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Where Minds Pollinate

Class 11
Mathematics

Chapter 3
Exercise 3.2
Notes

National Book Foundation

$\vec{EB} = (\vec{a}-\vec{b}) + (\vec{a}-\vec{b})$

$\boxed{\vec{EB} = 2\vec{a} - 2\vec{b}}$

$\vec{FA} = \vec{FO} + \vec{OA}$

$\boxed{\vec{FA} = \vec{a} - \vec{b}}$

$\vec{FC} = \vec{FO} + \vec{OC}$

$\vec{FC} = \vec{a} + \vec{a}$

$\boxed{\vec{FC} = 2\vec{a}}$ Answer.

$\perp$ orthagonal

$\vec{a}\cdot\vec{b} = 0$

Exercise 3.2

Concept: Dot Product/Scalar Product

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$. ∴ formula

if $\hat{i}$, is unit vector then

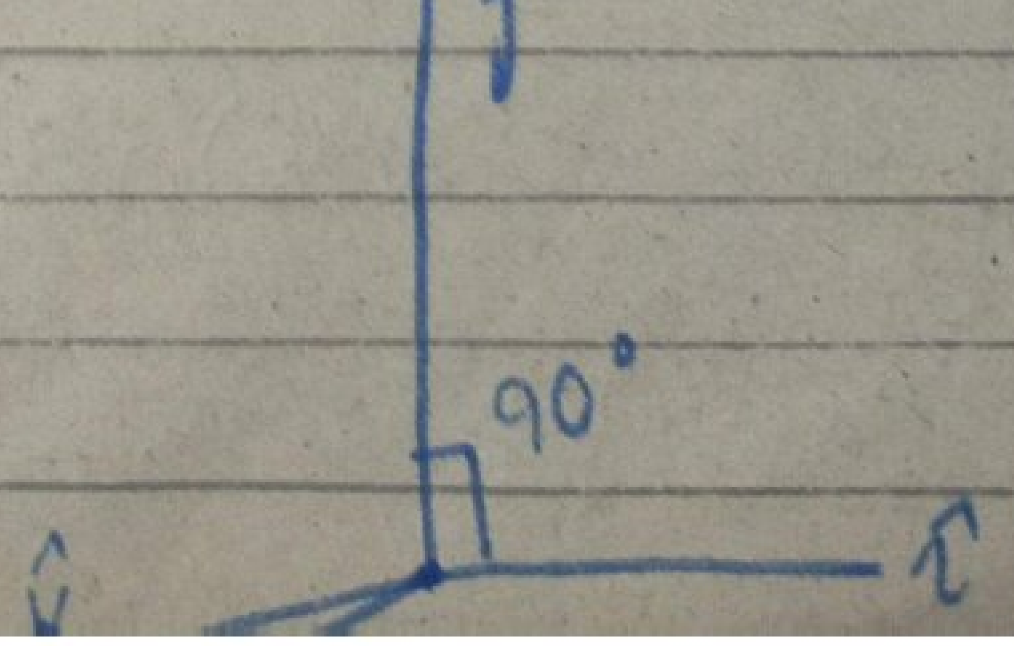
$\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0°$ ∴ $\theta = 0°$

$\boxed{\hat{i}\cdot\hat{i} = 1}$ similarly $\boxed{\hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1}$

so $\hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos 90°$

$\boxed{\hat{i}\cdot\hat{j} = 0}$ similarly

$\boxed{\hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0}$



$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ Commutative

Distributive

$a\cdot(b+c) = (\vec{a}\cdot\vec{b}) = \vec{a}\cdot\vec{c}$

(i) $\vec{a}\cdot\vec{b}$

$= (2\hat{i}-3\hat{j}+\hat{k})\cdot(\hat{i}-3\hat{j}+4\hat{k})$

$= 2(1)-3(-3)+1(4)$

$= 2+9+4$

$= \boxed{15}$ Answer

(ii) $2\vec{a}\cdot3\vec{b}$

Sol: $= 6(\vec{a}\cdot\vec{b})$

$= 6(15)$ ∴ using (i)

$= \boxed{90}$ Answer.

(iii) $(\vec{a}-\vec{b})\cdot\vec{c}$

$= \left[(2\hat{i}-3\hat{j}+\hat{k})-(\hat{i}-3\hat{j}+4\hat{k})\right]\cdot\vec{c}$

$= \left[2\hat{i}-3\hat{j}+\hat{k}-\hat{i}+3\hat{j}-4\hat{k}\right]\cdot\vec{c}$

$= (\hat{i}+0\hat{j}-3\hat{k})(-\hat{i}-2\hat{j}+5\hat{k})$

$= 1(-1)+0(-2)-3(5)$

$= -1-15$

$= -16$ Answer.

(iv) $(2\vec{a}+3\vec{b}-\vec{c})\cdot(\vec{a}+\vec{b})$

Sol: $= \left[2(2\hat{i}-3\hat{j}+\hat{k})+3(\hat{i}-3\hat{j}+4\hat{k})-(-\hat{i}-2\hat{j}+5\hat{k})\right]\cdot(3\hat{i}-6\hat{j}+5\hat{k})$

$= (4\hat{i}-6\hat{j}+2\hat{k}+3\hat{j}-9\hat{j}+12\hat{k}+\hat{i}+2\hat{j}-5\hat{k})\cdot(3\hat{i}-6\hat{j}+5\hat{k})$

$= (8\hat{i}-13\hat{j}+9\hat{k})\cdot(3\hat{i}-6\hat{j}+5\hat{k})$

$= 8(3)-13(-6)+9(5) = 24+78+45$

$= \boxed{147}$ Answer

(v) $\hat{i} \cdot \vec{a} + \hat{j} \cdot \vec{b} + \hat{k} \cdot \vec{c}$

$= \hat{i}(2\hat{i} - 3\hat{j} + \hat{k}) + \hat{j}(\hat{i} - 3\hat{j} + 4\hat{k}) + \hat{k}(-\hat{i} - 2\hat{j} + 5\hat{k})$

$= 2 - 3 + 5$ $\qquad \because \hat{i}\hat{i} = \hat{j}\hat{j} = \hat{k}\hat{k} = 1$

$= 4$ Answer $\qquad \because \hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$

Q2. (i) $\vec{a}$ and $3\vec{b}$

formula :

$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta$
$\theta = \cos^{-1} \dfrac{\vec{a} \cdot \vec{b}}{\|a\|\|b\|}$

Sol:- $\vec{a} \cdot 3\vec{b} = (\hat{j} - k) \cdot 3(3i - 4j + k)$

$\vec{a} \cdot 3\vec{b} = (\hat{j} - k) \cdot (9\hat{i} - 12\hat{j} + 3k)$

$\vec{a} \cdot 3\vec{b} = -12 - 3 \Rightarrow \boxed{-15}$

$|\vec{a}| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$3\vec{b} = 9\hat{i} - 12\hat{j} + 3k$

$|3\vec{b}| = \sqrt{(9)^2 + (-12)^2 + (3)^2} = \sqrt{81 + 144 + 9} = \sqrt{234}$

$|3\vec{b}| = 3\sqrt{2}\sqrt{13}$

now $\theta = \cos^{-1} \dfrac{\vec{a} \cdot 3\vec{b}}{|a||3b|}$

$\theta = \cos^{-1} \dfrac{(-15)}{(3\sqrt{2})^2(\sqrt{13})^2}$

$\boxed{\theta = \cos^{-1} \dfrac{-5}{2\sqrt{13}}}$ Answer

(ii) $(2\vec{a} - 3\vec{b})$ and $2\vec{c}$

Sol $2\vec{a} - 3\vec{b} = 2(\hat{j} - \hat{k}) - 3(3\hat{i} - 4\hat{j} + \hat{k})$

$2\vec{a} - 3\vec{b} = 2\hat{j} - 2\hat{k} - 9\hat{i} + 12\hat{j} - 3\hat{k}$

$2\vec{a} - 3\vec{b} = -9\hat{i} + 14\hat{j} - 5\hat{k}$

Now

$(2\vec{a} - 3\vec{b}) \cdot 2\vec{c} = (-9\hat{i} + 14\hat{j} - 5\hat{k}) \cdot (-2\hat{i} + 4\hat{j} - 8\hat{k})$

