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Where Minds Pollinate

Class 11
Mathematics

Chapter 3
Review Exercise 3
Notes

National Book Foundation

Review Ex 3

Q2 (i) **Parallel:-** $\vec{a} = \alpha\vec{b}$; α is any scalc

$3\hat{i} - 2\hat{j} + 6\hat{k} = \alpha(\hat{i} - d\hat{j} + 3d\hat{k})$

compare $\hat{i}, \hat{j}$ and $\hat{k}$ unit vectors.

$\boxed{3 = \alpha}$, $\cancel{+}2 = \cancel{-}d\alpha$

$2 = d(3) \Rightarrow \boxed{d = \frac{2}{3}}$ Answer

(ii) **Perpendicular:-**

$a \cdot b = 0$

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$(3\hat{i} - 2\hat{j} + 6\hat{k})\cdot(\hat{i} - d\hat{j} + 3d\hat{k}) = 0$

$3 + 2d + 18d = 0$

$20d = -3 \Rightarrow \boxed{d = \frac{-3}{20}}$ Ans

Q3. Sol $(\vec{a}+\vec{b})$ along $(\vec{a}-\vec{b}) = \dfrac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}$

$\vec{a}+\vec{b} = -2\hat{i} + 0\hat{j} + 8\hat{k}$

$\vec{a}-\vec{b} = -4\hat{i} + 4\hat{j} + 0\hat{k}$

$|\vec{a}-\vec{b}| = \sqrt{(-4)^2 + (4)^2 + (0)^2} = 4\sqrt{2}$.

Now putting values in above formula we get

$$= \frac{(-2\hat{i} + 0\hat{j} + 8\hat{k})\cdot(-4\hat{i} + 4\hat{j} + 0\hat{k})}{4\sqrt{2}}$$

$$= \frac{8+0+0}{4\sqrt{2}} = \frac{8}{4\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{\sqrt{2}}$$ Ans

Q4) $(\vec{u} - \vec{v})\cdot(\vec{u} + \vec{v}) = 18$

$\vec{u}\cdot\vec{u} + \cancel{\vec{u}\cdot\vec{v}} - \cancel{\vec{v}\cdot\vec{u}} - \vec{v}\cdot\vec{v} = 18$

$|\vec{u}|^2 - |\vec{v}|^2 = 18 \qquad \because \vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$

$|\vec{u}|^2 - 1 = 18$

$|\vec{u}|^2 = 18 + 1$

$\sqrt{|\vec{u}|^2} = \sqrt{19}$

$\boxed{|\vec{u}| = \sqrt{19}}$ Answer

Q5) Sol:- let $\vec{U} = \hat{i}+\hat{j}-\hat{k}$ and

$\vec{V} = (2\hat{i}-3\hat{j}+\hat{k}) + (d\hat{i}-2\hat{j}+3\hat{k})$

$= (2+d)\hat{i} - 5\hat{j} + 4\hat{k}$

Now given condition $\vec{U}\cdot\hat{V} = -1$

$\Rightarrow (\hat{i}+\hat{j}-\hat{k})\cdot\dfrac{(2+d)\hat{i}-5\hat{j}+4\hat{k}}{\sqrt{(2+d)^2+25+16}} = -1 \quad \therefore \hat{V} = \dfrac{\vec{V}}{|V|}$

$\dfrac{2+d-5-4}{\sqrt{4+d^2+4d+41}} = -1 \Rightarrow \left(\dfrac{d-4}{\sqrt{d^2+4d+45}}\right)^2 = (-1)^2$

$\cancel{d^2}+49-14d = \cancel{d^2}+4d+45$

$49-45 = 4d+14d$

$\dfrac{4}{18} = d \Rightarrow \boxed{d = \dfrac{2}{9}}$ Answer.

Q6 $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = 2|\vec{a}|^2$

Proof: let $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\vec{a}\times\hat{i} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\times\hat{i}$

$\vec{a}\times\hat{i} = a_x(\cancel{\hat{i}\times\hat{i}}) + a_y(\hat{j}\times\hat{i}) + a_z(\hat{k}\times\hat{i})$

$\vec{a}\times\hat{i} = -a_y\hat{k} + a_z\hat{j}$

$|\vec{a}\times\hat{i}|^2 = a_y^2 + a_z^2$ —— (1)

Similarly

Concept

$\hat{i}\times\hat{j} = k$

$\hat{j}\times\hat{k} = i$

$k\times\hat{i} = j$

$\hat{j}\times\hat{i} = -k$

$k\times\hat{j} = -i$

$\hat{i}\times k = -\hat{j}$

and

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = k\times k = 0$

$\vec{a} \times \hat{j} = a_x(\hat{i} \times \hat{j}) + a_z(\hat{k} \times \hat{j})$

