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Where Minds Pollinate

Class 11
Mathematics

Chapter 4
Exercise 4.5
Notes

National Book Foundation

Exercise # 4.5

Q1. Find the sum of each geometric series.

1. $16 + 16 + 16 + \dots$ to 11 terms.

Sol:-

We have $a_1 = 16$, $n = 11$, $r = 1$

As, $S_n = \dfrac{a_1(1 - r^n)}{1 - r}$ $r \neq 1$

$\because r = 1$

So, $sum = 16 \times 11 = 176$

2. $75 + 15 + 3 + \dots$ to 10 terms

Sol:-

We have, $a_1 = 75$, $n = 10$, $r = \dfrac{15}{75} = \dfrac{1}{5}$

As. $S_n = \frac{a_1(1-r^n)}{1-r}$

∴ $S_{10} = \frac{(75)(1-(1/5)^{10})}{1-\frac{1}{5}}$

$S_{10} = 93.75$

3. $a_1 = 5$, $r = 3$, $n = 12$

Sol:-

As. $S_n = \frac{a_1(1-r^n)}{1-r}$

∴ $S_{12} = \frac{(5)[1-(3)^{12}]}{1-3}$

$S_{12} = 1328600$

4. $a_1 = 256$, $r = 0.75$, $n = 9$

Sol:-

As, $S_n = \frac{a_1(1-r^n)}{1-r}$

∴ $S_9 = \frac{(256)[1-(0.75)^9]}{1-0.75}$

$S_9 = 947.11$

5. $a_1 = 7$, $r = 2$, $n = 14$

Sol:-

As, $S_n = \dfrac{a_1(1-r^n)}{1-r}$

$\therefore \quad S_{14} = \dfrac{(7)(1-(2)^{14})}{1-2}$

$S_{14} = 114681$

6. $a_1 = 12, \quad a_5 = 972, \quad r = -3$

Sol:-

As, $S_n = \dfrac{a_1 + a_n r}{1-r}$

$\therefore \quad S_n = \dfrac{12 + (972)(-3)}{1-(-3)}$

$S_n = -726$

7. $a_1 = 16, \quad r = -\dfrac{1}{2}, \quad n = 10$

Sol:-

As, $S_n = \dfrac{a_1(1-r^n)}{1-r}$

$S_{10} = \dfrac{(16)\left[1-(-0.5)^{10}\right]}{1-(-0.5)}$

$S_{10} = 10.66$

8. $a_1 = 243$, $r = -\frac{2}{3}$, $n = 5$

Sol:-

As, $S_n = \frac{a_1(1-r^n)}{1-r}$

$$\therefore \quad S_5 = \frac{(243)\left[1-(-2/3)^5\right]}{1-(-2/3)}$$

$$S_5 = 165$$

9. $a_1 = 343$, $a_4 = -1$, $r = -1/7$

Sol:-

As, $S_n = \frac{a_1(1-r^n)}{1-r}$

$\therefore$ OR $S_n = \frac{a_1 + a_n r}{1-r}$

$$\Rightarrow \quad S_n = \frac{343+(-1)(-1/7)}{1-(-1/7)}$$

$$S_n = 300.25$$

10. $a_3 = \frac{3}{4}$, $a_6 = \frac{3}{32}$, $n = 6$

Sol:-

As, $S_n = \frac{a_1 - a_n r}{1-r}$ —(i) also, $a_n = a_1 r^{n-1}$

$a_3 = a_1 r^{3-1}$

$a_3 = a_1 r^2$

$a_1 = \frac{3/4}{(r)^2}$

also. $a_6 = a_1 r^{6-1}$

$\frac{3}{32} = a_1 r^5$

$\Rightarrow \frac{\cancel{3}}{32} = \frac{\cancel{3}}{4\cancel{r^2}} \times r^5$

$\frac{1}{32} = \frac{1}{4} \times r^3$

$\frac{1}{32} = \frac{r^3}{4}$

$r = \sqrt[3]{\frac{1}{8}} \Rightarrow r = \frac{1}{2}, \; a_1 = 3$

eq.(i) $\Rightarrow$ $S_n = \frac{3 + (3/32)(1/2)}{1 - \frac{1}{2}}$

$S_n = 6$

Find a_1 for each geometric series.

1. $S_n = 244$, $r = -3$, $n = 5$

Sol:-

As, $S_n = \frac{a_1(1-r^n)}{1-r}$

$a_1 = \frac{S_n(1-r)}{1-r^n}$

$a_1 = \frac{244(1+3)}{1-(-3)^5}$

$a_1 = 4$

12. $S_n = 32,\ r = 2,\ n = 6$

Sol:-

As, $a_1 = \frac{S_n(1-r)}{1-r^n}$

$a_1 = \frac{32(1-2)}{1-(2)^6}$

$a_1 = 0.51$

13. $a_n = 324,\ r = 3,\ S_n = 484$

Sol:-

As, $S_n = \frac{a_1 - a_n r}{1-r}$

$\therefore\ a_1 = S_n(1-r) - a_n r$

$a_1 = 484(1-3) - (324)(3)$

$a_1 = 4$

Q14. Find fractional notation for infinite geometric series.

i) 0.444...

