \dots$ n-terms
As, $\frac{1}{a_n} = \frac{1}{a_1+(n-1)d}$	As, $\frac{1}{a_n} = \frac{1}{a_1+(n-1)d}$
$\frac{1}{a_n} = \frac{1}{1+(n-1)(3)}$	$\frac{1}{a_n} = \frac{1}{4+(n-1)(3)}$

$$\frac{1}{a_n} = \frac{1}{1+3n-3}$$

$$\frac{1}{a_n} = \frac{1}{3n-2}$$

$$\frac{1}{a_n} = \frac{1}{4+3n-3}$$

$$\frac{1}{a_n} = \frac{1}{3n+1}$$

$\therefore$ General term of given series is

$$T_n = \frac{1}{(3n-2)(3n+1)}$$

Replace "n" with "k"

$$\therefore \quad T_k = \frac{1}{(3k-2)(3k+1)}$$

Let $$\frac{1}{(3k-2)(3k+1)} = \frac{A}{3k-2} + \frac{B}{3k+1}$$

OR $1 = A(3k+1) + B(3k-2)$

Put $k = -1/3$ and $k = 2/3$

when $k = -1/3$

$$1 = A(0) + B\left[3(-1/3)-2\right]$$
$$1 = B(-3)$$
$$B = \frac{-1}{3}$$

$k = 2/3$

$$1 = A\left[3(2/3)+1\right] + B(0)$$
$$1 = A(3)$$
$$A = \frac{1}{3}$$

$\therefore \quad T_k = \dfrac{+1/3}{3k-2} + \dfrac{-1/3}{3k+1}$

$T_k = \dfrac{1}{3(3k-2)} - \dfrac{1}{3(3k+1)}$

As, $\quad S_n = \displaystyle\sum_{k=1}^{n} T_k$

$S_n = \displaystyle\sum_{k=1}^{n} \left(\frac{1}{3(3k-2)} - \frac{1}{3(3k+1)} \right)$

$S_n = \dfrac{1}{3}\displaystyle\sum_{k=1}^{n} \left(\frac{1}{(3k-2)} \right) - \frac{1}{3} \sum_{k=1}^{n} \left(\frac{1}{(3k+1)} \right)$

$S_n = \dfrac{1}{3}\displaystyle\sum_{k=1}^{n} \left(\frac{1}{3k-2} \right) - \frac{1}{3} \sum_{k=1}^{n} \left(\frac{1}{3k+1} \right)$

$S_n = \dfrac{1}{3}\left[\left(\dfrac{1}{1} + \dfrac{1}{4} + \dfrac{1}{7} + \dots + \dfrac{1}{3n-2} \right) - \left(\dfrac{1}{4} + \dfrac{1}{7} + \dfrac{1}{10} + \dots + \dfrac{1}{3n+1} \right) \right]$

$S_n = \dfrac{1}{3}\left[\left(\dfrac{1}{1} - \cancel{\dfrac{1}{4}} \right) + \left(\cancel{\dfrac{1}{4}} - \cancel{\dfrac{1}{7}} \right) + \dots + \left(\dfrac{1}{3n-2} - \dfrac{1}{3n+1} \right) \right]$

$S_n = \dfrac{1}{3}\left[1 - \dfrac{1}{3n+1} \right]$

$S_n = \dfrac{3}{3n+1}$

8. $\dfrac{1}{1\times 6} + \dfrac{1}{6\times 11} + \dfrac{1}{11\times 16} + \dots$

Sol:-

$= \dfrac{1}{1\times 6} + \dfrac{1}{6\times 11} + \dfrac{1}{11\times 16} + \dots$ n-terms let two harmonic progressions