$= -9(-2) + 14(4) - 5(-8)$

$= 18 + 56 + 40$

$(2\vec{a} - 3\vec{b}) \cdot 2\vec{c} = \boxed{114}$

$|2\vec{a} - 3\vec{b}| = \sqrt{(-9)^2 + (14)^2 + (-5)^2}$

$= \sqrt{81 + 196 + 25}$

$= \sqrt{302}$

and $|2\vec{c}| = \sqrt{(-2)^2 + (4)^2 + (-8)^2}$

$= \sqrt{84} = 2\sqrt{21}$

$\theta = \cos^{-1} \dfrac{(2\vec{a} - 3\vec{b}) \cdot (2\vec{c})}{|2\vec{a} - 3\vec{b}| \cdot |2\vec{c}|}$

$\theta = \cos^{-1} \dfrac{114}{2\sqrt{21}\sqrt{302}}$

$\boxed{\theta = \cos^{-1} \dfrac{57}{\sqrt{6342}}}$ Ans

(iii) $(-\vec{a} + \vec{c})$ and $(\vec{b} - 2\vec{c})$

Sol $(-\vec{a} + \vec{c}) = -\hat{j} + \hat{k} - \hat{i} + 2\hat{j} - 4\hat{k}$

$\boxed{(-\vec{a} + \vec{c}) = -\hat{i} + \hat{j} - 3\hat{k}}$

and $\vec{b}-2\vec{c} = 3\hat{i}-4\hat{j}+k-2(-\hat{i}+2\hat{j}-4\hat{k})$

$\vec{b}-2\vec{c} = 3\hat{i}-4\hat{j}+k+2\hat{i}-4\hat{j}+8k$

$\boxed{\vec{b}-2\vec{c} = 5\hat{i}-8\hat{j}+9k}$

$(-\vec{a}+\vec{c})\cdot(\vec{b}-2\vec{c}) = -1(5)+1(-8)-3(9)$

$= -5-8-27$

-40

$|-\vec{a}+\vec{c}| = \sqrt{(-1)^2+1^2+(-3)^2}$

$= \sqrt{1+1+9} = \sqrt{11}$

$|\vec{b}-2\vec{c}| = \sqrt{(5)^2+(-8)^2+(9)^2}$

$|\vec{b}-2\vec{c}| = \sqrt{25+64+81}$

$= \sqrt{170}$

$\theta = \cos^{-1}\dfrac{(-\vec{a}+\vec{c})\cdot(\vec{b}-2\vec{c})}{|-\vec{a}+\vec{c}|\,|\vec{b}-2\vec{c}|}$

$\theta = \cos^{-1}\dfrac{-40}{\sqrt{11}\sqrt{170}}$

$\boxed{\theta = \cos^{-1}\dfrac{-40}{\sqrt{1870}}}$

(iv) $(\vec{a}+\vec{b}+\vec{c})$ and $(\vec{a}-\vec{b}-\vec{c})$

for $\vec{a}+\vec{b}+\vec{c} = 2\hat{i}-\hat{j}-4k$

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{(2)^2+(-1)^2+(-4)^2} = \sqrt{21}$

$\vec{a}-\vec{b}\cancel{+\vec{c}} = \hat{j}-k-3\hat{i}+4\hat{j}-k+i-2\hat{j}+4\hat{k}$

$= -2\hat{i}+3\hat{j}+2k$

$|\vec{a}-\vec{b}-\vec{c}| = \sqrt{(-2)^2+(3)^2+(2)^2} = \sqrt{17}$

$(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}-\vec{b}-\vec{c}) = 2(-2)-1(3)-4(2) = -15$

$\theta = \cos^{-1} \dfrac{(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}-\vec{b}-\vec{c})}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{a}-\vec{b}-\vec{c}|}$

$\theta = \cos^{-1} \dfrac{-15}{\sqrt{21}\sqrt{17}} = \boxed{\dfrac{-15}{\sqrt{357}}}$ Answer

(v) $(\vec{a}-2\vec{b}+\vec{c})$ and $\vec{a}$

Sol $\vec{a}-2\vec{b}+\vec{c} = \hat{j}-\hat{k}-2(3\hat{i}-4\hat{j}+\hat{k})+(-\hat{i}+2\hat{j}-4\hat{k})$

$\vec{a}-2\vec{b}+\vec{c} = \hat{j}-\hat{k}-6\hat{i}+8\hat{j}-2\hat{k}-\hat{i}+2\hat{j}-4\hat{k}$

$\boxed{\vec{a}-2\vec{b}+\vec{c} = -7\hat{i}+11\hat{j}-7\hat{k}}$

$|\vec{a}-2\vec{b}+\vec{c}| = \sqrt{(-7)^2+(11)^2+(-7)^2}$

$= \sqrt{219}$

and $\vec{a} = \hat{j}-\hat{k}$

$\|\vec{a}\| = \sqrt{(1)^2+(-1)^2}$

$= \sqrt{2}$

$(\vec{a}-2\vec{b}+\vec{c})\cdot(\vec{a}) = (-7\hat{i}+11\hat{j}-7\hat{k})\cdot(\hat{j}-\hat{k})$

$= -7(0)+11(1)-7(-1)$

$= \boxed{18}$

$\theta = \cos^{-1} \dfrac{(\vec{a}-2\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}-2\vec{b}+\vec{c}|\cdot|\vec{a}|}$

$\theta = \cos^{-1} \dfrac{18}{\sqrt{219}\sqrt{2}} \Rightarrow \boxed{\theta = \cos^{-1} \dfrac{18}{\sqrt{438}}}$ A

$\vec{a}+\vec{b}+\vec{c}=0$

Q3 (i) $\vec{a}+\vec{b}=-\vec{c}$

Taking squre on b/s

$(\vec{a}+\vec{b})^2=(-\vec{c})^2$

$(\vec{a})^2+(\vec{b})^2+2\vec{a}\cdot\vec{b}=(\vec{c})^2$

$2\vec{a}\cdot\vec{b}=(\vec{c})^2-(\vec{a})^2-(\vec{b})^2$

OR

$(a+b)^2=(-c)^2$

$(a+b)\cdot(a+b)=(c)^2$

$a\cdot a+a\cdot b+b\cdot a+b\cdot b=c^2$

$|a|^2+a\cdot b+a\cdot b+|b|^2=|c|^2$

$|a|^2+2\vec{a}\cdot\vec{b}=|b|^2+|c|^2$

$2|\vec{a}||\vec{b}|=|\vec{c}|^2-|\vec{a}|^2-|\vec{b}|^2$ $\therefore (\vec{a})^2=|\vec{a}|^2$, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos$

$2(a)(b)=(4)^2-(2)^2-(3)^2 \Rightarrow a\cdot b=\frac{3}{2}$

$\cos\theta=\frac{16-4-9}{12}$

$\theta=\cos^{-1}\left(\frac{1}{4}\right)$ Answer

(ii) $|\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$ $\therefore$ squaring b/s

$\cancel{|\vec{a}|^2}+\cancel{|\vec{b}|^2}+2|\vec{a}||\vec{b}|=\cancel{|\vec{a}|^2}+\cancel{|\vec{b}|}-2|\vec{a}||\vec{b}|$

