Studybeesofficial

Where Minds Pollinate

Class 11
Mathematics

Chapter 4
Exercise 4.7
Notes

National Book Foundation

Exercise # 4.7

Evulate the sum.

iii

1. $\sum_{k=1}^{5} \frac{1}{2k}$

Sol:-

$$= \sum_{k=1}^{5} \frac{1}{2k}$$

$$= \frac{1}{2(1)} + \frac{1}{2(2)} + \frac{1}{2(3)} + \frac{1}{2(4)} + \frac{1}{2(5)}$$

$$= \frac{137}{120}$$

2. $\sum_{k=1}^{6} \frac{1}{2k+1}$

Sol:-

iv'

$$= \sum_{k=1}^{6} \frac{1}{2k+1}$$

$$= \frac{1}{2(1)+1} + \frac{1}{2(2)+1} + \frac{1}{2(3)+1} + \frac{1}{2(4)+1} + \frac{1}{2(5)+1} + \frac{1}{2(6)+1}$$

$$= \frac{43024}{45045} = 0.955$$

3. $\sum_{k=0}^{5} 2^k$

Sol:-

$$= \sum_{k=0}^{5} 2^k$$

$$= 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$

$$= 63$$

4. $\sum_{k=0}^{9} \pi k$

Sol:-

$$= \sum_{k=0}^{9} \pi k$$

$$= \pi \sum_{k=0}^{9} k$$

$$= \pi \left[0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9\right]$$

$$= 45\pi$$

5. $\sum_{k=1}^{8} \frac{k}{k+1}$

Sol:-

$$= \sum_{k=1}^{8} \frac{k}{k+1}$$

$$= \frac{1}{1+1} + \frac{2}{2+1} + \frac{3}{3+1} + \frac{4}{4+1} + \frac{5}{5+1} + \frac{6}{6+1} + \frac{7}{7+1} + \frac{8}{8+1}$$

$$= \frac{15551}{2520} = 6.17$$

6. $\sum_{k=1}^{7} (-1)^k 4^{k+1}$

iii) **Sol:-**

$$= \sum_{k=1}^{7} (-1)^k 4^{k+1}$$

$$= (-1)^1 4^{1+1} + (-1)^2 4^{2+1} + (-1)^3 4^{3+1} + (-1)^4 4^{4+1} + (-1)^5 4^{5+1} + (-1)^6 4^{6+1} (-1)^7 4^{7+1}$$

$$= -52432$$

7. $\sum_{k=0}^{5} (k^2 - 2k + 3)$

Sol:-

$$= \sum_{k=0}^{5} (k^2 - 2k + 3)$$

iv) $$= \left[(0)^2 - 2(0) + 3\right] + \left[(1)^2 - 2(1) + 3\right] + \left[(2)^2 - 2(2) + 3\right] + \left[(3)^2 - 2(3) + 3\right] + \left[(4)^2 - 2(4) + 3\right] + \left[(5)^2 - 2(5) + 3\right]$$

$$= 43$$

8. $\sum_{k=1}^{10} \frac{1}{k(k+1)}$

Sol:-

$$= \sum_{k=1}^{10} \frac{1}{k(k+1)}$$

$= \frac{1}{1(1+1)} + \frac{1}{2(2+1)} + \frac{1}{3(3+1)} + \frac{1}{4(4+1)} + \frac{1}{5(5+1)} + \frac{1}{6(6+1)}$

$+ \frac{1}{7(7+1)} + \frac{1}{8(8+1)} + \frac{1}{9(9+1)} + \frac{1}{10(10+1)}$

$= \frac{10}{11} \approx 0.909$

Rewrite the sum using sigma notation

9. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \cdots$

Sol:-

$= \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots \quad -(i)$

OR $= 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{4} + \cdots$

Let a progre. (arithmetic)

$1, 2, 3, \ldots$

Here, $a_1 = 1$, $d = 2 - 1 = 1$

As, $a_n = a_1 + (n-1)d$

$= 1 + (n-1)(1)$

$a_n = n$

Let another progression (harmonic)

