Studybeesofficial

Where Minds Pollinate

Class 11

Mathematics

Chapter 3

Exercise 3.3

Notes

National Book Foundation

Exercise 3.3

Q1) (i) $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} - 3\hat{j} + 7\hat{k}$

Sol:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -3 & 7 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \hat{i} \begin{vmatrix} 1 & -1 \\ -3 & 7 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & -1 \\ 1 & 7 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix}$$

$$= \hat{i}(7-3) - \hat{j}(14+1) + \hat{k}(-6-1)$$

$\vec{a} \times \vec{b} = 4\hat{i} - 15\hat{j} - 7\hat{k}$ —— (1) Ans

And

$$\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 7 \\ 2 & 1 & -1 \end{vmatrix}$$

$$\vec{b} \times \vec{a} = \hat{i}(3-7) - \hat{j}(-1-14) + \hat{k}(1+6)$$

$\boxed{\vec{b} \times \vec{a} = -4\hat{i} + 15\hat{j} + 7\hat{k}}$ —— (2)

From (1) & (2) It is clear that

$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$ Proved $\therefore$ property

(ii) $\vec{a} = 7\hat{i} + 3\hat{j} + 9\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 9 \\ 2 & -3 & 1 \end{vmatrix}$$

Condition for parallel: $\boxed{a \times b = 0}$

$\vec{a} \times \vec{b} = \hat{i}(3+27) - \hat{j}(7-18) + \hat{k}(-21-6)$

$\boxed{\vec{a} \times \vec{b} = 30\hat{i} + 11\hat{j} - 27\hat{k}}$ —— (1)

And $\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 7 & 3 & 9 \end{vmatrix}$

$= \hat{i}(-27-3) - \hat{j}(18-7) + \hat{k}(6+21)$

$\boxed{\vec{b} \times \vec{a} = -30\hat{i} - 11\hat{j} + 27\hat{k}}$ —— (2)

Hence from (1) & (2) $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$ Po

(iii) $\vec{a} = \hat{i} - 2\hat{k}$; $\vec{b} = 3\hat{i} + 2\hat{j}$.

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -2 \\ 3 & 2 & 0 \end{vmatrix}$

$= \hat{i}(0+4) - \hat{j}(0+6) + \hat{k}(2-0)$

$\boxed{\vec{a} \times \vec{b} = 4\hat{i} - 6\hat{j} + 2\hat{k}}$ —— (1)

And $\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 1 & 0 & -2 \end{vmatrix}$

$= \hat{i}(-4-0) - \hat{j}(-6-0) + \hat{k}(0-2)$

$\boxed{\vec{b} \times \vec{a} = -4\hat{i} + 6\hat{j} - 2\hat{k}}$ —— (2)

Hence from (1) & (2) it is clear that $(\vec{a} \times \vec{b}) = -(\vec{b} \times \vec{a})$ Proved.

Q2 (i) $\vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$; $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

Sol:- $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & -3 & 4 \end{vmatrix}$

$= \hat{i}(-24+6) - \hat{j}(12-4) + \hat{k}(-9+12)$

$\boxed{\vec{a} \times \vec{b} = -18\hat{i} - 8\hat{j} + 3\hat{k}}$

And $(\vec{a} \times \vec{b}) \cdot \vec{a} = -18(3) - 8(-6) + 3(2)$

$(\vec{a} \times \vec{b}) \cdot \vec{a} = -54 + 48 + 6$

$= 0$

Hence $(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$

$\boxed{(\vec{a} \times \vec{b}) \perp \vec{a}}$ Proved

And $(\vec{a} \times \vec{b}) \cdot \vec{b} = -18(2) - 8(-3) + 3(4)$

$= -36 + 24 + 12$

$(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$

$\boxed{(\vec{a} \times \vec{b}) \perp \vec{b} = 0}$ Hence proved

(ii) $\vec{a} = 4\hat{i} - 2\hat{j} + 3\hat{k}$; $\vec{b} = \hat{i} + \hat{j} - 3\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 3 \\ 1 & 1 & -3 \end{vmatrix} = \hat{i}(6-3) - \hat{j}(-12-3) + \hat{k}(4+2)$

$\boxed{\vec{a} \times \vec{b} = 3\hat{i} + 15\hat{j} + 6\hat{k}}$

And $(\vec{a} \times \vec{b}) \cdot \vec{a} = 3(4) + 15(-2) + 6(3)$

$(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$; then $\boxed{(\vec{a} \times \vec{b}) \perp \vec{a}}$

And $(\vec{a}\times\vec{b})\cdot\vec{b} = 3(1)+15(1)+6(-3)$
$= 0$ then $\boxed{(\vec{a}\times\vec{b}) \perp \vec{b}}$ proved

Q3 (i) $\vec{a} = 2\hat{i}-4\hat{j}+3\hat{k}$, $\vec{b} = \hat{i}-3\hat{j}+4\hat{k}$

Formula:- $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

$\because$ $\hat{n}$ is unit vector .

$\Rightarrow$ $|\vec{a}\times\vec{b}| = ||\vec{a}||\vec{b}||\,|\sin\theta|\,|\hat{n}|$

$\Rightarrow$ $|\hat{n}| = 1$.

$\Rightarrow$ $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$$\boxed{\frac{\vec{a}\times\vec{b}}{|\vec{a}||\vec{b}|} = \sin\theta}$$

Solution . $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 3 \\ 1 & -3 & 4 \end{vmatrix}$

$= \hat{i}(-16+9) - \hat{j}(8-3) + \hat{k}(-6+4)$
$= \boxed{-7\hat{i} - 15\hat{j} - 2\hat{k}}$

Now $|\vec{a}\times\vec{b}| = \sqrt{(-7)^2+(-5)^2+(-2)^2} = \sqrt{78}$

$|\vec{a}| = \sqrt{(2)^2+(-4)^2+(3)^2} = \sqrt{29}$

$|\vec{b}| = \sqrt{(1)^2+(-3)^2+(4)^2} = \sqrt{26}$

$$\Rightarrow \boxed{\sin\theta = \frac{\sqrt{78}}{\sqrt{26}\,\sqrt{29}}}$$ Answer .