$2\vec{a}\cdot\vec{b}+2\vec{a}\cdot\vec{b}=0 \Rightarrow 4\vec{a}\cdot\vec{b}=0$

$\vec{a}\cdot\vec{b}=\frac{0}{4} \Rightarrow \boxed{\vec{a}\cdot\vec{b}=0}$

$|\vec{a}||\vec{b}|\cos\theta=0$

$\cos\theta=\frac{0}{|\vec{a}||\vec{b}|}$

$\theta=\cos^{-1}(0)$

$\boxed{\theta=90^\circ}$ Answer

Q4 (i) we know that

$$(\vec{a} - d\vec{b})\cdot\vec{c} = 0$$

putting values

$$\left[\hat{i} - 3\hat{j} + 4\hat{k} - d(7\hat{i} - 9\hat{j} + \hat{k})\right]\cdot(3\hat{i} - 2\hat{j} + 5\hat{k}) = 0$$

$$\left[(1-7d)\hat{i} + (-3+9d)\hat{j} + (4-d)\hat{k}\right]\cdot(3\hat{i} - 2\hat{j} + 5\hat{k}) = 0$$

$$\Rightarrow (1-7d)3 + (-3+9d)(-2) + (4-d)(5) = 0$$

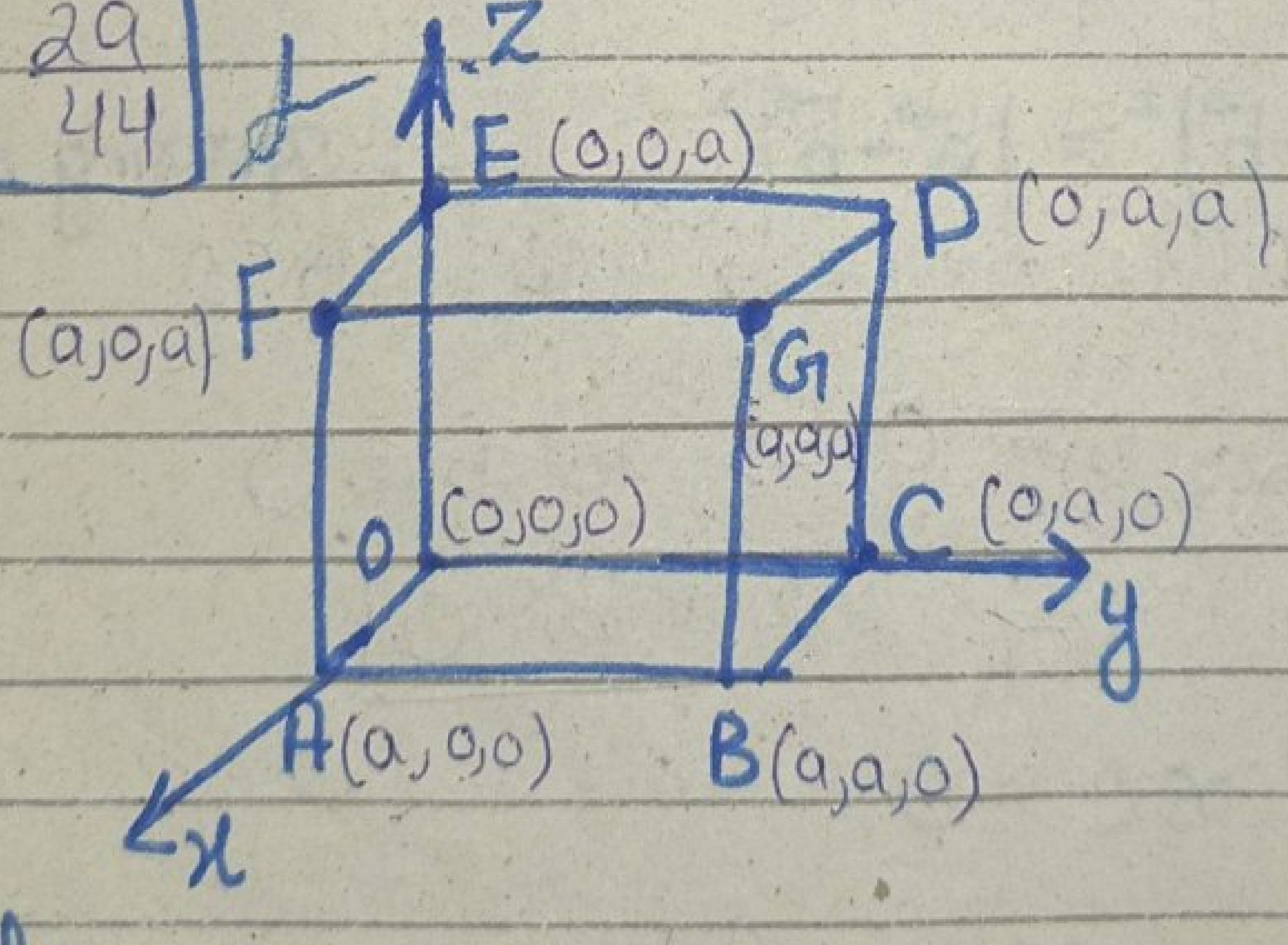
$$3 - 21d + 6 - 18d + 20 - 5d = 0$$

$$29 - 44d = 0$$

$$29 = 44d$$

$$\boxed{d = \frac{29}{44}}$$

(ii)



Proof:- The diagnal of the cube are OG and AD & CF & EB. The direction ratio of OG is $(a-0, a-0, a-0) = (a, a, a)$

The direction ratio of AD is
$(0-a, a-0, a-0) = (-a, a, a)$

let θ be the angle b/w OG and AD.

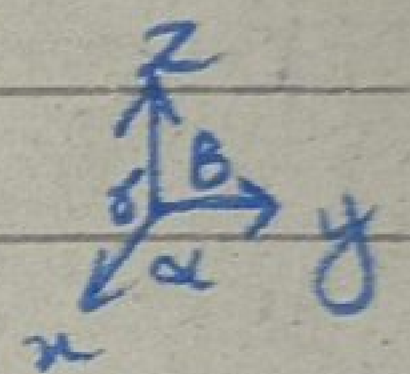
$$\cos\theta = \frac{\vec{OG}\cdot\vec{AD}}{|\vec{OG}|\cdot|\vec{AD}|} = \frac{a(-a)+a(a)+(a)(a)}{\left(\sqrt{a^2+a^2+a^2}\right)\left(\sqrt{(-a)^2+(a^2)+(a^2)}\right)}$$

$$\cos\theta = \frac{-a^2+a^2+a^2}{\sqrt{3a^2}\,\sqrt{3a^2}} = \frac{a^2}{(\sqrt{3a^2})^2} = \frac{a^2}{3a^2}$$

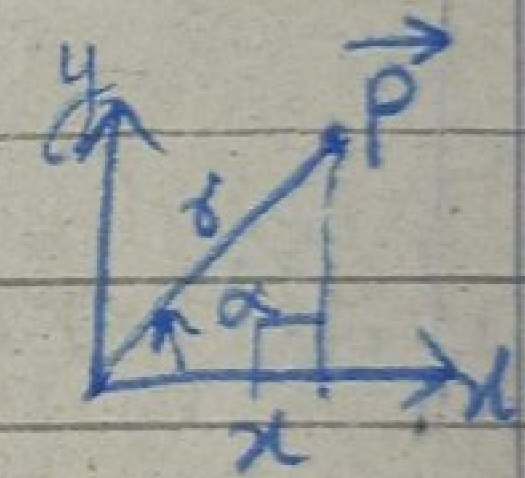
$$\boxed{\theta = \cos^{-1}\left(\frac{1}{3}\right)}$$ Proved

Q5)(i) Direction Cosine.

$$\cos\alpha = \frac{x}{|a|} \; ; \cos\beta = \frac{y}{|a|} \; ; \cos\gamma = \frac{z}{|a|}$$



sol:- $|a| = \sqrt{x^2+y^2+z^2} \Rightarrow |a| = \sqrt{(2)^2+(-3)^2+(4)^2}$



$|a| = \sqrt{29}$

$$\Rightarrow \boxed{\cos\alpha = \frac{2}{\sqrt{29}}} \; ; \boxed{\cos\beta = \frac{-3}{\sqrt{29}}} \; ; \boxed{\cos\gamma = \frac{4}{\sqrt{29}}}$$

Answer

$\cos\alpha = \frac{b}{p}$

$$\boxed{\cos\alpha = \frac{x}{|r|}}$$

Answer.

Check: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\left(\frac{2}{\sqrt{29}}\right)^2 + \left(\frac{-3}{\sqrt{29}}\right)^2 + \left(\frac{4}{\sqrt{29}}\right)^2$$

$$= \frac{4}{29} + \frac{9}{29} + \frac{16}{29}$$

$$= \frac{29}{29} = 1 = R.H.S$$ Hence proved.