$\vec{a} \times \hat{j} = a_x\hat{k} - a_z\hat{i}$

$|\vec{a} \times \hat{j}|^2 = a_x^2 + a_z^2$ — (2)

Similarly $\vec{a} \times \hat{k} = a_x(\hat{i} \times \hat{k}) + a_y(\hat{j} \times \hat{k}) = -a_x\hat{j} + a_y\hat{i}$

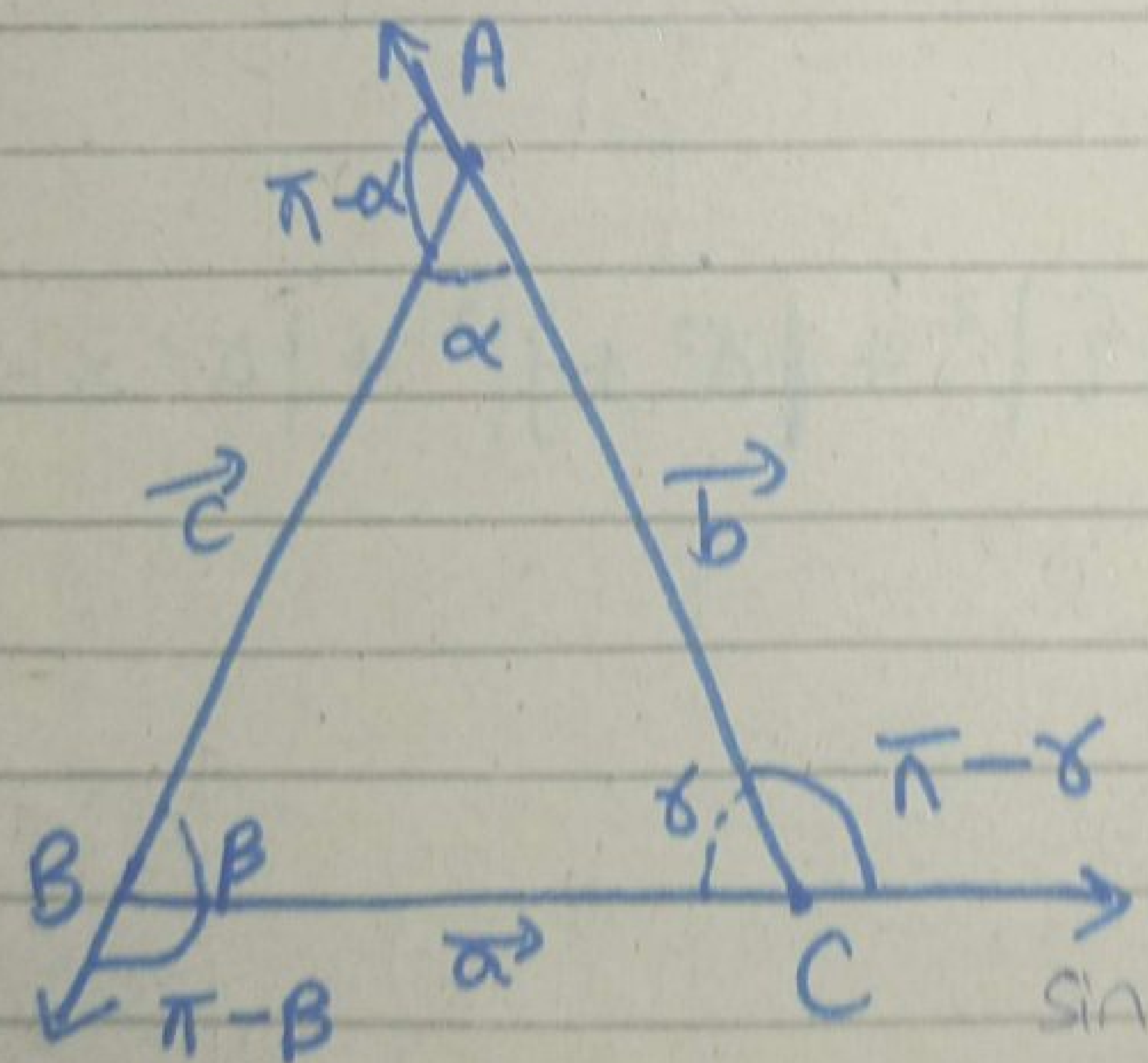
$|\vec{a} \times \hat{k}|^2 = a_x^2 + a_y^2$ — (3) Now add (1)(2)(3)