(ii) $\vec{a} = 4\hat{i} - 3\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 7\hat{j} + 5\hat{k}$

Sol:- $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 4 & -3 & 2 \\ 3 & -7 & 5 \end{vmatrix} = \hat{i}(-15+14) - \hat{j}[20-6] + \hat{k}(-28+9)$

$= \boxed{-\hat{i} - 14\hat{j} - 19\hat{k}}$

$|\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + (-14)^2 + (-19)^2} = 3\sqrt{62}$

$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (2)^2} = \sqrt{29}$

$|\vec{b}| = \sqrt{(3)^2 + (-7)^2 + (5)^2} = \sqrt{83}$

$\boxed{\sin\theta = \dfrac{3\sqrt{62}}{\sqrt{29}\sqrt{83}}}$ Answer

on end

4(i) Sol: Let $\vec{x}$

(ii) Sol:- let $\vec{a} = 5\hat{i} + 2\hat{j} - 3\hat{k}$ & $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\Rightarrow \vec{a} = \vec{x} + \vec{y} \Rightarrow \boxed{\vec{x} + \vec{y} = 5\hat{i} + 2\hat{j} - 3\hat{k}}$ — (1)

1st Condition $\quad \vec{x} = \underset{\text{parallel}}{\lambda}(\vec{b})$

$\boxed{\vec{x} = 2\lambda\hat{i} - \lambda\hat{j} + 3\lambda\hat{k}}$

put in eq (1)

$\vec{y} = 5\hat{i} + 2\hat{j} - 3\hat{k} - 2\lambda\hat{i} + \lambda\hat{j} - 3\lambda\hat{k}$

$\boxed{\vec{y} = (5 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (-3 - 3\lambda)\hat{k}}$

2nd Condition : $\quad \vec{y} \cdot \vec{b} = 0$

$(5 - 2\lambda)(2) + (2 + \lambda)(-1) + (-3 - 3\lambda)(3) = 0$

$10 - 4\lambda - 2 - \lambda - 9 - 9\lambda = 0$

$-1 - 14\lambda = 0$

$\boxed{\lambda = \frac{-1}{14}}$

Therefore putting values of λ in $\vec{x}$ & $\vec{y}$

$\vec{x} = 2\left(\frac{-1}{14}\right)\hat{i} - \left(\frac{-1}{14}\right)\hat{j} + 3\left(\frac{-1}{14}\right)\hat{k}$

$\boxed{\vec{x} = \frac{-1}{7}\hat{i} + \frac{1}{14}\hat{j} - \frac{3}{14}\hat{k}}$

And $\vec{y} = \left(5 + \frac{2}{14}\right)\hat{i} + \left(2 - \frac{1}{14}\right)\hat{j} + \left(-3 + \frac{3}{14}\right)\hat{k}$

$\boxed{\vec{y} = \frac{36}{7}\hat{i} + \frac{27}{14}\hat{j} - \frac{39}{14}\hat{k}}$

Answer

~~Prove~~

Q5 (i) $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

Proof:- $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

Taking square on b/s

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2.|\vec{b}|^2\sin^2\theta$$

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2.|\vec{b}|^2.(1-\cos^2\theta)$$

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2.|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$$

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (|\vec{a}||\vec{b}|\cos\theta)^2 \quad \therefore \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$$ Hence proved

(ii) Sol:- let $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

Given conditions

$\vec{d}\cdot\vec{a} = 0$; $\vec{d}\cdot\vec{b} = 0$; and $\vec{c}\cdot\vec{d} = 1$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}-2\hat{j})=0$; $(x\hat{i}+y\hat{j}+z\hat{k})(2\hat{i}+\hat{k})=0$; $(x\hat{i}+y\hat{j}+z\hat{k})\cdot(3\hat{j}+2\hat{k})$

$x(1)+y(-2)+z(0)=0$; $x(2)+y(0)+z(1)=0$; $0(x)+3(y)+2(z)=1$

$x-2y=0$ — (1) ; $2x+z=0$ — (2) ; $3y+2z=1$ — (3)

Solve eq (2) & (3)

$$4x+2z=0$$
$$\underline{\pm 3y \pm 2z = \pm 1}$$
$$4x-3y=-1 \quad \text{— (4)}$$

Solve eq (1) & (4)

$$4x-8y=0$$
$$\underline{\pm 4x \mp 3y = \mp 1}$$
$$-5y=1$$
$$\boxed{y=-\frac{1}{5}}$$

put y in eq (1)

$$x-2\left(\frac{-1}{5}\right)=0$$
$$x+\frac{2}{5}=0$$
$$\boxed{x=-\frac{2}{5}}$$

put x in eq (2)

$$2\left(\frac{-2}{5}\right)+z=0 \Rightarrow \boxed{z=\frac{4}{5}}$$

$$\vec{d} = \frac{-2}{5}\hat{i} - \frac{1}{5}\hat{j} + \frac{4}{5}\hat{k}. \text{ Ans}$$

① c) (i) Sol :- $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$.

Condition :- $\vec{a}\cdot\vec{b} = 3$

$1(x) - 2(y) + 3(z) = 3$

$x - 2y + 3z = 3$ —— ①

$\boxed{\vec{a}\times\vec{b} = \vec{c}}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ x & y & z \end{vmatrix} = \hat{i} + \hat{j} - \hat{k}$$

$\hat{i}(-2z - 3y) - \hat{j}(z - 3x) + \hat{k}(y + 2x) = \hat{i} + \hat{j} - \hat{k}$

Compare both sides

$-2z - 3y = 1$; $-z + 3x = 1$

$3y + 2z = -1$ —— ① ; $\boxed{3x - 1 = z}$ —— ②

and $2x + y = -1 \Rightarrow \boxed{y = -1 - 2x}$ —— ③

put eq ② & ③ in eq ①

$x - 2(-1 - 2x) + 3(3x - 1) = 3$

$x + 2 + 4x + 9x - 3 - 3 = 0$

$14x - 4 = 0$

$x = \frac{\cancel{4}^{2}}{\cancel{14}_{7}}$ $\boxed{x = \frac{2}{7}}$

put x in (2) & (3)