Projection of $\vec{a}$ along $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Projection of $\vec{b}$ along $\vec{a}$ = $\frac{\vec{a}\cdot\vec{b}}{|a|}$

(ii) Formula for Projection of $\vec{a}$ along $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

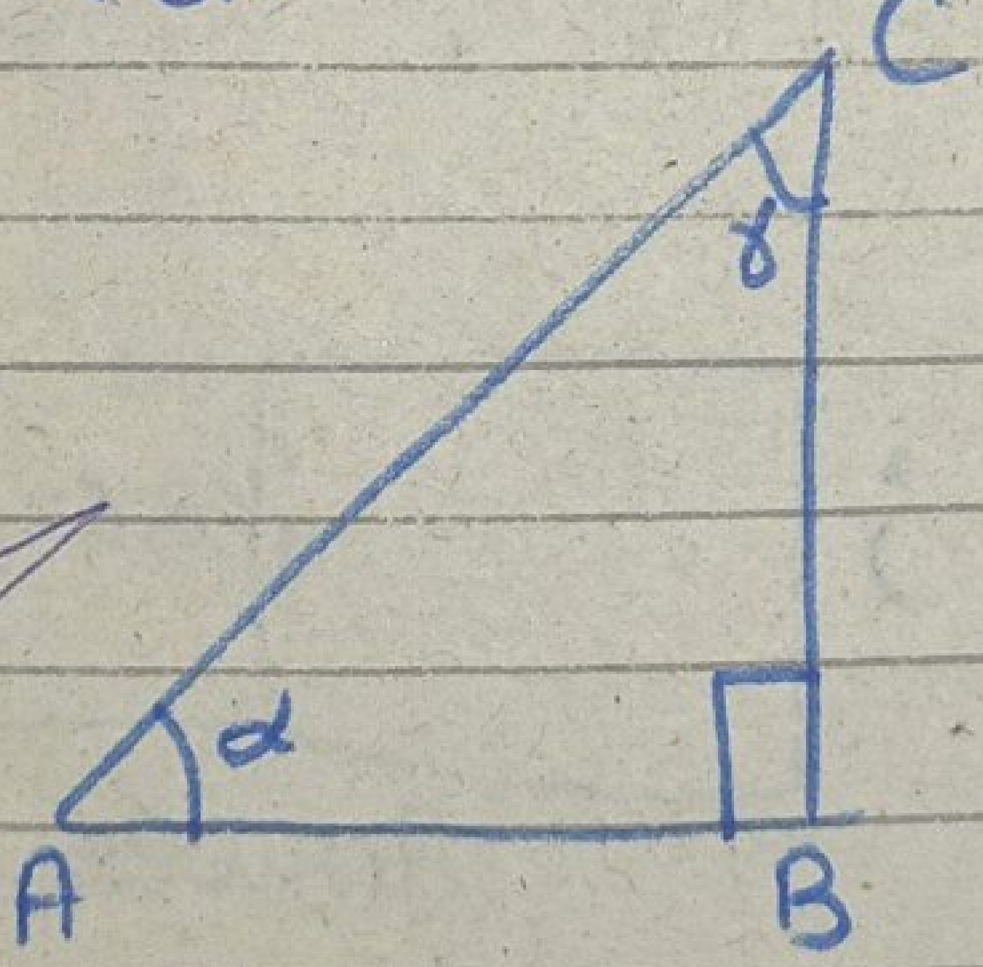
Sol:- $\frac{(3\hat{i}-2\hat{j}+\hat{k})\cdot(7\hat{i}-\hat{j}+8\hat{k}-\hat{i}+2\hat{j}-3\hat{k})}{|\vec{c}-\vec{a}|}$

$= \frac{(3\hat{i}-2\hat{j}+\hat{k})\cdot(6\hat{i}+\hat{j}+5\hat{k})}{\sqrt{(6)^2+1^2+(5)^2}}$

$= \frac{3(6)-2(1)+1(6)}{\sqrt{36+1+25}}$

$= \frac{18-2+5}{\sqrt{62}} = \frac{21}{\sqrt{62}}$ Ans.

Q6 (i) Proof:-



$\vec{AB} = \vec{OB} - \vec{OA} = 3\hat{i}+\hat{j}+4\hat{k}-0\hat{i}+\hat{j}+2\hat{k}$

$\vec{AB} = 3\hat{i}+2\hat{j}+6\hat{k}$

$\vec{BC} = \vec{OC} - \vec{OB} = 5\hat{i}+7\hat{j}+\hat{k}-3\hat{i}-\hat{j}-4\hat{k}$

$\vec{BC} = 2\hat{i}+6\hat{j}-3\hat{k}$

$|\vec{BC}| = \sqrt{(2)^2+(6)^2+(-3)^2} = 7.$

$\Rightarrow \vec{AB}\cdot\vec{BC} = (3\hat{i}+2\hat{j}+6\hat{k})\cdot(2\hat{i}+6\hat{j}-3\hat{k})$.

$= 3(2) + 2(6) + 6(-3)$

$\Rightarrow 0 \Rightarrow \vec{AB} \cdot \vec{BC}$.

$|\vec{AC}| = \sqrt{9 + 4 + 36} = \boxed{7}$

Hence $\vec{AB} \perp \vec{BC}$; $B = 90°$, its right angle triangle

$\tan\alpha = \frac{P}{B} = \frac{|\vec{BC}|}{|\vec{AB}|} = \frac{7}{7} = 1 \Rightarrow \alpha = \tan^{-1}(1) = 45°$

$B = 90°$, then we know that $\alpha + B + \gamma = 180°$

$45° + 90° + \gamma = 180° \Rightarrow \gamma = 180 - 90° - 45°$

$\boxed{\gamma = 45°}$. Answer.

ii Sol: we know that direction of cosine of vector $\vec{\gamma}$ is $l = \cos\alpha = \frac{x}{|\gamma|}$, $m = \cos\beta = \frac{y}{|\gamma|}$, $n = \cos\gamma = \frac{z}{|\gamma|}$

Given equally inclined $\Rightarrow \alpha = B = \gamma$ then $l = m = n$

then we know $l^2 + m^2 + n^2 = 1$

$l^2 + l^2 + l^2 = 1$ $\quad \because l = m = n$

$3l^2 = 1 \Rightarrow l^2 = \frac{1}{3} \Rightarrow \boxed{l = \pm\frac{1}{\sqrt{3}}}$

let $\gamma = l\hat{i} + m\hat{j} + n\hat{k} \Rightarrow \gamma = \pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}$

Given $|\vec{\gamma}| = 5$, then unit vector $\hat{\gamma} = \frac{\vec{\gamma}}{|\gamma|}$

$\Rightarrow \vec{\gamma} = \hat{\gamma} \cdot |\gamma|$

$\vec{\gamma} = 5\left[\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right]$ $\boxed{\vec{\gamma} = \pm\frac{5}{\sqrt{3}}\hat{i} \pm \frac{5}{\sqrt{3}}\hat{j} \pm \frac{5}{\sqrt{3}}\hat{k}}$

$(\vec{a})^2 = |a|^2$ $(\vec{a})^2 = |a|^2$

$\therefore (a)^2 = \vec{a}\cdot\vec{a} = |\vec{a}|^2$

7 (i) Sol Given $\vec{a}+\vec{b}+\vec{c} = 0$

$(\vec{a}+\vec{b}+\vec{c})^2 = (0)^2$ $\therefore$ squaring b/s

$(\vec{a})^2+(\vec{b})^2+(\vec{c})^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a} = 0$

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$

$(2)^2+(5)^2+(4)^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$

$2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = -4-25-16$

$\boxed{\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = \frac{-45}{2}}$

(ii) Proof: let $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$\Rightarrow \vec{r}\cdot\hat{i} = (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{i} = x$ — (1)

$\vec{r}\cdot\hat{j} = (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{j} = y$ — (2)

$\vec{r}\cdot\hat{k} = (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{k} = z$ — (3)

$\Rightarrow r = (\vec{r}\cdot\hat{i})\cdot\hat{i}+(\vec{r}\cdot\hat{j})\cdot\hat{j}+(\vec{r}\cdot\hat{k})\cdot\hat{k}$

R.H.S $= (\vec{r}\cdot\hat{i})\hat{i}+(\vec{r}\cdot\hat{j})\hat{j}+(\vec{r}\cdot\hat{k})\cdot\hat{k}$ $\therefore$ can put $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ Directly and then take dot with $\hat{i},\hat{j},\hat{k}$