$\frac{1}{1}, \frac{1}{6}, \frac{1}{11}, \ldots$

$\frac{1}{a_n} = \frac{1}{a_1 + (n-1)d}$

$\frac{1}{a_n} = \frac{1}{5n-4}$

$\frac{1}{6}, \frac{1}{11}, \frac{1}{16}, \ldots$

$\frac{1}{a_n} = \frac{1}{a_1+(n-1)d}$

$\frac{1}{a_n} = \frac{1}{5n+1}$

$$\therefore \quad T_n = \frac{1}{(5n+1)(5n-4)}$$

After desolving into partiol fractions

$$T_k = \frac{1}{5(5k-4)} - \frac{1}{5(5k+1)}$$

$$\therefore \quad S_n = \frac{1}{5} \sum_{k=1}^{n} \left[\frac{1}{5k-4} - \frac{1}{5k+1} \right]$$

$$= \frac{1}{5} \left[\left(\frac{1}{1} + \frac{1}{6} + \ldots + \frac{1}{5n-4} \right) - \left(\frac{1}{6} + \frac{1}{11} + \ldots + \frac{1}{5k+1} \right) \right]$$

$$S_n = \frac{1}{5} \left[\frac{1}{1} - \frac{1}{5n+1} \right]$$

$$S_n = \frac{n}{5n+1}$$

continued after chp. 5

let two harmonic progressions

$\frac{1}{1}, \frac{1}{6}, \frac{1}{11}, \ldots$	$\frac{1}{6}, \frac{1}{11}, \frac{1}{16}, \ldots$
$\frac{1}{a_n} = \frac{1}{a_1 + (n-1)d}$	$\frac{1}{a_n} = \frac{1}{a_1 + (n-1)d}$
$\frac{1}{a_n} = \frac{1}{5n-4}$	$\frac{1}{a_n} = \frac{1}{5n+1}$

$$\therefore \quad T_n = \frac{1}{(5n+1)(5n-4)}$$

After desolving into partiol fractions

$$T_k = \frac{1}{5(5k-4)} - \frac{1}{5(5k+1)}$$

$$\therefore \quad S_n = \frac{1}{5} \sum_{k=1}^{n} \left[\frac{1}{5k-4} - \frac{1}{5k+1} \right]$$

$$= \frac{1}{5} \left[\left(\frac{1}{1} + \frac{1}{6} + \ldots + \frac{1}{5n-4} \right) - \left(\frac{1}{6} + \frac{1}{11} + \ldots + \frac{1}{5k+1} \right) \right.$$

$$S_n = \frac{1}{5} \left[\frac{1}{1} - \frac{1}{5n+1} \right]$$

$$S_n = \frac{n}{5n+1}$$

Evulate the sum of the following series

10. $\sum_{k=3}^{n} \frac{1}{(k+1)(k+2)}$

Sol:-

$$= \sum_{k=3}^{n} \frac{1}{(k+1)(k+2)}$$

let $T_k = \frac{1}{(k+1)(k+2)}$ → general term of the series

Let $\frac{1}{(k+1)(k+2)} = \frac{A}{k+1} + \frac{B}{k+2}$ —(a)

$$1 = A(k+2) + B(k+1) \quad \text{—(i)}$$

Put $k=-1$ and $k=-2$ in (i)

when $k=-1$	when $k=-2$
$1 = A(-1+2) + B(-1+1)$	$1 = A(-2+2) + B(-2+1)$
$1 = A$	$1 = -B$
$\Rightarrow A = 1$	$\Rightarrow B = -1$

∴ eq (a) ⇒ $\frac{1}{(k+1)(k+2)} = \frac{1}{k+1} - \frac{1}{k+2}$ —(ii)

Put eq.(ii) in given sum.

$$\therefore \quad = \sum_{k=3}^{n} \left[\frac{1}{k+1} - \frac{1}{k+2} \right]$$

$$= \left[\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots + \frac{1}{n+1} \right] - \left[\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \dots + \frac{1}{n+2} \right]$$

$$= \left[\left(\frac{1}{4} - \cancel{\frac{1}{5}} \right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{6}} \right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{7}} \right) + \dots + \left(\cancel{\frac{1}{n+1}} - \frac{1}{n+2} \right) \right]$$

$$= \frac{1}{4} - \frac{1}{n+2}$$

$$= \frac{n+2-4}{4(n+2)} \Rightarrow \frac{n-2}{4(n+2)}$$

$\therefore$ sum of given summation is $\dfrac{(n-2)}{4(n+2)}$

11. $\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+2)}$

Sol:-

$$= \sum_{k=1}^{n} \frac{1}{k(k+2)} \quad \text{— (i)}$$

let $\dfrac{1}{k(k+2)} = \dfrac{A}{k} + \dfrac{B}{k+2}$ — (ii)