$z = 3\left(\frac{2}{7}\right) - 1$

$z = \frac{6}{7} - 1 = \frac{6-7}{7} = \boxed{\frac{-1}{7}}$

$y = -1 - 2\left(\frac{2}{7}\right) = \frac{-7-4}{7} = \frac{-11}{7}$

$$\boxed{\vec{b} = \frac{2}{7}\hat{i} - \frac{1}{7}\hat{j} - \frac{11}{7}\hat{k}}$$

(ii) Proof: $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ — (1) , $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ — (2)

eq (1) − eq (2)

$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = (\vec{c} \times \vec{d}) - (\vec{b} \times \vec{d})$

$\vec{a} \times (\vec{b} - \vec{c}) = (\vec{c} - \vec{b}) \times \vec{d}$ ∴ commutative property

$\vec{a} \times (\vec{b} - \vec{c}) = -\vec{d} \times (\vec{c} - \vec{b})$ ∴ $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

If $\vec{a} \times \vec{b} = 0$ then $\vec{a}$ parallel to $\vec{b}$

$\vec{a} \times (\vec{b} - \vec{c}) = \vec{d} \times (\vec{b} - \vec{c})$

$\vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = 0$

$(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$

$$\boxed{(\vec{a} - \vec{d}) \parallel (\vec{b} - \vec{c})}$$

7 (i) Proof: $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$

$\vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} = 0$

$a\cdot(\vec{b}-\vec{c}) = 0$

Given $\vec{a}$ is non zero i.e $\vec{a} \neq 0$

$\Rightarrow \vec{b}-\vec{c} = 0 \Rightarrow \boxed{\vec{b}=\vec{c}}$ proved this shows ther are perpend

And

$\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$

$\vec{a}\times\vec{b} - \vec{a}\times\vec{c} = 0$

$a\times(\vec{b}-\vec{c}) = 0$

$\vec{a} \neq 0 \quad \vec{b}-\vec{c} = 0$ this show there are parallel

$\boxed{\vec{b}=\vec{c}}$ Hence proved

(ii) Proof: $\vec{a}+\vec{b}+\vec{c} = 0 \Rightarrow \vec{a}+\vec{b} = -\vec{c}$ — (1)

Taking $\vec{a}$ cross with eq (1) on bothsides

$(\vec{a}+\vec{b})\times\vec{a} = -\vec{c}\times\vec{a} \Rightarrow (\vec{a}\times\vec{a}) + (\vec{b}\times\vec{a}) = -\vec{c}\times\vec{a}$

$+(\vec{a}\times\vec{b}) = +(\vec{c}\times\vec{a})$ — (4) $\therefore \vec{a}\times\vec{a} = 0$ & $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

$(\vec{a}\times\vec{b}) = (\vec{c}\times a)$ — (1)

Taking $\vec{b}$ cross with eq (1) on b/s

$(\vec{a}+\vec{b})\times\vec{b} = -\vec{c}\times\vec{b}$

$\vec{a}\times\vec{b} + \vec{b}\times\vec{b} = -\vec{c}\times\vec{b}$

$\boxed{\vec{a}\times\vec{b} = \vec{b}\times\vec{c}}$ — (2)

Taking $\vec{c}$ cross with eq (1) on b/s

$(\vec{a}+\vec{b})\times\vec{c} = -\vec{c}\times\vec{c}$

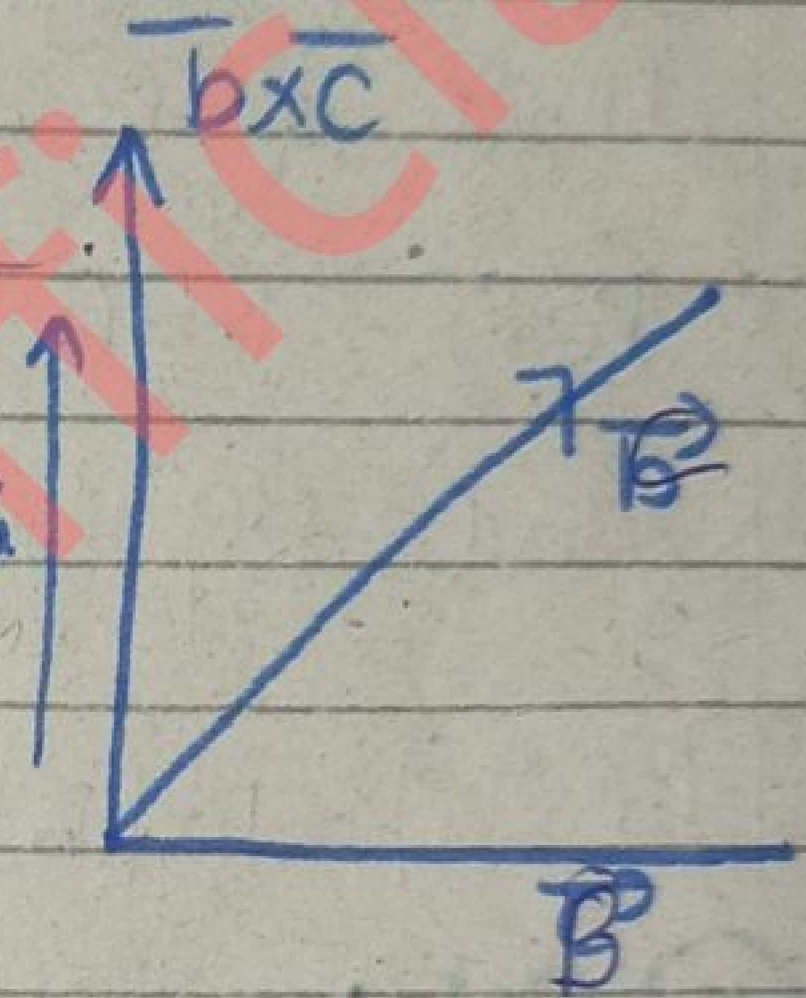
$\vec{a}\times\vec{c}+\vec{b}\times\vec{c} = 0$

$\vec{b}\times\vec{c} = -\vec{a}\times\vec{c}$ or $\boxed{\vec{b}\times\vec{c} = \vec{c}\times\vec{a}}$ — (3)

from eq (1) (2) & (3) we get

$\boxed{\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}}$

Q8) Proof Given $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$.