$= x\hat{i}+y\hat{j}+z\hat{k}$

$= r$ = L.H.S proved

Sol: Q 8.

let: $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+\hat{j}-3\hat{k}) = 0$ $\therefore r\cdot\vec{a} = 0$

then $x+y-3z = 0$ — (1)

and $(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+3\hat{j}+2\hat{k}) = 5$

then $x+3y-2z=5$ — (2) $\vec{r}\cdot\vec{b}=0$

And $(x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}+\hat{j}+4\hat{k})=8$

$2x+y+4z=8$ — (3) $\vec{r}\cdot\vec{c}=0$

Now solving eq (1) and (2)

$$x+3y-2z=5$$
$$\pm x \pm y \mp 3z = \pm 0$$
$$2y+z=5 \quad \text{— (4)}$$

and (2) and (3) solve

$$\cancel{2x}+6y-4z=10 \quad \therefore (2)\times 2$$
$$\pm \cancel{2x} \pm y \pm 4z = \pm 8$$
$$5y-8z=2 \quad \text{— (5)}$$

Now solve eq (4) and (5)

$$16y+8z=40$$
$$5y-8z=2$$
$$21y=42$$

$$y=\frac{42}{21} \Rightarrow \boxed{2=y}$$

put $y=2$ in eq (4)

$2(2)+z=5 \Rightarrow z=5-4 \Rightarrow \boxed{1}=z$

Now put y and z in eq (1)

$$x+2-3(1)=0$$
$$\boxed{x=1}$$

$\vec{r}=\hat{i}+2\hat{j}+\hat{k}$ Ans

Q9a) (i) $\vec{a}\cdot\vec{b} = \frac{1}{4}|\vec{a}+\vec{b}|^2 - \frac{1}{4}|\vec{a}-\vec{b}|^2$

Proof: R.H.S $= \frac{1}{4}\left[|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}\right] - \frac{1}{4}\left[|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}\right]$

$= \frac{1}{4}\{\cancel{|\vec{a}|^2}+\cancel{|\vec{b}|^2}+2\vec{a}\cdot\vec{b}-\cancel{|\vec{a}|^2}-\cancel{|\vec{b}|^2}+2\vec{a}\cdot\vec{b}]$

$= \frac{1}{4}(4\vec{a}\cdot\vec{b}) = \vec{a}\cdot\vec{b}$ = L.H.S Proved.

(ii) $|\vec{a}|^2+|\vec{b}|^2 = \frac{1}{2}|\vec{a}+\vec{b}|^2 + \frac{1}{2}|\vec{a}-\vec{b}|^2$

R.H.S.

$= \frac{1}{2}\left[|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}\right] + \frac{1}{2}\left[|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}\right]$

$= \frac{1}{2}\left[|\vec{a}|^2+|\vec{b}|^2+\cancel{2\vec{a}\cdot\vec{b}}+|\vec{a}|^2+|\vec{b}|^2-\cancel{2\vec{a}\cdot\vec{b}}\right]$

$= \frac{1}{2}\left[2|\vec{a}|^2+2|\vec{b}|^2\right] = \frac{1}{\cancel{2}}(\cancel{2})\left[|\vec{a}|^2+|\vec{b}|^2\right]$

$= |\vec{a}|^2+|\vec{b}|^2$ = L.H.S Proved!

Q10) (i) let $\vec{a}$ and $\vec{b}$ be the unit vecters then

$|\vec{a}|=1$ and $|\vec{b}|=1$

Given condition $|\vec{a}+\vec{b}|=1$

Taking square on b/s

$|\vec{a}+\vec{b}|^2 = (1)^2 \Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b} = 1.$

or $(a\cdot a)+a\cdot b+b\cdot a+b\cdot b=1$

$\Rightarrow 1+1+2|\vec{a}||\vec{b}|\cos\theta = 1$

$\Rightarrow 2+2(1)(1)\cos\theta = 1$

$\Rightarrow 2\cos\theta = 1-2 \Rightarrow \boxed{\cos\theta = \frac{-1}{2}}$

Now $|\vec{a}-\vec{b}| = \sqrt{3}$

square b/s

$|\vec{a}-\vec{b}|^2 = (\sqrt{3})^2$

$|\vec{a}-\vec{b}|^2 = 3$

L.H.S $= |\vec{a}-\vec{b}|^2$

$= |\vec{a}|^2+|\vec{b}|^2-2\{\vec{a}\cdot\vec{b}$

$= 1+1-2|\vec{a}||\vec{b}|\cos\theta$

$= 2-2|\vec{a}||\vec{b}|\cos\theta$

$= \cancel{2}-\cancel{2}(1)(1)\left(\frac{-1}{\cancel{2}}\right)$

$= 2+1$

$= 3 =$ R.H.S

$|a-b|^2$

$= \sqrt{(a\cdot b)\cdot(a-b)}$

$= \cancel{\sqrt{a\cdot a - a\cdot b - b\cdot a + b\cdot b}}$

$\Rightarrow \sqrt{a\cdot a - a\cdot b - b\cdot a + b\cdot b}$

$= \sqrt{|a|^2 - a\cdot b - a\cdot b + |b|^2}$

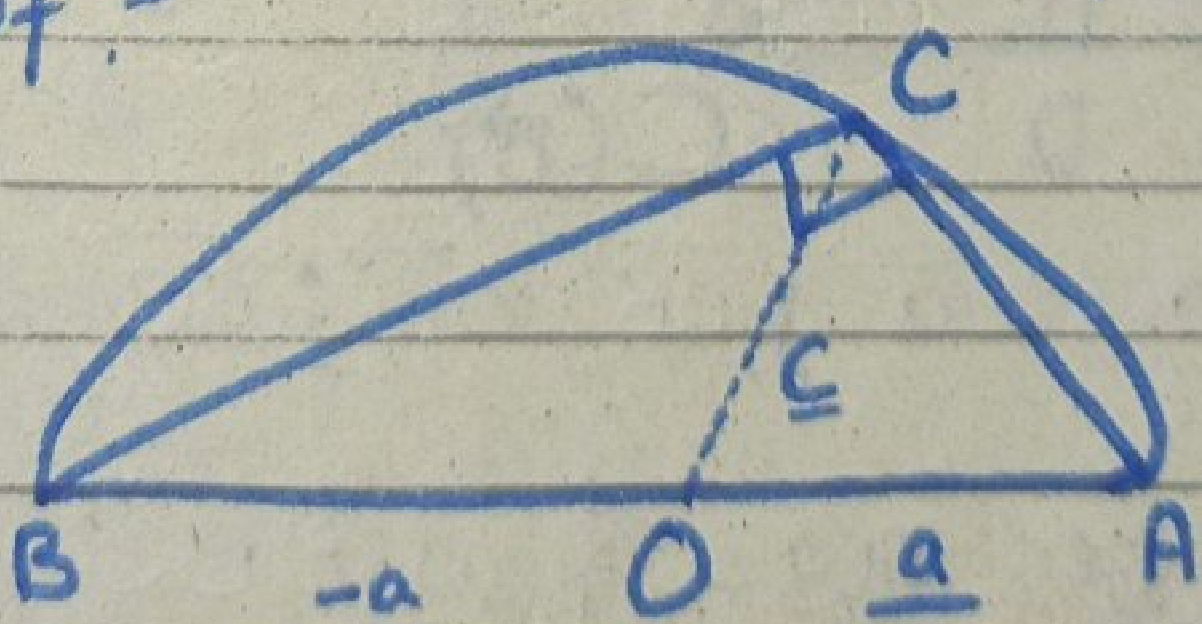
$= \sqrt{|1|^2 - 2(a\cdot b) + 1^2}$

$= \sqrt{2-2(|a||b|\cos\theta)}$

$\Rightarrow \sqrt{2-2(1\cdot 1 - \frac{1}{2})}$

$\boxed{= \sqrt{3} \text{ Proved}}$

(ii) Proof:-



~~Consider a semicircle having center~~ at origin O(0,0) and A, B are end points of diameter having position vector $\underline{c}$

Clearly radius of semicircle $= |\underline{a}| = |-\underline{a}| = |\underline{c}|$ ①