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$a_1 = 2 \;,\; d = 3-2 = 1$

iii) As, $\dfrac{1}{a_n} = \dfrac{1}{a_1+(n-1)d}$

$\dfrac{1}{a_n} = \dfrac{1}{2+(n-1)(1)}$

$\dfrac{1}{a_n} = \dfrac{1}{2+n-1}$

$\dfrac{1}{a_n} = \dfrac{1}{n+1}$

$\therefore$ n-term of series (i) $= T_n = n\left(\dfrac{1}{n+1}\right)$

Replace n with k

$\therefore T_k = k\left(\dfrac{1}{k+1}\right)$

iv) As, $S_n = \sum\limits_{k=1}^{n} T_k$

$\therefore S_n = \sum\limits_{k=1}^{n} \dfrac{k}{k+1}$

So, $= \dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} + \dfrac{4}{5} + \dots$

$= \sum\limits_{k=1}^{\infty} \dfrac{k}{k+1}$

10 $3+6+9+12+15$

Sol:-

$= 3+6+9+12+15$

It is an arithmetic series

$a_1 = 3$, $d = 6-3 = 3$,

$\therefore a_n = a_1 + (n-1)d$

$= 3+(n-1)(3)$

$= 3+3n-3$

$a_n = 3n$

$\therefore T_n = 3n$

OR $T_k = 3k$

As, $S_n = \sum_{k=1}^{5} T_k$

$= \sum_{k=1}^{5} 3k$

11. $-2+4-8+16-32+64$

Sol:-

$= -2+4-8+16-32+64$

$= +(-2)+4+(-8)+16+(-32)+64$

It is also geometric series

$a_1 = -2$, $r = \frac{4}{-2} = -2$

$\therefore \quad a_n = a_1 r^{n-1}$

$a_n = -2(-2)^{n-1}$

So, $\quad T_n = -2(-2)^{n-1}$

OR $\quad T_k = -2(-2)^{k-1}$

As, $\quad S_n = \sum_{k=1}^{6} T_k$

$\therefore \quad = \sum_{k=1}^{6} -2(-2)^{k-1}$

12. $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$

Sol:-

$= \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$

Let a progression (arithmetic)

$1, 2, 3, 4, \ldots$

$a_1 = 1, \quad d = 2 - 1 = 1$

As, $\quad a_n = a_1 + (n-1)d$

$= 1 + (n-1)(1)$

$= 1 + n - 1$

$a_n = n$

Let another progression

2, 3, 4, 5, ...

$a_1 = 2, \quad d = 3-2 = 1$

So, $a_n = a_1 + (n-1)d$

$= 2 + (n-1)(1)$

$= 2 + n - 1$

$a_n = n+1$

So, n-term of given series $= T_n = \dfrac{1}{n(n+1)}$

Replace n with k

$\therefore \quad T_k = \dfrac{1}{k(k+1)}$

$\therefore \quad = \sum_{k=1}^{\infty} \dfrac{1}{k(k+1)}$

13. Prove that $\sum_{k=1}^{n} k^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

Proof:-

As we know that, sum of m power of first n natural no. is

$$\sum_{k=1}^{n} \left[k^m - (k-1)^m\right] = n^m$$

Put $m = 4$

$$\sum_{k=1}^{n}\left[k^4-(k-1)^4\right] = n^4$$

iii $$\sum_{k=1}^{n}\left[k^4-(k-1)^2(k-1)^2\right] = n^4$$

$$\sum_{k=1}^{n}\left[k^4-(k^2-2k+1)(k^2-2k+1)\right] = n^4$$

$$\sum_{k=1}^{n}\left[k^4-(k^4-2k^3+k^2-2k^3+4k^2-2k+k^2-2k+1)\right] = n^4$$

$$\sum_{k=1}^{n}\left[\cancel{k^4}-\cancel{k^4}+2k^3-k^2+2k^3-4k^2+2k-k^2+2k-1\right] = n^4$$

$$\sum_{k=1}^{n}\left[4k^3-6k^2+4k-1\right] = n^4$$

$$4\sum_{k=1}^{n}k^3-6\sum_{k=1}^{n}k^2+4\sum_{k=1}^{n}k-\sum_{k=1}^{n}1 = n^4$$

iv $$4\sum_{k=1}^{n}k^3-\cancel{6}\left[\frac{n(n+1)(2n+1)}{\cancel{6}}\right]+\overset{2}{\cancel{4}}\left[\frac{n(n+1)}{\cancel{2}}\right]-n = n^4$$

$$4\sum_{k=1}^{n}k^3-n(n+1)(2n+1)+2n(n+1) = n^4+n$$

$$4\sum_{k=1}^{n}k^3-n(2n^2+n+2n+1)+(2n^2+2n) = n^4+n$$

$$4\sum_{k=1}^{n}k^3 = n^4+n(2n^2+3n+1)-(2n^2+2n)+n$$

$$4\sum_{k=1}^{n} k^3 = n^4 + 2n^3 + 3n^2 + n - 2n^2 - 2n + n$$

$$4\sum_{k=1}^{n} k^3 = n^4 + 2n^3 + n^2 + n$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n^2 + 2n + 1)}{4}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{(2)^2}$$

$$\therefore \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Find the sum to n terms of the series whose nth term are given.