$\Rightarrow \vec{a} \parallel (\vec{b}\times\vec{c})$

$\vec{a} = d(\vec{b}\times\vec{c})$

Taking absolute on both sides

$|\vec{a}| = |d||\vec{b}\times\vec{c}|$

$1 = |d||\vec{b}||\vec{c}|\sin\pi/6$

$1 = |d|(1)(1)(\frac{1}{2})$

$|d| = 2$ $\Rightarrow \boxed{d = \pm 2}$

Hence $\boxed{\vec{a} = \pm 2(\vec{b}\times\vec{c})}$ proved!

(ii) Proof R.H.S:- Expanding determinant

$= |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

$= |\vec{a}|^2|\vec{b}|^2 - (|\vec{a}||\vec{b}|\cos\theta)^2$

$= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$

$= |\vec{a}|^2|\vec{b}|^2\{1-\cos^2\theta\}$

$= |\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$= \left||\vec{a}||\vec{b}|\sin\theta\right|^2$

$= |\vec{a}\times\vec{b}|^2 = L.H.S$ proved.

dream

Q9) Sol (i) $\vec{a}\cdot\vec{b} = 60$

$|\vec{a}||\vec{b}|\cos\theta = 60 \Rightarrow (3)(5)\cos\theta = 60$

$\Rightarrow \cos\theta = \frac{60}{15} \Rightarrow \cos\theta = \frac{4}{1} \Rightarrow \cos\theta = \frac{B}{H}$

base $= 4$, hyp $= 1$ using pythagoras theorm

$(Hyp)^2 = B^2 + P^2 \Rightarrow (1)^2 = (P)^2 + (4)^2$

$(Per)^2 = 1 - 16 \Rightarrow \boxed{(Per) = \sqrt{-15}}$

$\Rightarrow \sin\theta = \frac{P}{H} \Rightarrow \boxed{\sin\theta = \frac{\sqrt{-15}}{1}}$

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta = (3)(5)(\sqrt{-15}) = 15\sqrt{-15}$ Ans

(ii) Sol $|\vec{a}\times\vec{b}| = 8$

$|\vec{a}||\vec{b}|\sin\theta = 8 \Rightarrow (2)(5)\sin\theta = 8$

$\sin\theta = \frac{\cancel{8}^{4}}{\cancel{10}_{5}} \Rightarrow \boxed{\sin\theta = \frac{4}{5}}$

$P = 4,\ H = 5$

$H^2 = B^2 + P^2 \Rightarrow \sqrt{B^2} = \sqrt{25 - 16}$

$\boxed{Base = \sqrt{9}} \Rightarrow \boxed{Base = 3}$

$\cos\theta = \frac{B}{H} \Rightarrow \boxed{\cos\theta = \frac{3}{5}}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\vec{a}\cdot\vec{b} = (2)(5)\left(\frac{3}{5}\right) \Rightarrow \boxed{\vec{a}\cdot\vec{b} = 6}$

Answer

Q10) (i) Sol Area of Parallelogram $= |\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & -2 & 7 \end{vmatrix}$$

$$= \hat{i}(-21+2) - \hat{j}(14-1) + \hat{k}(-4+3)$$

$$\vec{a} \times \vec{b} = -19\hat{i} - 13\hat{j} - \hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-19)^2 + (-13)^2 + (-1)^2}$$

$$= \sqrt{531}$$

$$\boxed{|\vec{a} \times \vec{b}| = 3\sqrt{59}}$$

(ii) Sol:- let $A(1,-1,1)$, $B(2,1,2)$, $C(3,0,-1)$

$$\vec{AB} = \vec{OB} - \vec{OA} = (2,1,2) - (1,-1,1) = (1,2,1)$$

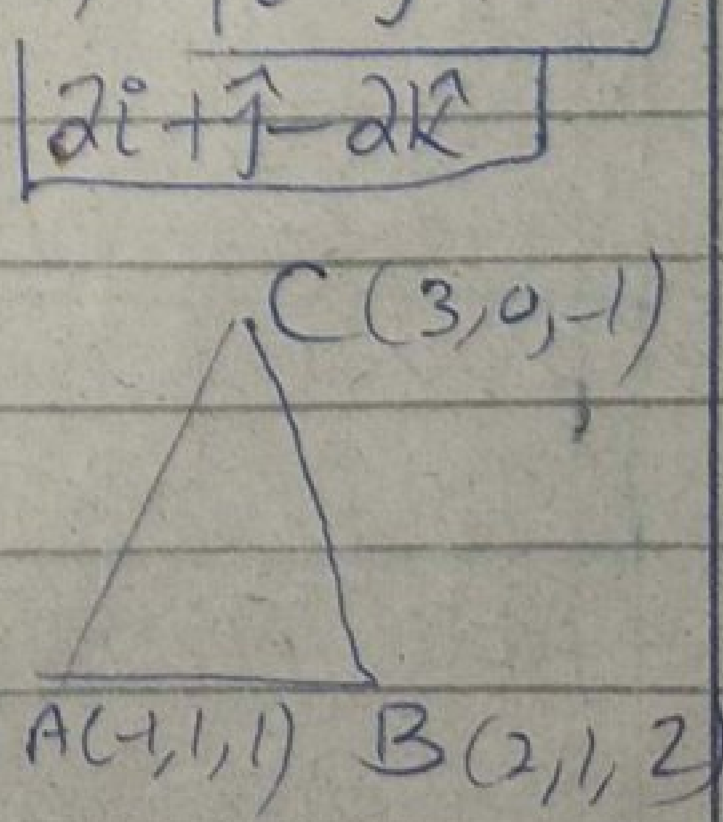
$$\boxed{\vec{AB} = \hat{i} + 2\hat{j} + \hat{k}}$$

any two adjacent sides

$$\vec{BC} = (3,0,-1) - (2,1,2) = (1,-1,-3) = \boxed{\hat{i} - \hat{j} - 3\hat{k}}$$

$$\vec{AC} = (3,0,-1) - (1,-1,1) = (2,1,-2) = \boxed{2\hat{i} + \hat{j} - 2\hat{k}}$$