Now $\vec{AC} = c - a$ And $\vec{BC} = \underline{c} - (-\underline{a}) = \underline{c} + \underline{a}$

Consider $\vec{AC} \cdot \vec{BC} = (\underline{c} - \underline{a}) \cdot (\underline{c} + \underline{a})$

$= |\underline{c}|^2 + \underline{c} \cdot \underline{a} - \underline{a} \cdot \underline{c} - |\underline{a}|^2$ $\quad \because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

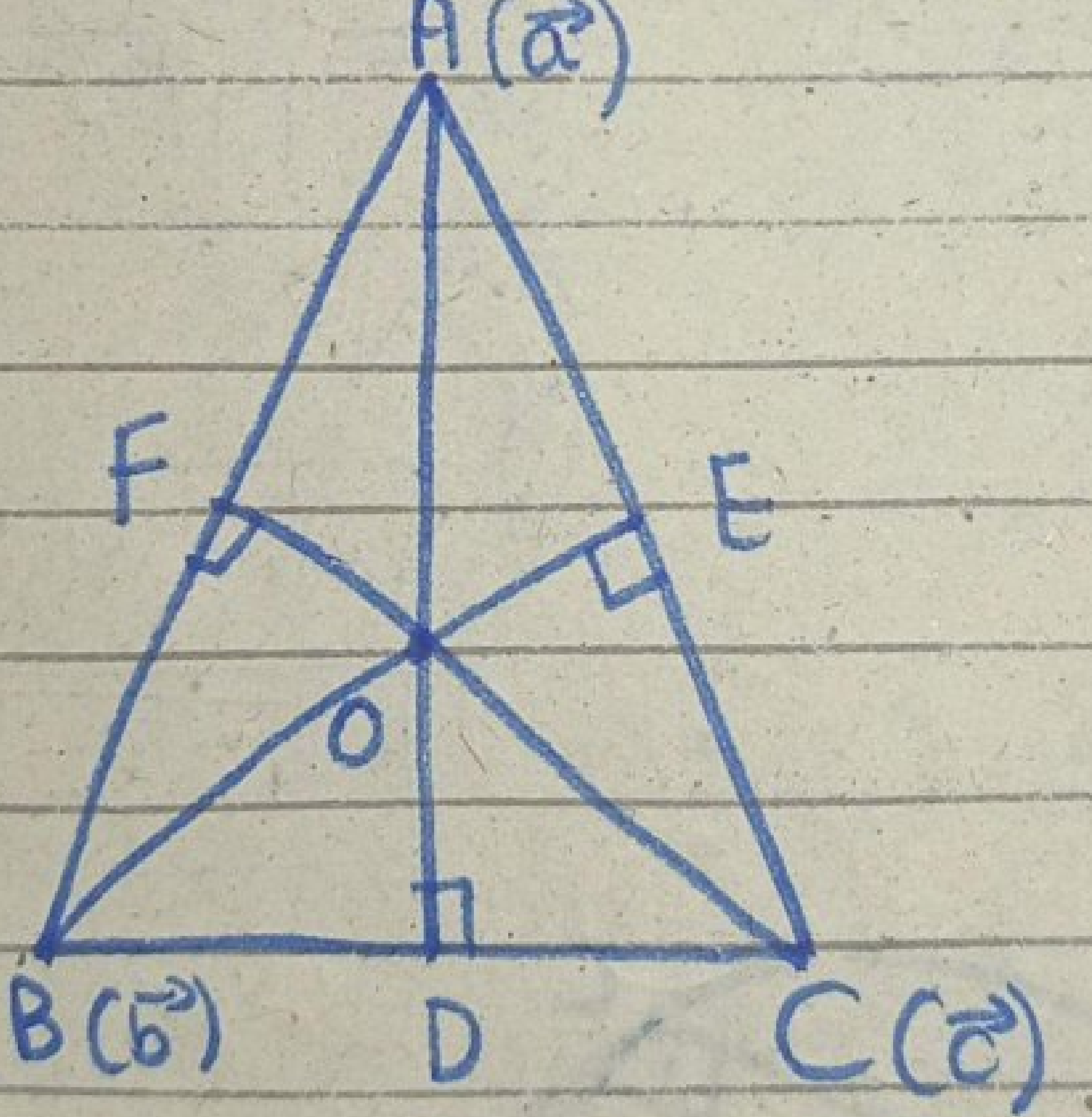
$= |\underline{c}|^2 - |\underline{c}|^2$ $\quad \therefore$ using ①

$= 0$

This show that $\vec{AC}$ perpendicular $\vec{BC}$.

I.e $\angle ACB = 90°$ proved

Q11 ①



Proof: let $\vec{OA} = \vec{a}$; $\vec{OB} = \vec{b}$; $\vec{OC} = \vec{c}$ be the position vectors of ΔABC. let O be the concurrent point

$\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ be the altitude of triangle from fig $\vec{OA} \parallel \vec{AD}$ then

$\vec{AD} = \lambda \vec{OA} = \lambda \vec{a}$

$\vec{AD} \perp \vec{BC}$ then $\vec{AD} \cdot \vec{BC} = 0 \Rightarrow \lambda \vec{a} \cdot (\vec{c} - \vec{b}) = 0$

$\vec{a} \cdot (\vec{c} - \vec{b}) = \frac{0}{\lambda} \Rightarrow \vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b} = 0$ — (1)

from fig $\vec{BE} \parallel \vec{OB}$ then $\vec{BE} = \lambda \vec{OB} \cdot \lambda \vec{b}$

$\vec{BE} \perp \vec{CA}$ then $\vec{BE} \cdot \vec{CA} = 0 \Rightarrow \lambda \vec{b} \cdot (\vec{a} - \vec{c}) = 0$

$\vec{b} \cdot (\vec{a} - \vec{c}) = \frac{0}{\lambda} \Rightarrow \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{c} = 0$ — (2)

Adding (1) and (2)

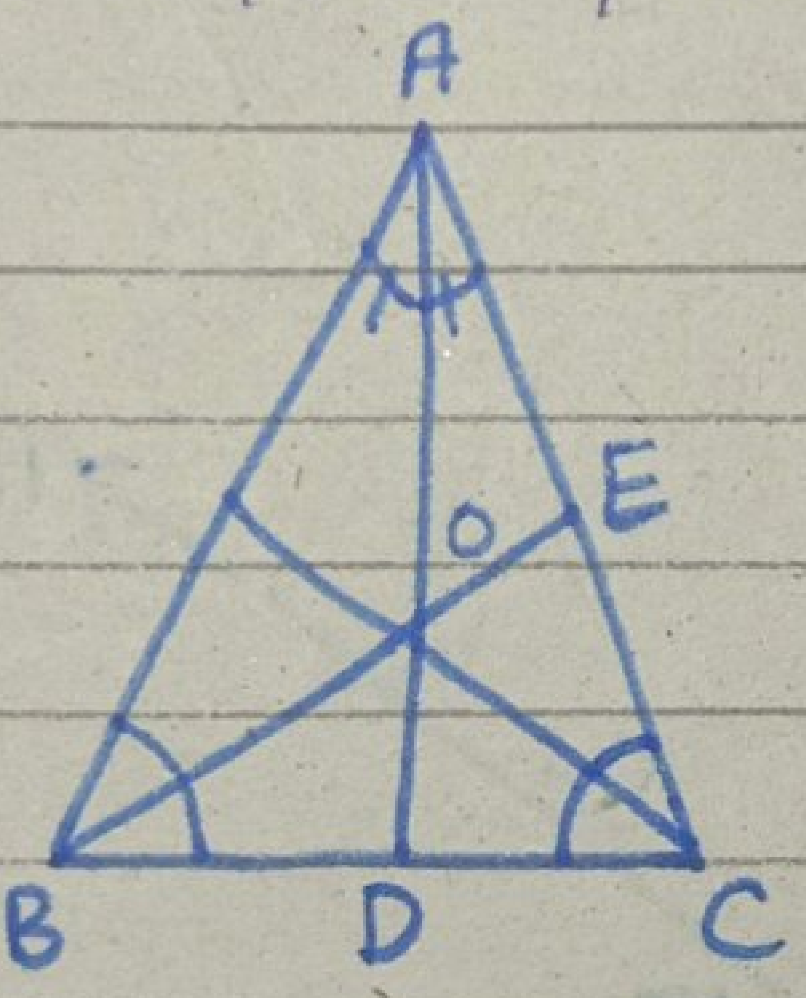
$\vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{c} = 0$