14. $n + 1$

Sol:-

n term of series is

$$T_n = n + 1$$

Replace n with k

$$\therefore T_k = k + 1$$

So, in sigma notation

$$S_n = \sum_{k=1}^{n} T_k$$

$$\therefore \quad S_n = \sum_{k=1}^{n} (k+1)$$

iii OR $$S_n = \sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$S_n = \frac{n(n+1)}{2} + n$$

$$= \frac{n(n+1)+2n}{2}$$

$$\therefore \quad S_n = \frac{n[n+1+2]}{2}$$

$$S_n = \frac{n(n+3)}{2}$$

15. $n^2 + 2n$

Sol:-

n- term of series is:

iv

$$T_n = n^2 + 2n$$
$$T_n = n(n+2)$$

Replace n with k

$$\therefore \quad T_k = k(k+2)$$

As, $$S_n = \sum_{k=1}^{n} T_k$$

$$S_n = \sum_{k=1}^{n} k(k+2)$$

$$= \sum_{k=1}^{n} k^2 + 2k$$

$$S_n = \sum_{k=1}^{n} k^2 + 2\sum_{k=1} k$$

$$S_n = \frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1) + 3n(2n+1)}{6}$$

$$= \frac{n\left[(n+1)(2n+1) + 3(2n+1)\right]}{6}$$

$$= \frac{n(2n^2 + n + 2n + 1 + 6n + 6)}{6}$$

$$\therefore S_n = \frac{n(2n^2 + 9n + 7)}{6}$$

16. $3n^2 + 2n + 1$

Sol:-

n-term of series is:-

$$T_n = 3n^2 + 2n + 1$$

Replace n with k

$$\therefore T_k = 3k^2 + 2k + 1$$

As, $$S_n = \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} (3k^2 + 2k + 1)$$

$$S_n = 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \cancel{3}\left(\frac{n(n+1)(2n+1)}{\cancel{6}_2}\right) + \cancel{2}\left(\frac{n(n+1)}{\cancel{2}}\right) + n$$

$$S_n = \frac{n(n+1)(2n+1)}{2} + n(n+1) + n$$

$$S_n = \frac{n(n+1)(2n+1) + 2n(n+1) + 2n}{2}$$

$$= \frac{n[(n+1)(2n+1) + 2(n+1) + 2]}{2}$$

iii

$$S_n = \frac{n(2n^2 + n + 2n + 1 + 2n + 2 + 2)}{2}$$

$$S_n = \frac{n(2n^2 + 5n + 5)}{2}$$

Sum the following series upto n-terms.

17. $2^2 + 5^2 + 8^2 + \dots$

Sol:-

Given series, $2^2 + 5^2 + 8^2 + \dots T_n$

Let a progression (arithmetic)

$$2, 5, 8, \dots$$

$a_1 = 2$, $d = 5 - 2 = 3$

iv As, $a_n = a_1 + (n-1)d$

$$= 2 + (n-1)(3)$$

$$= 2 + 3n - 3$$

$$a_n = 3n - 1$$

n-term of given series will be $= (3n-1)^2 = T_n$

Replace n with k

$$\therefore T_k = (3k-1)^2$$

As, $S_n = \sum_{k=1}^{n} (3k-1)^2$

$$S_n = \sum_{k=1}^{n} (9k^2 + 1 - 6k)$$

$$= 9\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1 - 6\sum_{k=1}^{n} k$$

$$S_n = 9\left[\frac{n(n+1)(2n+1)}{6}\right] + n - 6\left[\frac{n(n+1)}{2}\right]$$

$$= \frac{3n(n+1)(2n+1)}{2} + n - 3n(n+1)$$

$$S_n = \frac{3n(n+1)(2n+1) + 2n - 6n(n+1)}{2}$$

$$= \frac{n\left[3(n+1)(2n+1) + 2 - 6(n+1)\right]}{2}$$

$$S_n = \frac{n\left[(3n+3)(2n+1) + 2 - 6n - 6\right]}{2}$$

$$= \frac{n(6n^2 + 3n + \cancel{6n} + 3 + 2 - \cancel{6n} - 6)}{2}$$

$$S_n = \frac{n(6n^2 + 3n - 1)}{2}$$

8. $2^2 + 4^2 + 6^2 + \ldots$

Sol:-

Given series, $2^2 + 4^2 + 6^2 + \ldots T_n$

let a progression (arithmetic)