$$\text{Area of triangle} = \frac{|\vec{AB} \times \vec{AC}|}{2}$$



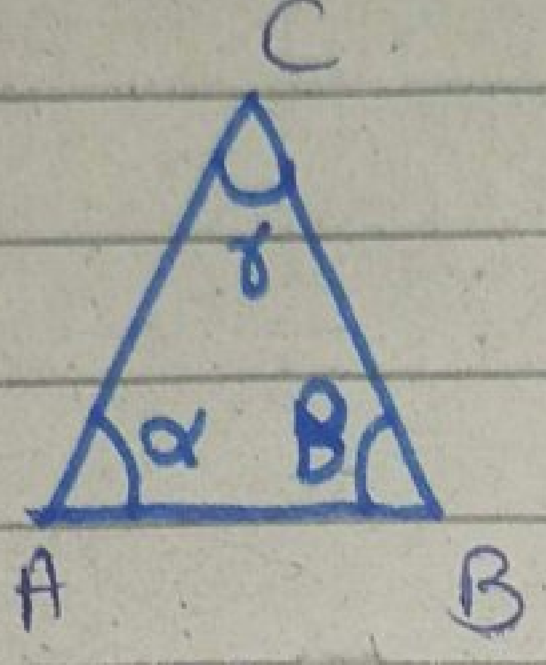
$$(\vec{AB} \times \vec{AC}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(-4-1) - \hat{j}(-2-2) + \hat{k}(1-4) = -5\hat{i} + 4\hat{j} - 3\hat{k}$$

$|\vec{AB}\times\vec{AC}| = \sqrt{(-5)^2+(4)^2+(-3)^2} = \sqrt{25+16+9}$

$\frac{|\vec{AB}\times\vec{AC}|}{2} = \frac{\sqrt{50}}{2} = \frac{5\sqrt{2}}{2} = \boxed{\frac{5}{\sqrt{2}}}$ Ans

Interior angles



$\vec{BA}\cdot\vec{AC} = |\vec{BA}||\vec{AC}|\cos\alpha$

$\vec{BA} = -\hat{i}-2\hat{j}-\hat{k}$

therefore

$-1(2)-2(1)-1(-2) = \left(\sqrt{(-1)^2+(-2)^2+(-1)^2}\right)\left(\sqrt{(2)^2+(1)^2+(-2)^2}\right)\cos\alpha$

$-2-\cancel{2}+\cancel{2} = \left(\sqrt{1+4+1}\right)\left(\sqrt{4+1+4}\right)\cos\alpha$

$\cos\alpha = \frac{-2}{\sqrt{6}\sqrt{9}} \Rightarrow \frac{-2}{\sqrt{3}\sqrt{2}\cdot 3} \Rightarrow \boxed{\alpha = \cos^{-1}\frac{-\sqrt{2}}{3\sqrt{3}}}$

And $\vec{AB}\cdot\vec{BC} = |\vec{AB}||\vec{BC}|\cos\beta$

$1(1)+2(-1)+1(-3) = \sqrt{1+4+1}\,\sqrt{1+1+9}\,\cos\beta$

$1-2-3 = \sqrt{6}\cdot\sqrt{11}\,\cos\beta$

$\boxed{\beta = \cos^{-1}\left(\frac{-4}{\sqrt{6}\sqrt{11}}\right)}$

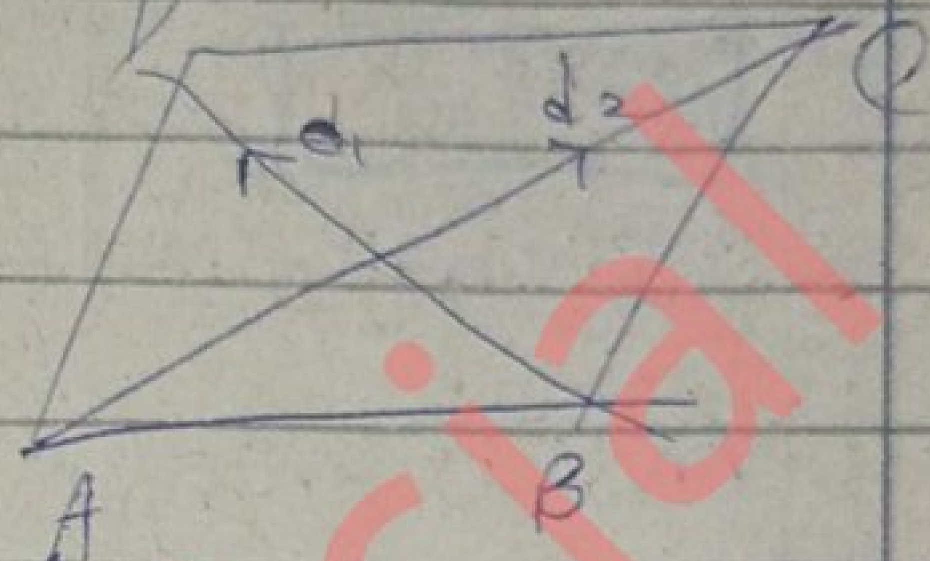
And $\vec{AC}\cdot\vec{CB} = |\vec{AC}||\vec{CB}|\cos\gamma$

$2(-1)+1(1)-2(3) = \sqrt{4+1+4}\,\sqrt{1+1+9}\,\cos\gamma$

$-2+1-6 = \sqrt{9}\,\sqrt{11}\,\cos\gamma$

$\boxed{\gamma = \cos^{-1}\frac{-7}{3\sqrt{11}}}$ Answer

Q11 (i) Area of Parallelogram $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$



$$d_1 \times d_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$= \hat{i}(4-6) - \hat{j}(12+2) + \hat{k}(-9-1)$