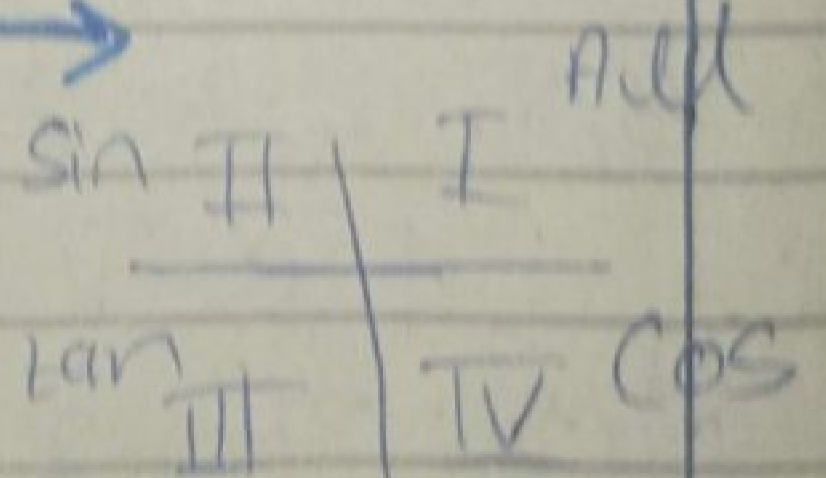
$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = a_y^2 + a_z^2 + a_x^2 + a_z^2 + a_x^2 + a_y^2$

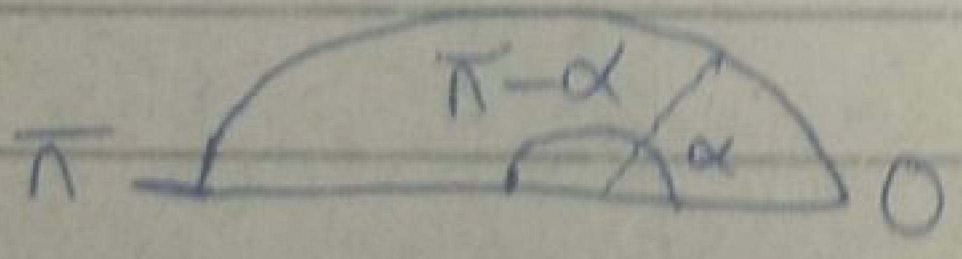
$= 2a_x^2 + 2a_y^2 + 2a_z^2 = 2(a_x^2 + a_y^2 + a_z^2)$

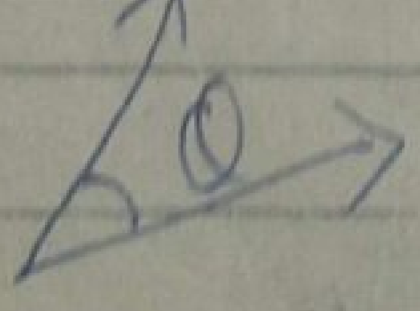
$= \boxed{2|\vec{a}|^2}$ Hence proved.

Q7.









$\sin(\alpha - \beta)$

$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin(\pi - \alpha) =$

$= \sin\pi\cos\alpha - \cos\pi\sin\alpha = \sin\alpha$

Suppose $\vec{a}, \vec{b}$ and $\vec{c}$ are along the sides BC, CA, and AB respectively of the triangle ABC

$\therefore \vec{a} + \vec{b} + \vec{c} = 0$ $\quad$ ∵ by using vectors

$\Rightarrow \vec{b} + \vec{c} = -\vec{a}$ —— (ii)

Take cross product with $\vec{c}$

$(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{c}) = -(\vec{a} \times \vec{c})$

$(\vec{b} \times \vec{c}) = \vec{c} \times \vec{a}$ $\quad$ $\therefore \vec{c} \times \vec{c} = 0$

$\therefore \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

Taking magnitude on b/s

$|\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$

$|\vec{b}||\vec{c}| \sin(\pi - \alpha) = |\vec{c}||\vec{a}| \sin(\pi - \beta)$

$b\cancel{c} \sin\alpha = \cancel{c}a \sin\beta$

$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$ —— (1)

Similarly by taking cross product of (i) with $\underline{b}$, we have

$(\vec{b} \times \vec{b}) + (\vec{c} \times \vec{b}) = -(\vec{a} \times \vec{b})$

