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Where Minds Pollinate

Class 11
Mathematics

Chapter 4
Exercise 4.2
Notes

National Book Foundation

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Exercise # 4.2

Q1. Find first four term of each arthmetic sequence.

i) $a_1 = 4, \quad d = 3$

Sol:-

As, $a_n = a_1 + (n-1)d$

$a_1 = 4 + (1-1)(3) \Rightarrow a_1 = 4$

$a_2 = 4 + (2-1)(3) \Rightarrow a_2 = 7$

$a_3 = 4 + (3-1)(3) \Rightarrow a_3 = 10$

$a_4 = 4 + (4-1)(3) \Rightarrow a_4 = 13$

ii) $a_1 = 7, d = 5$

Sol:-

As, $a_n = a_1 + (n-1)d$

$a_1 = 7$

$a_2 = 7 + (2-1)(5) \Rightarrow a_2 = 12$

$a_3 = 7 + (3-1)(5) \Rightarrow a_3 = 17$

$a_4 = 7 + (4-1)(5) \Rightarrow a_4 = 22$

iii) $a_1 = 16, \ d = -2$

Sol:-

As, $a_n = a_1 + (n-1)d$

$a_1 = 16$

$a_2 = 16 + (+2-1)(-2) \rightarrow a_2 = 14$

$a_3 = 16 + (3-1)(-2) \Rightarrow a_3 = 12$

$a_4 = 16 + (4-1)(-2) \Rightarrow a_4 = 10$

iv) $a_1 = 38, \ d = -4$

Sol:-

As, $a_n = a_1 + (n-1)d$

$a_1 = 38$

$a_2 = 38 + (2-1)(-4) \Rightarrow a_2 = 34$

$a_3 = 38 + (3-1)(-4) \Rightarrow a_3 = 30$

$a_4 = 38 + (4-1)(-4) \Rightarrow a_4 = 26$

) $a_1 = \frac{3}{4}, \ d = \frac{1}{4}$

Sol:-

As, $a_n = a_1 + (n-1)d$

$a_1 = \frac{3}{4}$

$a_2 = \frac{3}{4} + (2-1)\left(\frac{1}{4}\right) \Rightarrow a_2 = 1$

$a_3 = \frac{3}{4} + (3-1)\left(\frac{1}{4}\right) \Rightarrow a_3 = 5/4$

$a_4 = \frac{3}{4} + (4-1)\left(\frac{1}{4}\right) \Rightarrow a_4 = 3/2$

vi) $a_1 = \frac{3}{8}$, $d = \frac{5}{8}$

As, $a_n = a_1 + (n-1)d$

$a_1 = 3/8$

$a_2 = \frac{3}{8} + (2-1)\left(\frac{5}{8}\right) \Rightarrow a_2 = 1$

$a_3 = \frac{3}{8} + (3-1)\left(\frac{5}{8}\right) \Rightarrow a_3 = \frac{13}{8}$

$a_4 = \frac{3}{8} + (4-1)\left(\frac{5}{8}\right) \Rightarrow a_4 = \frac{9}{4}$

Q2. Find the next three terms in each arthmetic sequence.

i) 5, 9, 13, ...

Sol:-

Each term is obtained by adding $(9-5) = 4$

$\therefore$ 5, 9, 13, 17, 21, 25, ...

ii) 11, 14, 17, ...

Sol:-

Each new term is obtained by adding (14 - 11)

$= 3$

$\therefore$ 11, 14, 17, 20, 23, 26, ...

iii) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

Sol:-

Each new term is obtained by adding $\left(\frac{3}{2} - \frac{1}{2}\right)$

$= 1$

$\therefore \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \ldots$

v) $-5.4, -1.4, +2.6, \ldots$

Sol:-

Each new term is obtained by adding $(-1.4 - (-5.4))$

$= 4 = 2.6$

$\therefore -5.4, -1.4, 2.6, 6.6, 10.6, 14.6, \ldots$

13. Find 11^{th} term of arthmetic sequence

0.07, 0.12, 0.7, ...

Sol:-

Common difference $= d = 0.12 - 0.07$

$d = 0.05$

and $a_1 = 0.07$

As, $a_n = a_1 + (n-1)d$

So. $a_{11} = 0.07 + (11-1)(0.05^-)$

$a_{11} = 0.57$

Q4. The third term of an arthmetic sequence is 14 and nineth term is -1. Find first four terms of sequence.