$= -2\hat{i} - 14\hat{j} - 10\hat{k}$

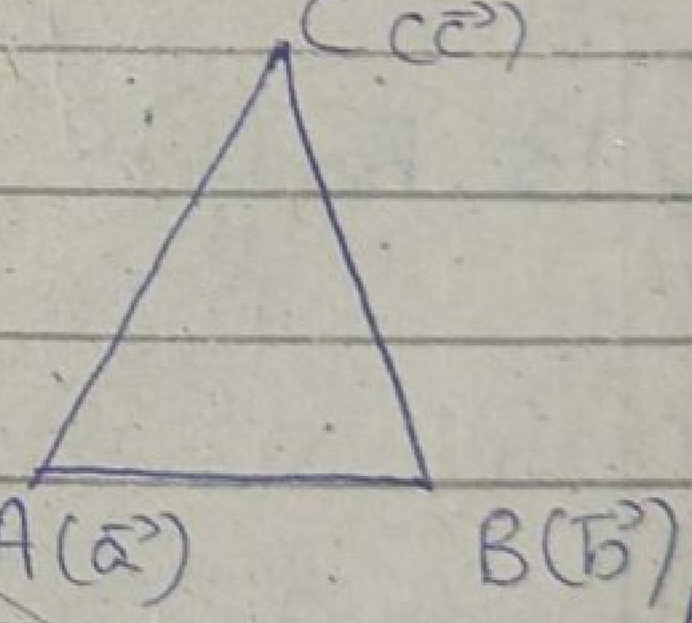
$|\vec{d_1} \times \vec{d_2}| = \sqrt{(-2)^2 + (-14)^2 + (-10)^2} = \sqrt{300}$

Area $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}| = \frac{1}{2}\sqrt{300}$

$\boxed{\text{Area} = \sqrt{75}}$

(ii) Proof :- Area of triangle $= \frac{1}{2}|\overline{AB} \times \overline{AC}|$

$\Rightarrow \overline{AB} = \vec{b} - \vec{a}$, $\vec{AC} = \vec{c} - \vec{a}$



Area $= \frac{1}{2}|(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})|$

$= \frac{1}{2}|(\vec{b} \times \vec{c}) - (\vec{b} \times \vec{a}) - (\vec{a} \times \vec{c}) + \cancel{(\vec{a} \times \vec{a})}|$

$= \frac{1}{2}|(\vec{b} \times \vec{c}) + (\vec{a} \times \vec{b}) + (\vec{c} \times \vec{a}) + 0|$

$$\boxed{\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$$

Q12 Sol:- $\vec{a} \cdot \vec{b} = 0$

$|\vec{a}||\vec{b}|\cos\theta = 0$

if $|\vec{a}| \neq 0$, $|\vec{b}| \neq 0$

then

It is not possible $\bar{a}$ & $\bar{b}$ are parallel & perpend then at same time

$\theta / 4$ mis

condition (

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\boxed{\theta = 90^\circ}$

$\vec{a}$ and $\vec{b}$ are perpendicular

And $\vec{a} \times \vec{b} = 0$

$|\vec{a}||\vec{b}|\sin\theta = 0$

if $|\vec{a}| \neq 0$, $|\vec{b}| \neq 0$

then $\sin\theta = 0$

$\theta = \sin^{-1}(0)$

$\boxed{\theta = 0}$

$\vec{a}$ and $\vec{b}$ are parallel

Ex3.2

Q13 Given that $|\vec{a}| = \frac{1}{\sqrt{7}}|\vec{b}|$ — (1)

and resultant of $\vec{a}$ & $\vec{b}$ is $\perp$ to $\vec{a}$.

$(\vec{a}+\vec{b})\cdot\vec{a} = 0$

$\vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{a} = 0$

$|a|^2 + \vec{b}\cdot\vec{a} = 0$

$\bar{a}\cdot\bar{b} = -|a|^2$ — (2) → $|a| = \frac{1}{\sqrt{7}}|\vec{b}|$

$\sqrt{7}|a| = |\vec{b}|$ — (3)

now taking

⇒ Eq $(7\vec{a}+\vec{b})\cdot\vec{b}$

$7(\vec{a}\cdot\vec{b}) + \vec{b}\cdot\vec{b}$

$= 7(\vec{a}\cdot\vec{b}) + |b|^2$

using (2) & (3)