Sol:-

$0.444... = 0.4 + 0.04 + 0.004 + ...$

Here, $r = \frac{0.04}{0.4} = 0.1 \Rightarrow -1 < 0.1 < 1$

and $a_1 = 0.4$

$\therefore S_\infty = \frac{a_1}{1-r} \Rightarrow \frac{0.4}{1-0.1} = \frac{4}{9}$

$\therefore 0.44... = \frac{4}{9}$

ii) 9.999...

Sol:-

$9.99... = 9 + 0.9 + 0.09 + 0.009 + ...$

Here, $r = \frac{0.9}{9} = 0.1$, $a_1 = 9$

$\because S_\infty = \frac{a_1}{1-r} \Rightarrow \frac{9}{1-0.1} = 10$

$\therefore \quad 9.999... = 10$

iii) $0.555...$

Sol:-

$$0.555... = 0.5 + 0.05 + 0.005 + ...$$

Here, $r = \frac{0.05}{0.5} = 0.1$, $a_1 = 0.5$

$$\because \quad S_\infty = \frac{a_1}{1-r} \Rightarrow \frac{0.5}{1-0.1}$$

$$S_\infty = \frac{5}{9}$$

$$\therefore \quad 0.555... = \frac{5}{9}$$

iv) $0.666...$

Sol:-

$$0.666... = 0.6 + 0.06 + 0.006 + ...$$

Here, $r = \frac{0.06}{0.6} = 0.1$, $a_1 = 0.6$

$$\because \quad S_\infty = \frac{a_1}{1-r} \Rightarrow \frac{0.6}{1-0.1} = \frac{2}{3}$$

$$\therefore \quad 0.66... = \frac{2}{3}$$

v) $0.1515\ldots$

Sol:-

$$0.\overline{15} = 0.15 + 0.0015 + 0.000015 + \ldots$$

Her $\Rightarrow r = \dfrac{0.0015}{0.15} = 0.1$, $a_1 = 0.15$

$$\therefore \quad S_\infty = \frac{a_1}{1-r} \Rightarrow \frac{0.15}{1-0.1} \Rightarrow S_\infty = \frac{5}{33}$$

$$\therefore \quad 0.1515\ldots = \frac{5}{33}$$

vi) $0.121212\ldots$

Sol:-

$$0.\overline{12} = 0.12 + 0.0012 + 0.000012 + \ldots$$

Here, $r = \dfrac{0.0012}{0.12} = 0.1$, $a_1 = 0.12$

$$\because \quad S_\infty = \frac{0.12}{1-0.1} \Rightarrow \frac{4}{33}$$

$$\therefore \quad 0.121212\ldots = \frac{4}{33}$$

Q15. To test its elasticity, a rubber ball is dropped into a 30ft hollow tube that is clibrated so that the scientist can measure the height of each subsequent bounce. The scientist found that each bounce, the ball rises to a height 2/5 the height of previous bounce. How far the ball travel before it stops bouncing?

Sol:-

Height of tube = 30ft
Rise of ball after each bounce = 2/5

So, height of ball in 1st bounce (up and down) will be $(30)\left(\frac{2}{5}\right) = 12ft$

Similarly, $12 + 4.8 + 1.92 + \ldots$

Here, $r = \frac{4.8}{12} = 0.4 \Rightarrow -1 < r < 1$

and $a_1 = 12$

$\because S_\infty = \frac{a_1}{1-r} \Rightarrow S_\infty = \frac{12}{1-0.4} = 20ft$

for (up and down) $= 20(2) = 40ft$

So, total height travelled $= 40 + 30 = 70ft$

Q16. A hot-air ballon rises 80ft in the first minute of flight. If in each succeeding minute the ballon rises only 90% as far as in the previous minute, what will be its maximum altitude if it allowed to rise without limit?

Sol:-

Rise of ballon in first min. $= a_1 = 80ft$

Each rise in succeeding min. $= \frac{90}{100} = 0.9$

$\therefore \quad 80 + 72 + 64.8 + \ldots$

$r = 0.9 \Rightarrow -1 < r < 1 \quad , a_1 = 80$

$\because \quad S_\infty = \frac{a_1}{1-r} = \frac{80}{1-0.9} = 800$

So, ballon will gain an altitude of 800ft.