Sol:-

Given:- $a_3 = 14$, $a_9 = -1$

As, $a_n = a_1 + (n-1)d$

OR $a_3 = a_1 + (3-1)d$, $a_9 = a_1 + (9-1)d$

$14 = a_1 + 2d$ $\quad -1 = a_1 + 8d$

$a_1 = 14 - 2d$ $\quad -1 = (14-2d) + 8d$

$-1 = 14 + 6d$

$d = \frac{-1-14}{6}$

$d = \frac{-5}{2}$

$\therefore a_1 = 14 - 2\left(\frac{-5}{2}\right)$

$a_1 = 19$

So, 19, 16.5, 14, 11.5, ...

Q5. Find an arthmetic sequence for $a_{17} = -40$ and $a_{28} = -73$. Find a_1 and d. write first five terms of the sequence.

Sol:-

Given:- $a_{17} = -40$, $a_{28} = -73$

As, $a_n = a_1 + (n-1)d$

$\therefore \quad a_{17} = a_1 + (17-1)d$

$-40 = a_1 + 16d$

$a_1 = -40 - 16d$

$\therefore \quad a_1 = -40 - 16(-3)$

$a_1 = 8$

So, 8, 5, 2, −1, −4, ...

The fifth term of an arthmetic sequence is 19 and 11^{th} term is 43. Find first term and 87^{th} term.

Sol:-

Given :- $a_5 = 19, \quad a_{11} = 43$

As, $a_n = a_1 + (n-1)d$

$\therefore \quad a_5 = a_1 + (5-1)d$, $a_{11} = a_1 + (11-1)d$

$19 = a_1 + 4d$, $43 = a_1 + 10d$

$a_1 = 19 - 4d$, $\therefore 43 = 19 - 4d + 10d$

$\Rightarrow a_1 = 19 - 4(4)$ $d = \frac{43 - 19}{6}$

$\therefore a_1 = 3$ $d = 4$

and $a_{87} = 3 + (87-1)(4)$

$a_{87} = 347$

Q7. Which term of the sequence $-6, -2, 2, \ldots$ is 70?

Sol:-

The given sequence is arithmetic sequence as common difference $= d = 4$

$\therefore \quad a_1 = -6 \;, \quad d = 4$

As, $a_n = a_1 + (n-1)d$

Let $\quad 70 = a_n$

$\therefore \quad 70 = -6 + (n-1)(4)$

$70 + 6 = 4n - 4$

$76 + 4 = 4n$

$n = 20$

Therefore, 70 is the 20^{th} term of the sequence.

Q8 Which term of the sequence $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, \ldots$ is $\frac{-105}{2}$?

Sol:-

Common difference $= d = \frac{3}{2} - \frac{5}{2} = -1$

$\therefore \quad a_1 = \frac{5}{2} \;, \quad d = -1$

As, $a_n = a_1 + (n-1)d$

Let $\quad \frac{-105}{2} = a_n$

$$\therefore \frac{-105}{2} = \frac{5}{2} + (n-1)(-1)$$

$$\frac{-105}{2} - \frac{5}{2} = 1 - n$$

$$-55 = 1 - n$$

$$n = 56$$

$\therefore \frac{-105}{2}$ is 56^{th} term of the sequence

7. If $\frac{1}{a}, b, \frac{1}{c}$ are in A.P. show that common difference is $\frac{a+c}{2ac}$ for b.

Proof:-

Given arithmetic progression

$$\frac{1}{a}, b, \frac{1}{c}$$

Here, Difference of 2^{nd} and $1^{st} = b - \frac{1}{a} = d$

$$d = \frac{ab-1}{a}$$

Difference of 3^{rd} and $4^{th} = d = \frac{1}{c} - b$

$$d = \frac{1-cb}{c}$$

$$\therefore \frac{1-cb}{c} = \frac{ab-1}{a}$$

$$\therefore \quad \frac{1}{b} - b = b - \frac{1}{a}$$

$$\therefore \quad b + b = \frac{1}{c} + \frac{1}{a}$$

$$2b = \frac{c+a}{ac}$$

$$b = \frac{c+a}{2ac}$$

Hence proved.

Q10. During a free fall, a sky diver falls 16 feet in the first second, 48 feet in the 2nd second and 80 feet in the 3rd second. If he contniues to fall at this rate, how many feet he will fall during the 8th second

Sol:-

The variation in the height of free fall is arithmetic sequence

$\therefore$ 16 ft, 48 ft, 80 ft, ...

common-difference = 48 - 16 $\Rightarrow$ 32

and $a_1 = 16$

so, at 8th second i.e a_8

As, $a_n = a_1 + (n-1)d$

$$a_8 = 16 + (8-1)(32)$$

$a_8 = 240$ feet at 8th second.

If Rs. 1,000 is saved on August 1, Rs. 3,000 on August 2, Rs. 5,000 on August 3 and so on. How much is saved till August 20?

Sol:-

According to the given condition, the data in arthmetic progression will be:-

1,000, 3,000, 5,000, ...