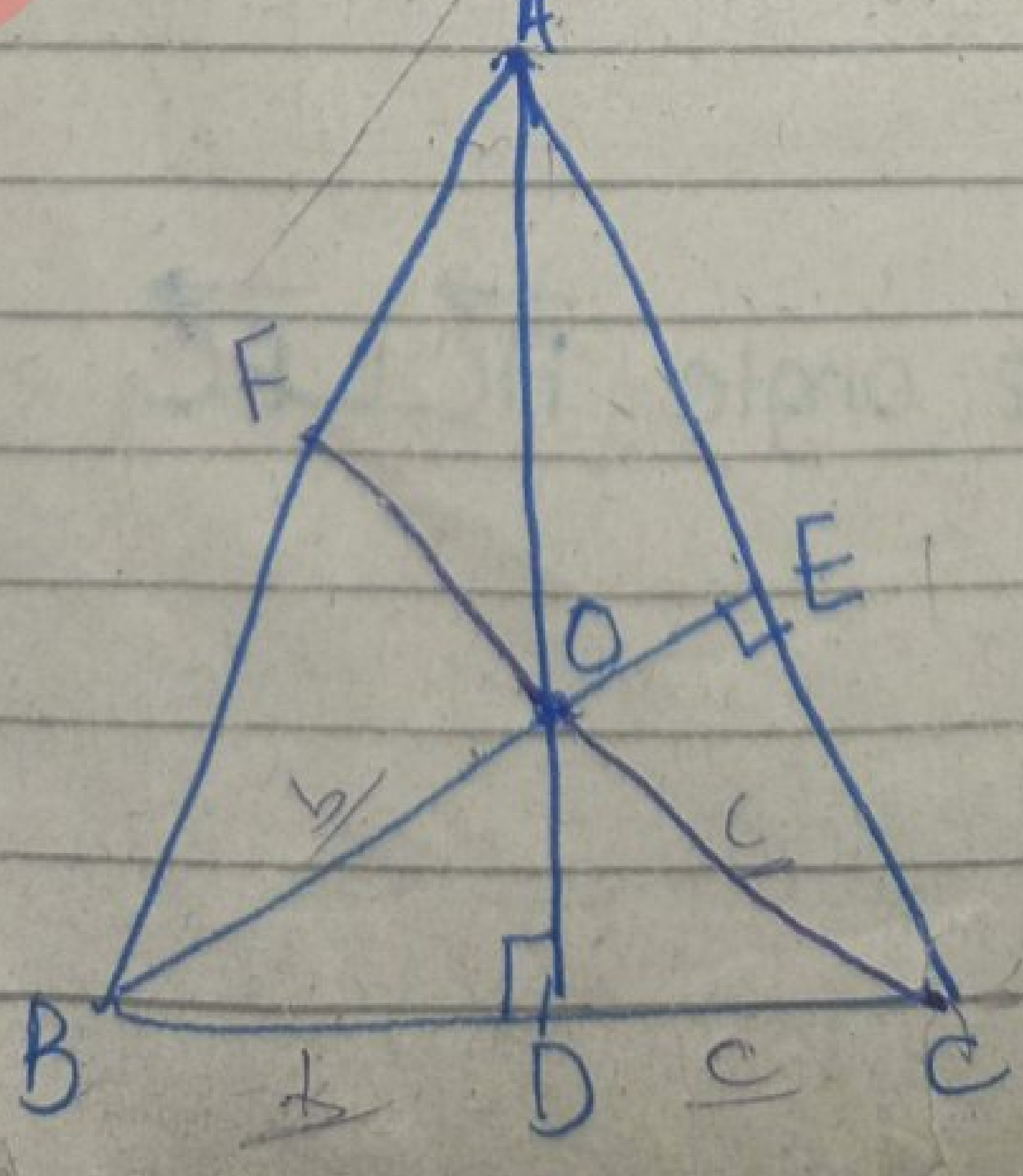
$7(-|a|^2) + (\sqrt{7}|a|)^2$

$= -7|a|^2 + 7|a|^2 = 0$

$(7\vec{a}+\vec{b})\cdot\vec{b} = 0$

$7a+b \perp \vec{b}$ proved

Q11 (i)



In ΔABC

$\overline{AD}$ is the altitude from A on $\overline{BC}$

$\overline{BE}$ " " " B on CA

Both meet at point O. join C with O and extend it to side AB on F.

as we know that :

$\overrightarrow{OA} \parallel \overrightarrow{AD}$

$\Rightarrow \overrightarrow{AD} = d\overrightarrow{OA}$

$\boxed{\overrightarrow{AD} = d \cdot \vec{a}}$ — (i)

But $\overrightarrow{AD} \perp \overrightarrow{BC}$

$\Rightarrow \overrightarrow{AD} \cdot \overrightarrow{BC} = 0$

$\Rightarrow d\vec{a} \cdot (\overrightarrow{OC} - \overrightarrow{OB}) = 0$

$\Rightarrow d\vec{a} \cdot (\vec{c} - \vec{b}) = 0$

$\Rightarrow \bar{a} \cdot \bar{c} - \bar{a} \cdot \bar{b} = \frac{0}{d} = 0$

$\boxed{\bar{a} \cdot \bar{c} - \bar{a} \cdot \bar{b} = 0}$ — (ii)

again

$\overrightarrow{BE} \parallel \overrightarrow{OB}$

$\Rightarrow \overrightarrow{BE} = d\overrightarrow{OB}$

$\boxed{\overrightarrow{BE} = d \cdot \vec{B}}$ — (iii)

but $\overrightarrow{BE} \perp \overrightarrow{CA}$

$\overrightarrow{BE} \cdot \overrightarrow{CA} = 0$

$d \cdot \vec{b} \cdot (\vec{a} - \vec{c}) = 0$

$\bar{b} \cdot \bar{a} - \bar{b} \cdot \bar{c} = \frac{0}{d} = 0$

$\boxed{\bar{a} \cdot \bar{b} - \bar{b} \cdot \bar{c} = 0}$ — (iv)

By solving eq (ii) & (iv)

~~$\bar{a} \cdot \bar{c} - \bar{a} \cdot \bar{b} + \bar{a} \cdot \bar{b}$~~

$\bar{a} \cdot \bar{c} - \bar{a} \cdot \bar{b} + \bar{a} \cdot \bar{b} - \bar{b} \cdot \bar{c} = 0$

$\bar{a} \cdot \bar{c} - \vec{b} \cdot \vec{c} = 0$

$(\bar{a} - \bar{b}) \cdot \vec{c} = 0$

$\vec{c} \cdot (\vec{a} - \vec{b}) = 0$

we can write

$\vec{c} \cdot (\vec{a} - \vec{b}) = \frac{0}{d}$

$d\vec{c} \cdot (\vec{a} - \vec{b}) = 0$

$d(\overrightarrow{OC}) \cdot (\overrightarrow{BA}) = 0$

But $\overrightarrow{OC} \parallel \overrightarrow{CF}$

$\overrightarrow{CF} \cdot \overrightarrow{BA} = 0$

$\overline{CF} \perp \overline{BA}$

Hence CF is the third altitude of triangle which passes through O

Hence altitude are concurrent.

Q15 (i) $\vec{F} = ?$ $|\vec{M}| = \sqrt{57}$, $\vec{M} = 6\hat{i} - 21\hat{j} - 6\hat{k}$

, $P(2,1,-3)$, $Q(-1,-1,1)$

$\vec{r} = \vec{PQ} = \vec{OQ} - \vec{OP} = (-1-2, -1-1, 1+3)$

$\Rightarrow \boxed{\vec{r} = -3\hat{i} - 2\hat{j} + 4\hat{k}}$

$\hat{M} = \frac{\vec{M}}{|M|} = \frac{6\hat{i} - 21\hat{j} - 6\hat{k}}{\sqrt{(6)^2 + (-21)^2 + (-6)^2}} = \frac{6\hat{i} - 21\hat{j} - 6\hat{k}}{\sqrt{513}}$

$\Rightarrow \hat{M} = \frac{\vec{M}}{|M|} \Rightarrow \boxed{\vec{M} = |M|\hat{M}}$ putting values

$\vec{M} = \cancel{\sqrt{57}} \cdot \frac{\cancel{3}(2\hat{i} - 7\hat{j} - 2\hat{k})}{\cancel{3 \cdot \sqrt{57}}}$

$\boxed{\vec{M} = 2\hat{i} - 7\hat{j} - 2\hat{k}}$ now let $\boxed{\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}}$

$\cancel{\vec{M} = \vec{r} \times \vec{F}}$ $\hookrightarrow$

$\vec{M} = \vec{r} \times \vec{F}$

$$2\hat{i} - 7\hat{j} - 2\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & 4 \\ x & y & z \end{vmatrix} = \hat{i}(-2z-4y) - \hat{j}(-3z-4x) + \hat{k}(-3y+2x)$$