$2, 4, 6, 8, \ldots$

$a_1 = 2$, $d = 4 - 2 = 2$

As, $a_n = a_1 + (n-1)d$

$= 2 + (n-1)(2)$

$a_n = 2n$

$\therefore$ n-term of given series $= T_n = (2n)^2 = 4n^2$

Replace n with k

$\therefore \quad T_k = 4k^2$

As, $\quad S_n = \sum_{k=1}^{n} T_k$

$S_n = \sum_{k=1}^{n} 4k^2$

$S_n = 4 \sum_{k=1}^{n} k^2 \Rightarrow \quad S_n = \cancel{4}^{2}\left[\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right]$

$$S_n = \frac{2n(n+1)(2n+1)}{3}$$

$$= \frac{2n(2n^2+n+2n+1)}{3}$$

$$S_n = \frac{2n(2n^2+3n+1)}{3}$$

$$S_n = \frac{4n^3+6n^2+2n}{3}$$

19. $\quad 1^3 + 3^3 + 5^3 + \dots$

Sol:-

Given series. $\quad 1^3 + 3^3 + 5^3 + \dots T_n$

Let a arithmetic progression

$1, 3, 5, \dots$

Here, $a_1 = 1, \quad d = 3 - 1 = 2$

As. $a_n = a_1 + (n-1)d$

$= 1 + (n-1)(2)$

$= 1 + 2n - 2$

$\therefore \quad a_n = 2n - 1$

So, n-term of given series $= T_n = (2n-1)^3$

Replace n with k

$\therefore \quad T_k = (2k-1)^3$

$= (2k)^3 - (1)^3 - 3(2k)^2(1) + 3(2k)(1)^2$

$T_k = 8k^3 - 1 - 3(4k^2) + 6k$

$T_k = 8k^3 - 1 - 12k^2 + 6k$

$T_k = 8k^3 + 6k - 12k^2 - 1$

$T_k = 8k^3 - 12k^2 + 6k - 1$

As, $S_n = \sum_{k=1}^{n} T_k \Rightarrow S_n = \sum_{k=1}^{n} (8k^3 - 12k^2 + 6k - 1)$

$S_n = 8\sum_{k=1}^{n} k^3 - 12\sum_{k=1}^{n} k^2 + 6\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1$

$= 8\left[\frac{n(n+1)}{2}\right]^2 - \cancel{12}^{\,2}\left[\frac{n(n+1)(2n+1)}{\cancel{6}}\right] + \cancel{6}^{\,3}\left[\frac{n(n+1)}{\cancel{2}}\right] - n$

$= \cancel{8}^{\,2}\left[\frac{n^2(n+1)^2}{\cancel{4}}\right] - 2n(n+1)(2n+1) + 3n(n+1) - n$

$S_n = 2n^2(n^2+2n+1) - 2n(n+1)(2n+1) + 3n(n+1) - n$

$S_n = n\left[2n(n^2+2n+1) - 2(n+1)(2n+1) + 3(n+1) - 1\right]$

$= n\left[2n^3 + 4n^2 + 2n - 2(2n^2 + n + 2n + 1) + 3n + 3 - 1\right]$

$= n\left[2n^3 + \cancel{4n^2} + 2n - \cancel{4n^2} - 6n - 2 + 3n + 3 - 1\right]$

$S_n = n\left[2n^3 - n\right]$

$S_n = n^2\left[2n^2 - 1\right]$

20. $2 + 5 + 10 + 17 + \dots$ to n terms

Sol:-

$= 2 + 5 + 10 + 17 + \dots$ to n-terms

$= \left[(1)^2+1\right] + \left[(2)^2+1\right] + \left[(3)^2+1\right] + \left[(4)^2+1\right] + \dots \left[n^2+1\right]$