Here, $a_1 = 1{,}000$ and $d = 3000 - 1000 \Rightarrow d = 2{,}000$

As. $a_n = a_1 + (n-1)d$

$a_{20} = 1000 + (20-1)(2000)$

$a_{20} = 39{,}000$

So. on 20 August there will Rs. 39,000 saved

12. A gardner is making a triangular planning with 35 plants in first row, 31 in second row, 27 in third row and so on. If the pattern is consistent. How many plants will be there in eight row?

Sol:- According to the given condition

35, 31, 27, ...

Here, $a_1 = 35$, $d = 31 - 35 \Rightarrow d = -4$

Ans. $a_n = a_1 + (n-1)d$

$\therefore \quad a_8 = 35 + (8-1)(-4)$

$a_8 = 7$

Therefore, in 8th row there will be 7 plants.

Q13. Fin A.M between

i) 7 and 17

Sol:-

Given:- $a=7$, $b=17$

Arthmetic mean between a and b' $= \frac{a+b}{2}$

$$A.M = \frac{7+17}{2}$$

$$A.M = 12$$

ii) $3+3\sqrt{2}$ and $7-3\sqrt{2}$

Sol:-

$$a = 3+3\sqrt{2} \quad , \quad b = 7-3\sqrt{2}$$

$$A.M = \frac{a+b}{2} = \frac{3+3\sqrt{2}+7-3\sqrt{2}}{2}$$

$$A.M = 5$$

iii) $7\sqrt{5}$ and $\sqrt{5}$

Sol:-

$$a = 7\sqrt{5} \quad , \quad b = \sqrt{5}$$

As, $A.M = \frac{a+b}{2} = \frac{7\sqrt{5}+\sqrt{5}}{2}$

$$= \frac{8\sqrt{5}}{2}$$

$$A.M = 4\sqrt{5}$$

iv) $2y+5$ and $5y+3$

Sol:-

$$a = 2y+5 \quad , \quad b = 5y+3$$

As, $A.M = \frac{a+b}{2} = \frac{2y+5+5y+3}{2}$

$$= \frac{7y+8}{2}$$

$$A.M = \frac{7}{2}y + 4$$

14. Find 'b' if 10 is A.M between b and 20.

Sol:-

Given:- $A.M = 10$, $a = 20$ and $c = b$

$\because$ $A.M = \frac{a+c}{2} \Rightarrow 10 = \frac{20+b}{2}$

$2(10) = 20+b$

$20-20 = b$

$b = 0$

Q15. Find x and y if 2 and 13 are two arithmetic means between x and y.

Sol:-

According to the given condition, the arithmetic sequence will be:-

$x, 2, 13, y$

Here, $a_1 = x$ and $d = 13-2 = 11$

As, $a_n = a_1 + (n-1)d$

$\therefore a_2 = a_1 + (2-1)d$

$\Rightarrow 2 = a_1 + (11)$

$a_1 = 2-11$

$a_1 = x = -9$

and $a_4 = a_1 + (4-1)d$

$y = -9 + 3(11)$

$y = 24$

Q16. Find arithmetic mean between 5 and 17.

Sol:-

Let x and y be arithmetic mean.

$\therefore$ 5, x, y, 17 is arithmetic sequence

Here, $d = x - 5 \Rightarrow x = d + 5 \quad -(i)$

$a_3 = a_1 + (3-1)d$

$y = 5 + 2d \quad -(ii)$

$a_2 = a_1 + (2-1)d$

also, $x = a_4 = a_1 + (4-1)d$

$17 = 5 + 3d$

$3d = 17 - 5$

$d = 4$

Put d in (i) and (ii)

$\therefore \quad x = 4 + 5 \Rightarrow x = 9$

and $y = 5 + 2(4) \Rightarrow y = 13$

Q17. Find three arithmetic means between 2 & -18.

Sol:-

let x, y and z be three arithmetic means

$\therefore$ 2, x, y, z, -18 is arithmetic sequence.

As, $a_n = a_1 + (n-1)d$

$\therefore \quad a_2 = a_1 + (2-1)d$

$x = 2 + d \quad -(i)$

$a_3 = a_1 + (3-1)d$

$y = 2 + 2d \quad -(ii)$

$a_4 = a_1 + (4-1)d$

$z = 2 + 3d \quad -(iii)$

and $a_5 = a_1 + (5-1)d$

$-18 = 2 + 4d$

$d4 = -18 - 2$

$\therefore d = -5$

$\therefore$ eq.(i) $\Rightarrow$ $x = 2 - 5$

$x = -3$

eq.(ii) $\Rightarrow$ $y = 2 + 2(-5)$

$y = -8$

eq.(iii) $\Rightarrow$ $z = 2 + 3(-5)$

$z = -12$