$1 = A(k+2) + Bk$

Put $k = 0$ and $k = -2$

when $k = -2$	when $k = 0$
$1 = A(-2+2) + B(-2)$	$1 = A(0+2) + B(0)$

$1 = -2B$ | $1 = 2A$

$B = -\frac{1}{2}$ | $A = \frac{1}{2}$

$$\text{eq. (ii)} \Rightarrow \frac{1}{k(k+2)} = \frac{1/2}{k} + \frac{-1/2}{k+2}$$

$$\frac{1}{k(k+2)} = \frac{1}{2k} - \frac{1}{2(k+2)}$$

so. expression (i) will become.

$$= \sum_{k=1}^{n} \left[\frac{1}{2k} - \frac{1}{2(k+2)} \right]$$

$$= \frac{1}{2} \sum_{k=1}^{n} \left[\frac{1}{k} - \frac{1}{k+2} \right]$$

$$= \frac{1}{2} \left[\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right) - \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n+2} \right) \right.$$

$$= \frac{1}{2} \left[\left(\frac{1}{1} - \cancel{\frac{1}{3}} \right) + \left(\frac{1}{2} - \frac{1}{4} \right) + \left(\cancel{\frac{1}{3}} - \frac{1}{5} \right) + \dots + \left(\frac{1}{n} - \frac{1}{n+2} \right) \right]$$

$$= \frac{1}{2} \left[1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \right]$$

$$= \frac{n(3n+5)}{2(n+1)(n+2)}$$

12. $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots$ to infinity

Sol:-

$$= \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots \text{ to infinity}$$

let two harmonic progressions i.e.

$\frac{1}{1}, \frac{1}{4}, \frac{1}{7}, \dots$

As, $\frac{1}{a_n} = \frac{1}{a_1+(n-1)d}$

$\frac{1}{a_n} = \frac{1}{1+(n-1)(3)}$

$\frac{1}{a_n} = \frac{1}{3n-2}$

$\frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \dots$

As, $\frac{1}{a_n} = \frac{1}{a_1+(n-1)d}$

$\frac{1}{a_n} = \frac{1}{4+(n-1)(3)}$

$\frac{1}{a_n} = \frac{1}{3n+1}$

So, the general term i.e T_k of the given series will be

$$T_k = \frac{1}{(3k-2)(3k+1)} \quad \text{--- (i)}$$

$$\text{As,} = \sum_{k=1}^{\infty} \left[\frac{1}{(3k-2)(3k+1)} \right] \quad \text{--- (ii)}$$

Resolving eq. (i) into partial fraction,

$$\frac{1}{(3k-2)(3k+1)} = \frac{A}{3k-2} + \frac{B}{3k+1}$$

$1 = A(3k+1) + B(3k-2)$

Put $k = -1/3$ and $k = 2/3$

when $k = -1/3$

$1 = A(0) + B\left[3(-1/3)-2\right]$

$1 = B(-3)$

$B = -1/3$

when $k = 2/3$

$1 = A\left[3(2/3)+1\right] + B(0)$

$1 = A(3)$

$A = 1/3$

$\therefore$ eq.(i) $\Rightarrow$ $$\frac{1}{(3k-2)(3k+1)} = \frac{1/3}{3k-2} + \frac{-1/3}{3k+1}$$

$$\frac{1}{(3k-2)(3k+1)} = \frac{1}{3(3k-2)} - \frac{1}{3(3k+1)}$$

$$\therefore \quad = \sum_{k=1}^{\infty}\left[\frac{1}{3(3k-2)} - \frac{1}{3(3k+1)}\right]$$

$$= \frac{1}{3}\sum_{k=1}^{\infty}\left[\frac{1}{3k-2} - \frac{1}{3k+1}\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{1} + \frac{1}{4} + \frac{1}{7} + \frac{1}{10} + \dots\infty\right) - \left(\frac{1}{4} + \frac{1}{7} + \frac{1}{10} + \frac{1}{13} + \dots\infty\right)\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{1} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right) + \left(\cancel{\frac{1}{7}} - \cancel{\frac{1}{10}}\right) + \left(\frac{1}{10} - \frac{1}{13}\right) + \dots\infty\right]$$

$$= \frac{1}{3}(1)$$

$= \frac{1}{3}$ → sum of given series.