OR $= \sum_{k=1}^{n} (k^2+1)$

$$\therefore \; S_n = \sum_{k=1}^{n} (k^2+1)$$

$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1$$

$$S_n = \frac{n(n+1)(2n+1)}{6} + n$$

$$= \frac{n(2n^2+n+2n+1)+6n}{6}$$

$$= \frac{n(2n^2+3n+1+6)}{6}$$

$$S_n = \frac{n(2n^2+3n+7)}{6}$$

21. $1\times 4 + 2\times 7 + 3\times 10 + \ldots$

Sol:-

$$(1\times 4) + (2\times 7) + (3\times 10) + \ldots T_n$$

Let two arithmetic progressions

$1, 2, 3, \ldots$	$4, 7, 10, \ldots$
$a_1 = 1,\; d = 2-1 = 1$	$a_1 = 4,\; d = 7-4 = 3$
As, $a_n = a_1 + (n-1)d$	As, $a_n = a_1 + (n-1)d$
$= 1 + (n-1)(1)$	$= 4 + (n-1)(3)$
$a_n = n$	$a_n = 3n+1$

So, n-term of given series $= T_n = n(3n+1)$

$$T_n = 3n^2 + n$$

Replace n with k

$$\therefore \; T_k = 3k^2 + k$$

As, $S_n = \sum_{k=1}^{n} T_k$

$$= \sum_{k=1}^{n} (3k^2 + k)$$

$$S_n = 3\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$$

$$S_n = 3\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{n(n+1)}{2}$$

$$S_n = \frac{n(2n^2+3n+1)}{2} + \frac{n^2+n}{2}$$

$$S_n = \frac{n(2n^2+3n+1)+n^2+n}{2}$$

$$S_n = \frac{n[2n^2+3n+1+n+1]}{2}$$

$$S_n = \frac{n(2n^2+4n+2)}{2}$$

$$S_n = n(n^2+2n+1)/2$$

$$S_n = n(n+1)^2/2$$

2. $1\times3\times5 + 3\times5\times7 + 5\times7\times9 + \dots$

Sol:-

$= (1\times3\times5)+(3\times5\times7)+(5\times7\times9)+\dots T_n$

Let three arithmetic progressions

$1, 3, 5, \dots$	$3, 5, 7, \dots$	$5, 7, 9, \dots$
$a_1 = 1,\ d = 3-1 = 2$	$a_1 3,\ d = 5-3 = 2$	$a_1 = 5,\ d = 7-5 = 2$
$a_n = a_1 + (n-1)d$	$a_n = a_1 + (n-1)d$	$a_n = a_1 + (n-1)d$
$= 1+(n-1)(2)$	$= 3+(n-1)(2)$	$= 5+(n-1)(2)$
$a_n = 2n-1$	$a_n = 2n+1$	$a_n = 2n+3$

$\therefore$ n-term of series $= T_n = (2n+1)(2n-1)(2n+3)$

$T_n = 8n^3+12n^2-2n-3$

Replace n with k

$\therefore\ T_k = 8k^3+12k^2-2k-3$

As, $S_n = \sum_{k=1}^{n} T_k$

$$S_n = 8\sum_{k=1}^{n} k^3 + 12\sum_{k=1}^{n} k^2 - 2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 3$$

iii
$$S_n = 8\left[\frac{n(n+1)}{2}\right]^2 + 12\left[\frac{n(n+1)(2n+1)}{6}\right] - 2\left[\frac{n(n+1)}{2}\right] - 3n$$

on solving it we get

$$S_n = n\left[2n^3 + 8n^2 + 7n - 2\right]$$

Sum to n-terms of the following series

23. $1 + 2\times2 + 3\times2^2 + 4\times2^3 + \dots$

Sol:-

$= (1\times2^0) + (2\times2) + (3\times2^2) + (4\times2^3) + \dots$ n-terms

$\because$ it is a arithmetico-geometric series

So, $S_n = \frac{a}{1-r} + \frac{dr(1-r^n)}{(1-r)^2} - \frac{(a+nd)r^n}{1-r}$

Here, $r = 2$, $d = 1$, $a = 1\times2^0 = 1$

iv
$\therefore S_n = \frac{1}{1-2} + \frac{(1)(2)[1-(2)^n]}{(1-2)^2} - \frac{[1+n(1)](2)^n}{1-2}$