$\vec{c} \times \vec{b} = \vec{b} \times \vec{a}$, Take magnitude

$|\vec{c} \times \vec{b}| = |\vec{b} \times \vec{a}|$

$|\vec{c}||\vec{b}| \sin(\pi - \alpha) = |\vec{b}||\vec{a}| \sin(\pi - \gamma)$

$c\cancel{b} \sin\alpha = \cancel{b}a \sin\gamma$

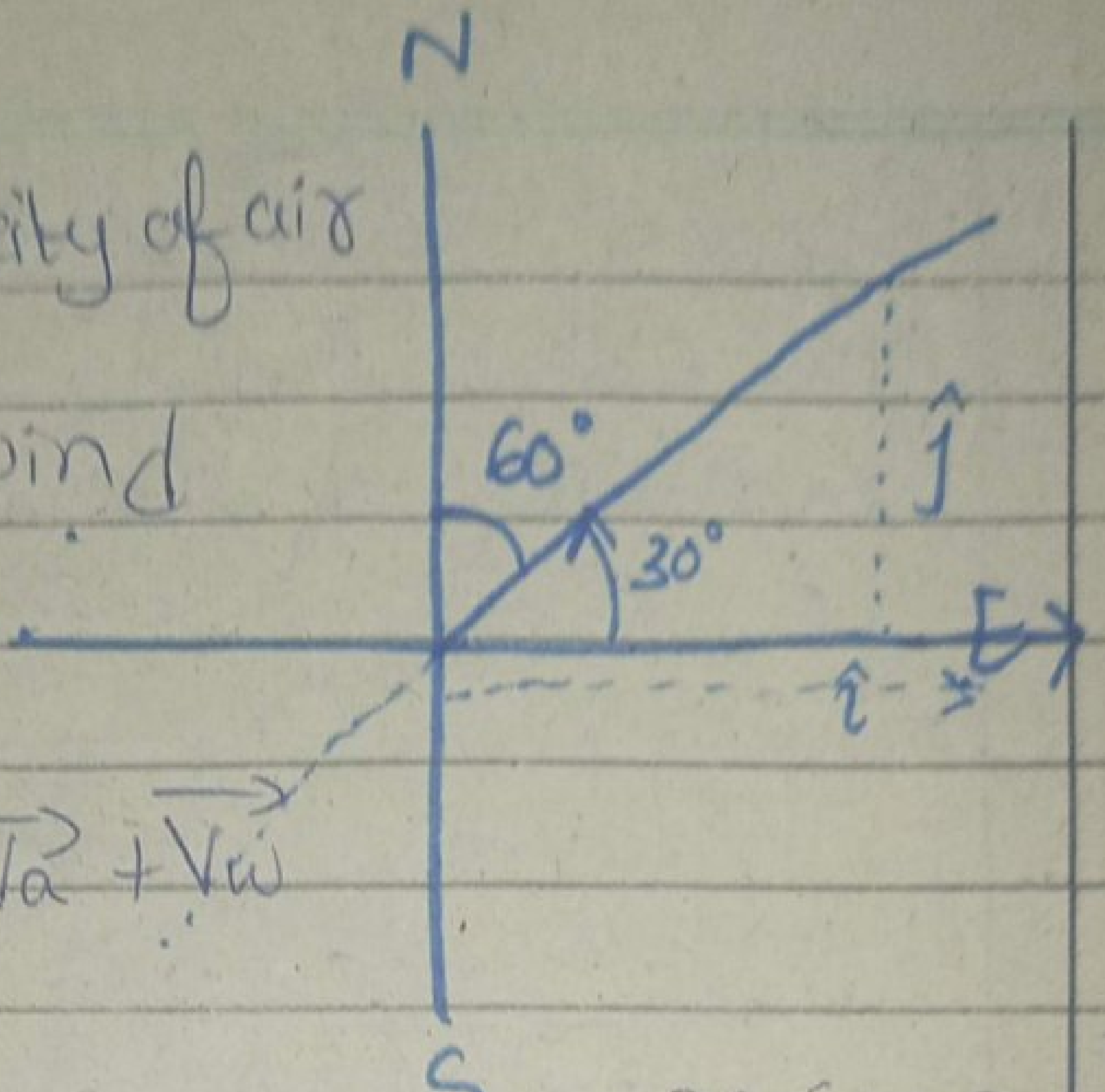
$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$ —— (2) combine (1) & (2) we get

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ Proved.

Q8) Sol $\vec{V_a}$ = velocity of air

$\vec{V_w}$ = velocity of wind

$\vec{V_g}$ = velocity of ground.