iii) 13. $\frac{1}{5.11} + \frac{1}{7.13} + \frac{1}{9.15} + \ldots$ to n-terms

Sol:-

$$= \frac{1}{5\cdot 11} + \frac{1}{7\cdot 13} + \frac{1}{9.15} + \ldots \text{ to n-terms}$$

let two harmonic progressions i.e.

$\frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$	$\frac{1}{11}, \frac{1}{13}, \frac{1}{15}, \ldots$
As, $\frac{1}{a_n} = \frac{1}{a_1 + (n-1)d}$	As, $\frac{1}{a_n} = \frac{1}{a_1 + (n-1)d}$
$\frac{1}{a_n} = \frac{1}{5+(n-1)(2)}$	$\frac{1}{a_n} = \frac{1}{11+(n-1)(2)}$
$\frac{1}{a_n} = \frac{1}{2n+3}$	$\frac{1}{a_n} = \frac{1}{2n+9}$

So, the n^{th} term in the form of k will be

$$T_k = \frac{1}{(2k+3)(2k+9)}$$

let $\frac{1}{(2k+3)(2k+9)} = \frac{A}{2k+3} + \frac{B}{2k+9}$

Put $k = -3/2$ and $k = -9/2$

when $k = -3/2$

$$1 = A(2(-3/2)+9) + B(0)$$

$$1 = A(6)$$

$$A = \frac{1}{6}$$

when $k = -9/2$

$$1 = A(0) + B(2(-9/2)+3)$$

$$1 = B(-6)$$

$$B = -\frac{1}{6}$$

$$\therefore \quad T_k = \frac{1}{6(2k+3)} - \frac{1}{6(2k+9)}$$

As $$S_n = \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} \left[\frac{1}{6(2k+3)} - \frac{1}{6(2k+9)} \right]$$

$$= \frac{1}{6}\left[\left(\frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \dots \frac{1}{2n+3} \right) - \left(\frac{1}{11} + \frac{1}{13} + \frac{1}{15} + \frac{1}{17} + \dots + \frac{1}{2n+9} \right) \right]$$

$$= \frac{1}{6}\left[\left(\frac{1}{5} - \cancel{\frac{1}{11}} \right) + \left(\frac{1}{7} - \frac{1}{13} \right) + \left(\frac{1}{9} - \frac{1}{15} \right) + \left(\cancel{\frac{1}{11}} - \frac{1}{17} \right) + \dots + \left(\cancel{\frac{1}{2n-1}} - \cancel{\frac{1}{2n+3}} \right) + \left(\frac{1}{2n+1} - \frac{1}{2n+7} \right) + \left(\frac{1}{2n+3} - \frac{1}{2n+9} \right) \right]$$

$$= \frac{1}{6}\left[\frac{1}{5} + \frac{1}{7} + \frac{1}{9} - \frac{1}{2n+5} - \frac{1}{2n+7} - \frac{1}{2n+9} \right]$$

$$= \frac{1}{6}\left[\frac{143}{315} - \left(\frac{(2n+5)(2n+7)+(2n+7)(2n+9) + (2n+5)(2n+9)}{(2n+5)(2n+7)(2n+9)}\right)\right]$$

$$= \frac{1}{6}\left[\frac{143}{315} - \left\{\frac{4n^2+14n+10n+35+4n^2+18n+14n+63 + 4n^2+18n+10n+45}{(2n+5)(2n+7)(2n+9)}\right\}\right]$$

$$= \frac{1}{6}\left[\frac{143}{315} - \frac{12n^2+84n+143}{(2n+5)(2n+7)(2n+9)}\right]$$

$$= \frac{1}{6}\left[\frac{143(2n+5)(2n+7)(2n+9) - 315(12n^2+84n+143)}{315(2n+5)(2n+7)(2n+9)}\right]$$

$$= \frac{1}{6}\left[\frac{143(4n^2+24n+35)(2n+9) - 3780n^2 + 26460n \ldots 45045}{\cancel{315(2n+5)(2n+7)(2n+9)}}\right]$$

$$= \frac{1}{6}\left[\frac{143(8n^3+84n^2+186n+315) - 315(12n^2 \cancel{+84n+143})}{315(2n+5)(2n+7)(2n+9)}\right]$$