$(\vec{a} - \vec{b}) \cdot \vec{c} = 0 \Rightarrow \lambda \vec{c} \cdot (\vec{a} - \vec{b}) = 0$

$\vec{CF} \cdot \vec{BA} = 0 \Rightarrow \vec{CF} \perp \vec{BA}$

here $\vec{CF} = \lambda \vec{c} = \lambda \vec{OC}$ then $\vec{CF} \parallel \vec{OC}$ Hence

(ii)



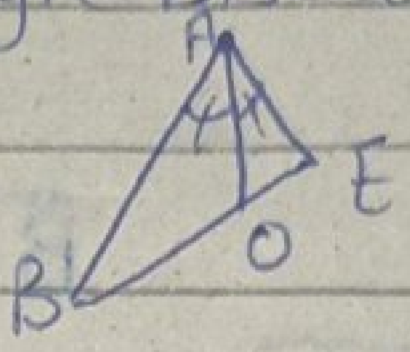
Proof: In ΔABC, let AD, BE are two angle bisectors. They meet at O.

we have to prove that $\frac{AC}{CD} = \frac{AO}{OD}$

Construct CO to meet the intersecting point O from C

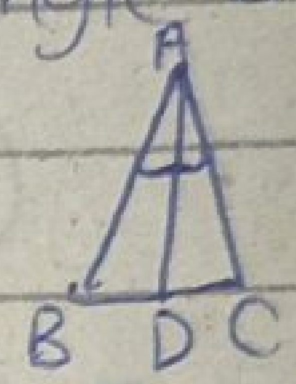
In ΔABE

$\dfrac{AB}{AE} = \dfrac{BO}{OE}$ ∴ by using angle bisecter theorm



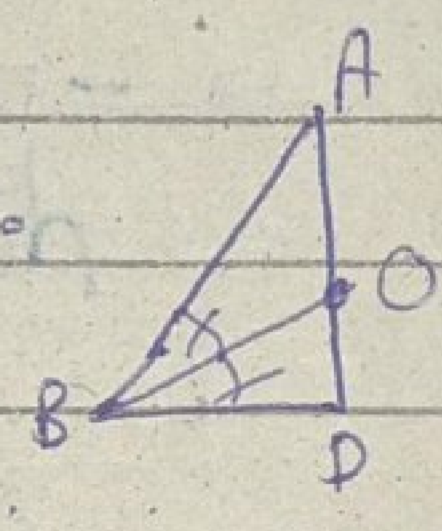
Also In ΔABC

$\dfrac{AB}{AC} = \dfrac{BD}{CD} \Rightarrow \dfrac{AB}{BD} = \dfrac{AC}{CD}$ — (1) ∴ using angle bisecter theorm



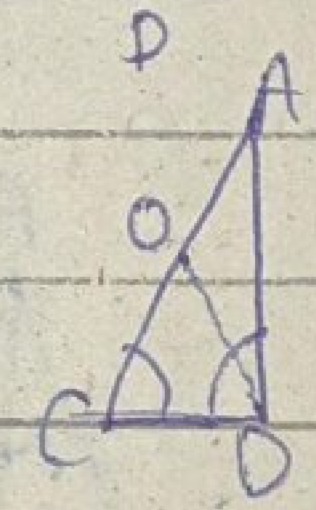
In ΔABD,

$\dfrac{AB}{BD} = \dfrac{AO}{OD}$ — (2) ∴

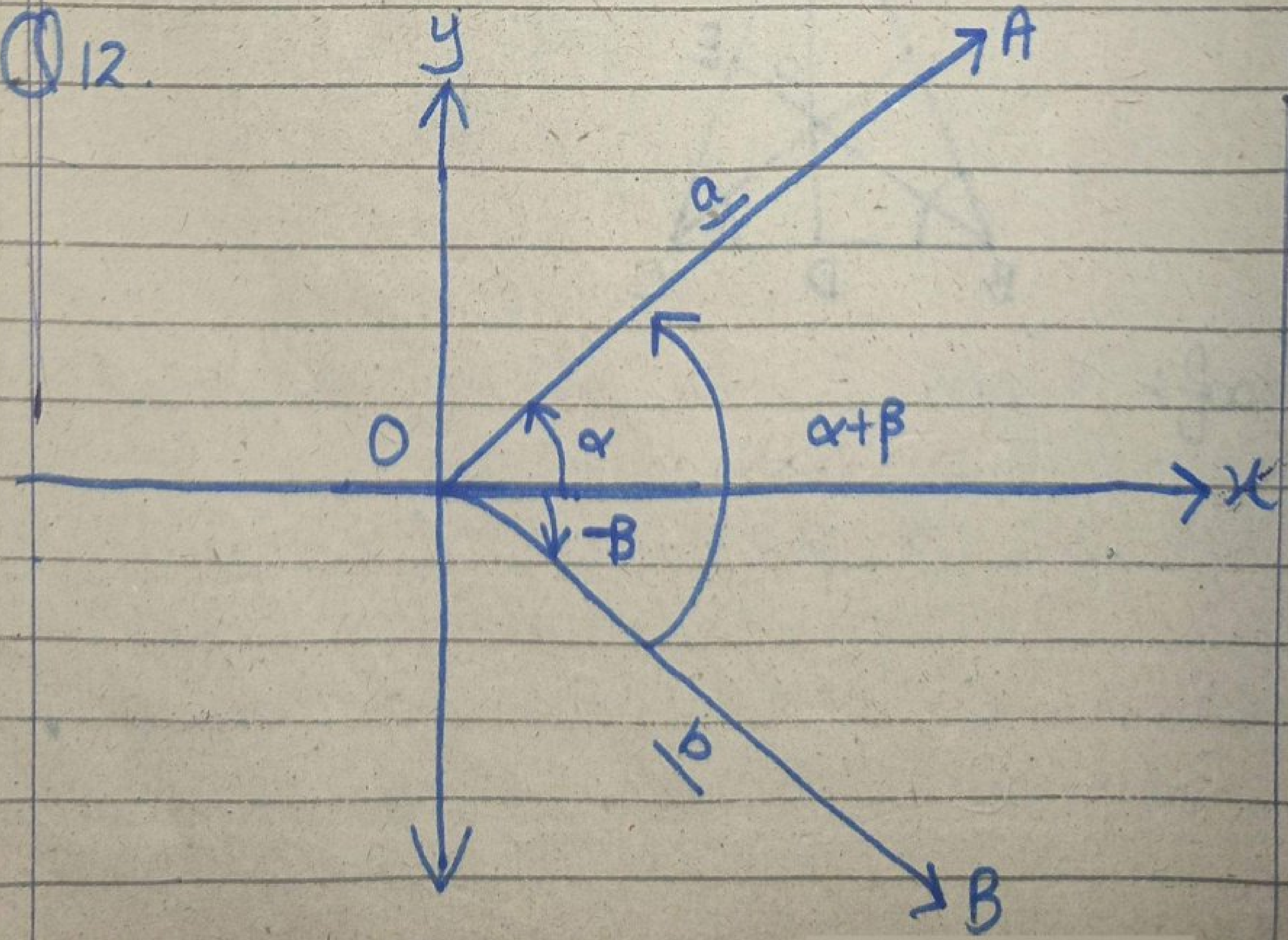


from (1) & (2) we get

$\dfrac{AC}{CD} = \dfrac{AO}{OD}$ proved



Q12.