⇒ Formula :- $\vec{V_g} = \vec{V_a} + \vec{V_w}$

90-60=30°

1st

$\vec{V_a} = (r, \theta) = (200, 30°)$

Resolve into components

$\vec{V_a} = 200(\cos 30° \hat{i} + \sin 30° \hat{j})$

$\vec{V_a} = 200(0.8660\hat{i} + 0.5\hat{j})$

$\boxed{\vec{V_a} = 173.2\hat{i} + 100\hat{j}}$

2nd $\boxed{\vec{V_w} = 40\hat{i}}$

And $\vec{V_g} = \vec{V_a} + \vec{V_w} = 173.2\hat{i} + 100\hat{j} + 40\hat{i}$

$\boxed{\vec{V_g} = 213.2\hat{i} + 100\hat{j}}$

$|\vec{V_g}| = \sqrt{(213.2)^2 + (100)^2}$

$\boxed{|\vec{V_g}| = 235.48\ km/h}$

And $\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{100}{213.2}\right)$

$\boxed{\theta = 25.12°}$

Since angle is measuring from east so

$$\theta = 90° - 25.12°$$

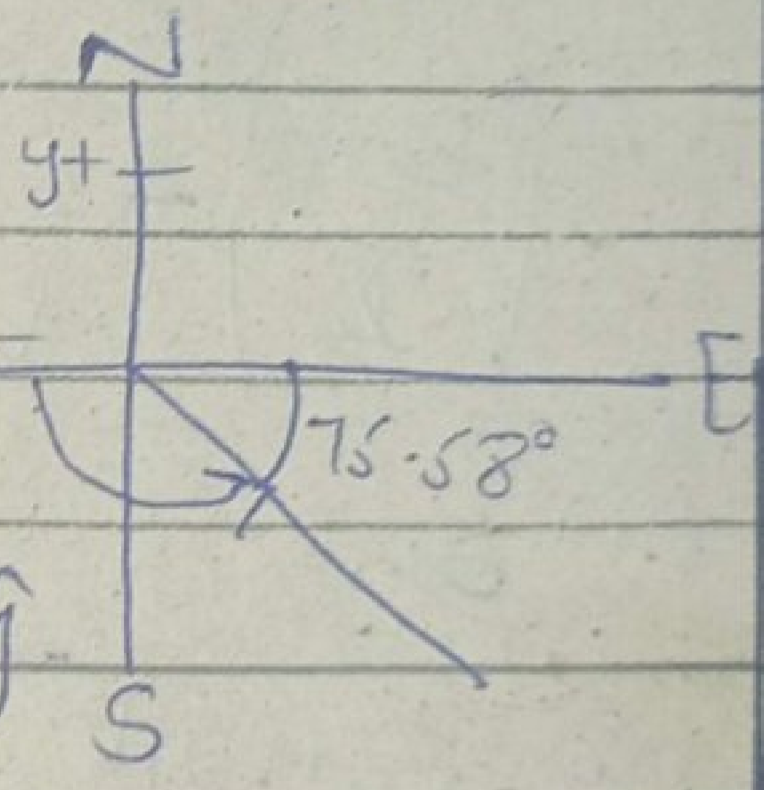
$$\boxed{\theta = 64.8°}$$ Answers

Q9. Sol $\vec{V_g} = (250, 100°) = 250(\cos 100° \hat{i} + \sin 100° \hat{j})$

$$\boxed{\vec{V_g} = -43.41\hat{i} + 246.20\hat{j}}$$

$$\vec{V_w} = (20, 30°) = 20(\cos 30° \hat{i} + \sin 30° \hat{j})$$

$$\boxed{\vec{V_w} = 17.32\hat{i} + 10\hat{j}}$$, $V_a = ?$



Formula: $\vec{V_g} = \vec{V_a} + \vec{V_w}$

or $\vec{V_a} = \vec{V_g} - \vec{V_w}$

$$\vec{V_a} = (-43.41 - 17.32)\hat{i} + (246.20 - 10)\hat{j}$$

$$\vec{V_a} = -(60.73)\hat{i} + 236.20\hat{j}$$

$$\boxed{|\vec{V_a}| = 243.8 \text{ Km/h}}$$ Now $\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

$$\theta = \tan^{-1}\left(\frac{236.20}{-60.73}\right) = -\tan^{-1}(3.8893)$$

$$= -75.58$$

Since angle measuring from west

$$= 180 - 75.58°$$

$$\boxed{\theta = 104.42°}$$