14. $\displaystyle\sum_{k=1}^{n} \frac{1}{9k^2+3k-2}$

Sol:-

$$= \sum_{k=1}^{n} \frac{1}{9k^2+3k-2}$$

$$\therefore \quad T_k = \frac{1}{9k^2+3k-2}$$

$$T_k = \frac{1}{9k^2+6k-3k-2}$$

$$T_k = \frac{1}{3k(3k+2) - 1(3k+2)}$$

$$T_k = \frac{1}{(3k-1)(3k+2)}$$

Resolving it into partial fractions

$$\frac{1}{(3k-1)(3k+2)} = \frac{A}{3k-1} + \frac{B}{3k+2}$$

$$1 = A(3k+2) + B(3k-1)$$

Put $k = -2/3$ and $k = 1/3$

when $k = 1/3$

$$1 = A\left[3(1/3) + 2\right] + B(0)$$

$$1 = A(3)$$

$$A = \frac{1}{3}$$

when $k = -2/3$

$$1 = A(0) + B\left[3\left(\frac{-2}{3}\right) - 1\right]$$

$$1 = B(-3)$$

$$B = -\frac{1}{3}$$

$$\therefore \quad T_k = \frac{1}{3(3k-1)} - \frac{1}{3(3k+2)}$$

As. $$S_n = \sum_{k=1}^{n} \left[\frac{1}{3(3k-1)} - \frac{1}{3(3k+2)}\right]$$

$$S_n = \frac{1}{3}\left[\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{8} + \frac{1}{11} + \dots + \frac{1}{3n-1}\right) - \left(\frac{1}{5} + \frac{1}{8} + \frac{1}{11} + \frac{1}{14} + \dots + \frac{1}{3n+2}\right)\right]$$

iii)

$$S_n = \frac{1}{3}\left[\left(\frac{1}{2} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{8}} - \cancel{\frac{1}{11}}\right) + \dots + \left(\frac{1}{3n-1} - \frac{1}{3n+2}\right)\right]$$

$$S_n = \frac{1}{3}\left(\frac{1}{2} - \frac{1}{3n+2}\right)$$

$$S_n = \frac{1}{3}\left(\frac{3n + \cancel{2} - \cancel{2}}{2(3n+2)}\right)$$

$$S_n = \frac{\cancel{3}n}{\cancel{3}\,2(3n+2)}$$

$$S_n = \frac{n}{6n+4}$$

15. $\displaystyle\sum_{k=2}^{n} \frac{1}{k^2 - k}$

iv)

Sol:-

$$= \sum_{k=2}^{n} \frac{1}{k^2 - k}$$

$$= \sum_{k=2}^{n} \frac{1}{k(k-1)}$$

let $\dfrac{1}{k(k-1)} = \dfrac{A}{k} + \dfrac{B}{k-1}$

$$1 = A(k-1) + Bk$$

Put $k = 1$ and $k = 0$

when $k = 0$	when $k = 1$
$1 = A(0-1) + B(0)$	$1 = A(1-1) + B(1)$

$A = -1$ and $B = 1$

$$\therefore \frac{1}{k(k-1)} = -\frac{1}{k} + \frac{1}{k-1}$$

$$\Rightarrow = \sum_{k=2}^{n} \left(\frac{1}{k-1} - \frac{1}{k} \right)$$

$$= \left[\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n-1} \right) - \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n} \right) \right]$$

$$= \left[\left(\frac{1}{1} - \cancel{\frac{1}{2}} \right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} \right) + \left(\cancel{\frac{1}{3}} - \frac{1}{4} \right) + \dots + \left(\frac{1}{n-1} - \frac{1}{n} \right) \right]$$

$$= 1 - \frac{1}{n}$$

$$= \frac{n-1}{n}$$