On solving

$S_n = 1 + 2^n(n-1)$

24. $1 + 4y + 7y^2 + 10y^3 + \dots$

Sol:-

$= (1\times y^0) + (4\times y) + (7\times y^2) + (10\times y^3) + \dots$

as, $S_n = \frac{a}{1-r} + \frac{dr(1-r^n)}{(1-r)^2} - \frac{(a+nd)r^n}{1-r}$

Here, $a = 1, d = 3, r = y$

$$S_n = \frac{1}{1-y} + \frac{(3)y[1-(y)^n]}{(1-y)^2} - \frac{[1+n(3)](y)^n}{1-y}$$

$$S_n = \frac{1+2y-(1-3n)y^n-(2+3n)y^{n+1}}{(1-y)^2}$$

25. $1 + \frac{4}{7} + \frac{7}{7^2} + \frac{10}{7^3} + \dots$

Sol:-

$$= \left(1 \times \frac{1}{7^0}\right) + \left(4 \times \frac{1}{7}\right) + \left(7 \times \frac{1}{7^2}\right) + \left(10 \times \frac{1}{7^3}\right) + \dots$$

As, $S_n = \frac{a}{1-r} + \frac{dr(1-r^n)}{(1-r)^2} - \frac{(a+nd)r^n}{1-r}$

Here, $a = 1, d = 3, r = 1/7$

$$\therefore S_n = \frac{1}{1-1/7} + \frac{(3)(1/7)[1-(1/7)^n]}{(1-1/7)^2} - \frac{[1+n(3)](1/7)^n}{1-1/7}$$

$$S_n = \frac{3.7^{n+1} - 42n - 21}{12.7^n}$$

26. $1 + \frac{7}{2} + \frac{13}{4} + \frac{19}{8} + \frac{25}{16} + \dots$

Sol:-

$$= \left(1 \times \frac{1}{2^0}\right) + \left(7 \times \frac{1}{2}\right) + \left(13 \times \frac{1}{2^2}\right) + \left(25 \times \frac{1}{2^3}\right) + \dots$$

as, $S_n = \frac{a}{1-r} + \frac{dr(1-r^n)}{(1-r)^2} - \frac{(a+nd)r^n}{1-r}$

Here, $a = 1, d = 6, r = \frac{1}{2}$

$$S_n = \frac{1}{1-\frac{1}{2}} + \frac{(6)(1/2)[1-(1/2)^n]}{(1-\frac{1}{2})^2} - \frac{[1+n(6)](1/2)^n}{1-\frac{1}{2}}$$

$$S_n = \frac{2^{n+1} + 12.2^n - 12n - 14}{2^n}$$

iii Find the sum to infinity of the following series.

27. $5 + \frac{7}{3} + \frac{9}{9} + \frac{11}{27} + \dots$

Sol:-

$$= \left(5 \times \frac{1}{3^0}\right) + \left(7 \times \frac{1}{3}\right) + \left(9 \times \frac{1}{3^2}\right) + \left(11 \times \frac{1}{3^3}\right) + \dots$$

as. $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Here, $a = 5$, $d = 2$, $r = \frac{1}{3} < 1$

$$\therefore S_\infty = \frac{5}{1-\frac{1}{3}} + \frac{(2)(1/3)}{(1-\frac{1}{3})^2} = 9$$

28. $1 + \frac{2}{5} + \frac{3}{25} + \frac{4}{125} + \dots$

Sol:- $= \left(1 \times \frac{1}{5^0}\right) + \left(2 \times \frac{1}{5}\right) + \left(3 \times \frac{1}{5^2}\right) + \dots$

iv as, $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Here, $a = 1$, $d = 1$, $r = \frac{1}{5} < 1$

$$\therefore S_\infty = \frac{1}{1 - 1/5} + \frac{(1)(1/5)}{(1-1/5)^2} = \frac{25}{16}$$

29. $1 + 4x + 7x^2 + 10x^3 + \dots$

Sol:-

$$= (1 \times x^0) + (4 \times x) + (7 \times x^2) + (10 \times x^3) + \dots$$

The sum of this series exist if $|x| < 1$

$\therefore S_\infty = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$

Here, $a = 1$, $d = 3$, $r = x$

$\therefore S_\infty = \dfrac{1}{1-x} + \dfrac{3x}{(1-x)^2} = \dfrac{1+2x}{(1-x)^2}$

30. $3 + \dfrac{6}{10} + \dfrac{9}{100} + \dfrac{12}{1000} + \dots$

Sol:-

$= \left(3 \times \dfrac{1}{10^0}\right) + \left(6 \times \dfrac{1}{10^2}\right) + \left(12 \times \dfrac{1}{10^3}\right) + \dots$

as, $S_\infty = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$

Here, $a = 3$, $d = 3$, $r = \dfrac{1}{10} < 1$

$\therefore S_\infty = \dfrac{3}{1 - 1/10} + \dfrac{(3)(1/10)}{(1-1/10)^2} = \dfrac{100}{27}$