Now compare b/s

$2 = -2z - 4y \quad , \quad 7 = -4x - 3z \quad , \quad -2 = -3y + 2x$

or $2y + z = -1 \rightarrow (1)$ or $\boxed{4x + 3z = -7} \rightarrow (2)$ $\boxed{2x - 3y = -2} \rightarrow (3)$

Convert equations into matrix

$$\left[\begin{array}{ccc|c} 2 & -3 & 0 & -2 \\ 0 & 2 & 1 & -1 \\ 4 & 0 & 3 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 0 & -1 \\ 0 & 2 & 1 & -1 \\ 4 & 0 & 3 & -7 \end{array}\right] \frac{1}{2}R_1$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 0 & -1 \\ 0 & 2 & 1 & -1 \\ 0 & 6 & 3 & -3 \end{array}\right] R_3 - 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 0 & -1 \\ 0 & 1 & 1/2 & -1/2 \\ 0 & 6 & 3 & -3 \end{array}\right] \frac{1}{2}R_2$$

And $\left[\begin{array}{ccc|c} 1 & -3/2 & 0 & -1 \\ 0 & 1 & 1/2 & -1/2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $R_3 - 6R_2$

Hence system of eq has infinite salutions so let $\boxed{z = t}$, $y + \frac{1}{2}z = -\frac{1}{2}$

$$\boxed{y = -\frac{1}{2} - \frac{1}{2}t}$$

And $x - \frac{3}{2}y = -1$ or $x = -1 + \frac{3}{2}y$

$$x = -1 + \frac{3}{2}\left(-\frac{1}{2} - \frac{1}{2}t\right) = -1 - \frac{3}{4} - \frac{3}{4}t$$

$$\boxed{x = -\frac{7}{4} - \frac{3}{4}t}$$

$$\boxed{\vec{F} = \left(-\frac{7}{4} - \frac{3}{4}t\right)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}t\right)\hat{j} + t\hat{k}}$$

(ii) Sol $P(-1, 2, -3)$ and $F = 2\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{M} = 3\hat{i} - 2\hat{j} + \hat{k}$.

let $Q(x, y, z) \Rightarrow \vec{r} = \vec{PQ} = \vec{OQ} - \vec{OP}$

$= (x+1, y-2, z+3)$

we know that

$\vec{M} = \vec{r} \times \vec{F}$

$$3\hat{i} - 2\hat{j} + k = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x+1 & y-2 & z+3 \\ 2 & 2 & -3 \end{vmatrix} = \hat{i}(-3y + 6 - 2z - 6)$$

$-\hat{j}(-3x-3-2z-6)+\hat{k}(2x+z+2y+4)$

Compare both sides

$\boxed{3=-3y-2z}$ — (1) $z=-3x-2z-9$

$\boxed{11=-3x-2z}$ — (2)

and $1=2x+2y+6$

$\boxed{-5=2x+2y}$ — (3)

Solve eq (1) and (2)

$-3y-2z=3$

$\underset{+}{-}3x\ \underset{+}{-}2z=\underset{-}{+}11$

$3x-3y=-8$ — (4)

Solve eq (3) & (4)

$6x+6y=-15$ $\therefore 3\times(2)$

$6x\cancel{-6y}=-16$ $\therefore 2\times(4)$

$12x=-31 \Rightarrow \boxed{x=\frac{-31}{12}}$ put in (3)

$\Rightarrow 2\left(\frac{-31}{12}\right)+2y=-5$

$\frac{-31}{6}+2y=-5 \Rightarrow 2y=-5+\frac{31}{6} \Rightarrow$

$\frac{-30+31}{6} \Rightarrow 2y=\frac{1}{6}$ $\boxed{y=\frac{1}{12}}$

put value of y in eq (1)

$3 = -3\left(\frac{1}{12}\right) - 2z \Rightarrow -2z = 3 + \frac{1}{4}$

$-2z = \frac{13}{4} \Rightarrow \boxed{z = \frac{-13}{8}}$

Hence $\boxed{Q\left(\frac{-31}{12}, \frac{1}{12}, \frac{-13}{8}\right)}$ A

Q16) (i) $\vec{r} = 2\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{F} = 3\hat{i} + 2\hat{j} + 7\hat{k}$

$M = ?$

we know that

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 3 & 2 & 7 \end{vmatrix}$$

$\vec{M} = \hat{i}(21-10) - \hat{j}(14-15) + \hat{k}(4-9)$

$\boxed{\vec{M} = 11\hat{i} + \hat{j} - 5\hat{k}}$ Answer

Q17) Case 1

$\vec{F_1} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ $Q(0, 2, 3)$

$P_1(4, 5, 3)$

$\Rightarrow \vec{r} = \vec{P_1Q} = \vec{OQ} - \vec{OP_1}$

$\vec{r} = (0-4, 2-5, 3-3) = (-4, -3, 0)$

$$\vec{M_1} = \vec{r} \times \vec{F_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -3 & 0 \\ 3 & 4 & 5 \end{vmatrix}$$

$\vec{M_1} = \hat{i}(-15-0) - \hat{j}(-20-0) + \hat{k}(-16+9)$

$\boxed{\vec{M_1} = -15\hat{i} + 20\hat{j} - 7\hat{k}}$

Net $\vec{M} = \vec{M_1} + \vec{M_2}$

Case 2

$F_2 = 9\hat{i} + 2\hat{j} + 7\hat{k}$

$P_2(-4, -1, 3)$

$\vec{r} = \vec{P_2Q} = \vec{OQ} - \vec{OP_2} = (0+4, 2+1, 3-3)$

$\vec{r} = (4, 3, 0)$

$$\vec{M_2} = \vec{r} \times \vec{F_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ 9 & 2 & 7 \end{vmatrix}$$

$\vec{M_2} = \hat{i}(21-0) - \hat{j}(28-0) + \hat{k}(8-27)$

$$\boxed{\vec{M_2} = 21\hat{i} - 28\hat{j} - 19\hat{k}}$$

$$\boxed{\vec{M} = 6\hat{i} - 8\hat{j} - 26\hat{k}}$$