Proof:- Consider two unit vectors $\underline{a}$ and $\underline{b}$ making angle α and $-\beta$ with +ve x-axis

then:- $\underline{a} = OA = \cos\alpha\hat{i} + \sin\alpha\hat{j}$

$\underline{b} = OB = \cos(-\beta)\hat{i} + \sin(-\beta)\hat{j}$

$\underline{b} = \cos\beta\hat{i} - \sin\beta\hat{j}$

Now

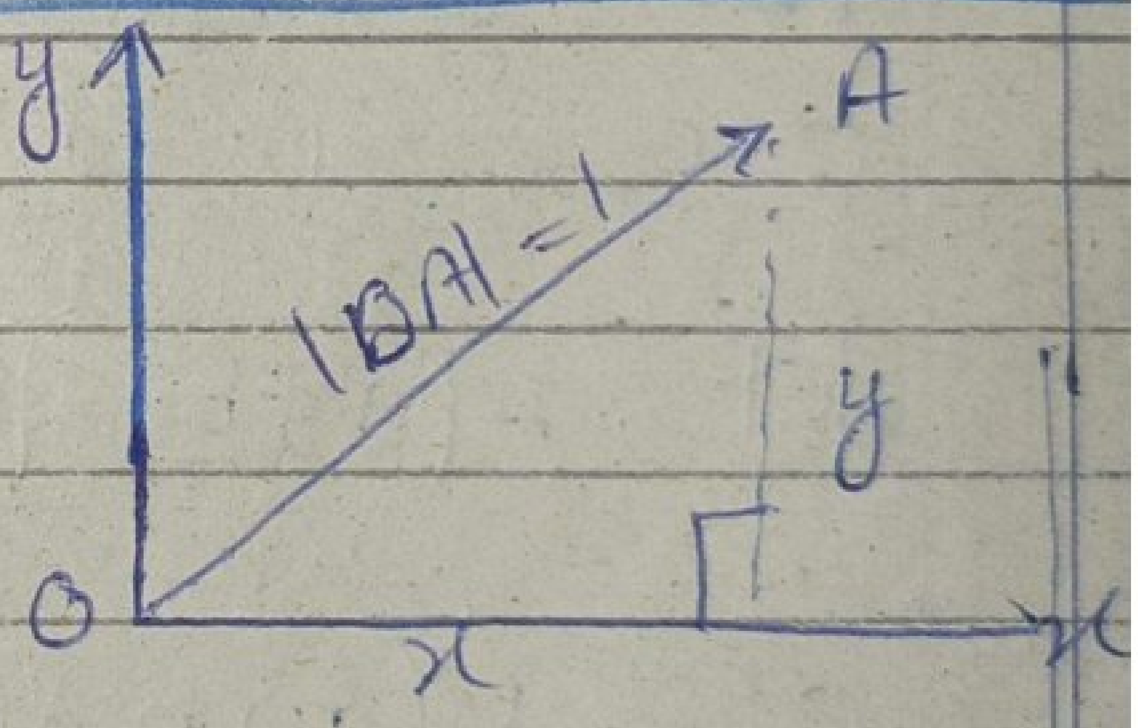
$\underline{a} \cdot \underline{b} = (\cos\alpha\hat{i} + \sin\alpha\hat{j}) \cdot (\cos\beta\hat{i} - \sin\beta\hat{j})$

$|\underline{a}||\underline{b}|\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

∴ $|\underline{a}| = 1$ and $|\underline{b}| = 1$ because unit vectors

$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$ Proved

Key points



$OA = x\hat{i} + y\hat{j}$

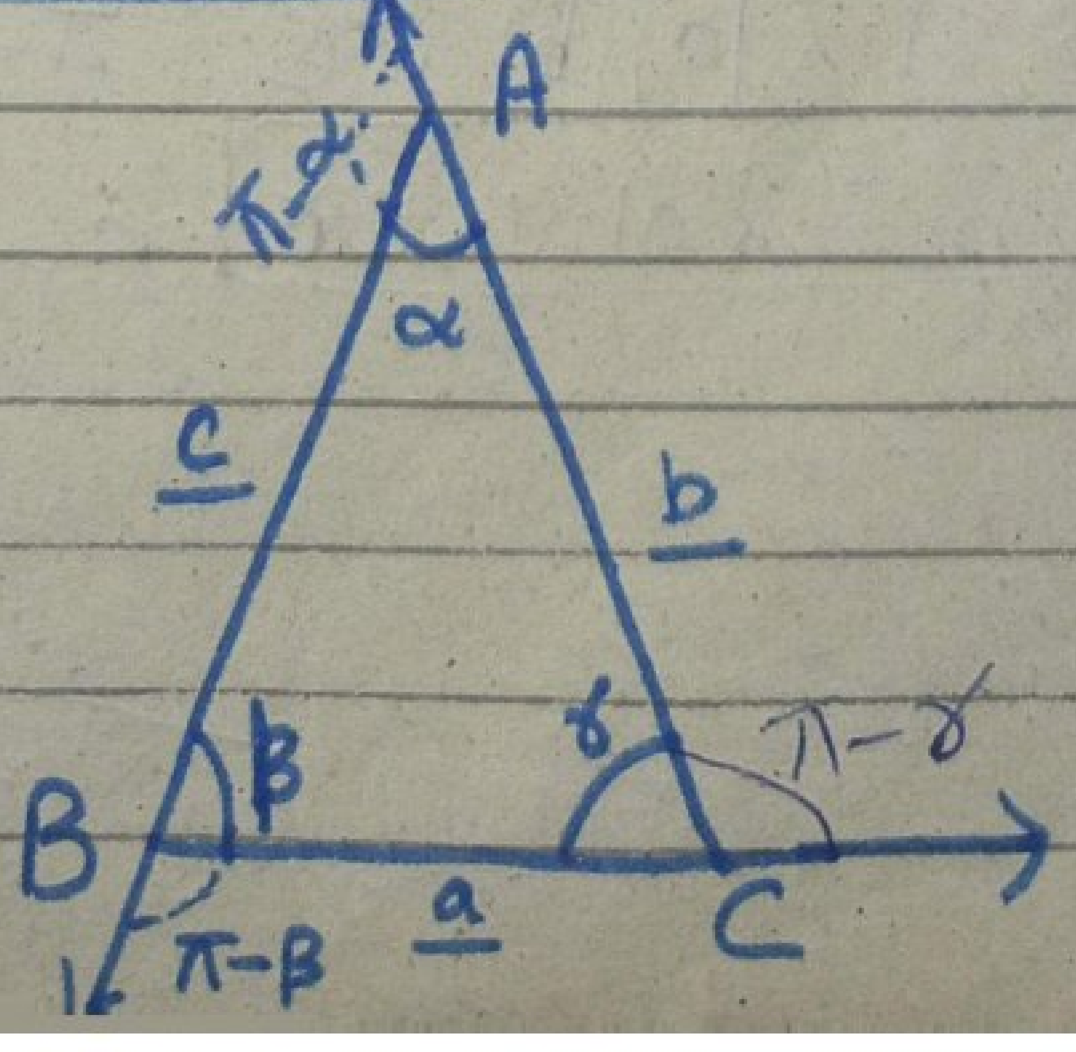
$\Rightarrow \sin\alpha = \frac{P}{H} = y$

$\boxed{y = \sin\alpha}$

and $\cos\alpha = \frac{B}{P} = x$

$\boxed{x = \cos x}$

(ii)



Proof Consider $\underline{a}$, $\underline{b}$ and $\underline{c}$ are vectors along the sides BC, CA and AB respectively. Also let $|\underline{a}| = a$ then from triangle $\underline{a} + \underline{b} + \underline{c} = 0$

$\Rightarrow \underline{b} = -\underline{a} - \underline{c}$

$\underline{b} \cdot \underline{b} = (-\underline{a} - \underline{c}) \cdot \underline{b}$ ∴ Taking dot with $\underline{b}$.

$\underline{b} \cdot \underline{b} = -\underline{a} \cdot \underline{b} - \underline{c} \cdot \underline{b}$

$|\underline{b}|^2 = -|\underline{a}||\underline{b}| \cos(\pi - \gamma) - |\underline{b}||\underline{c}| \cos(\pi - \alpha)$

$|\underline{b}|^2 = |\cancel{\underline{b}}| \left[\underline{a} \cos\gamma + \underline{c} \cos\alpha \right]$

$\boxed{\underline{b} = \underline{a} \cos\gamma + \underline{c} \cos\alpha}$ Hence proved

∴ $\cos(\pi - \gamma) = -\cos\gamma$.

Now $\underline{c} = -\underline{a} - \underline{b}$

$\underline{c} \cdot \underline{c} = -(\underline{a} + \underline{b}) \cdot \underline{c}$

$|\underline{c}|^2 = -(\underline{a} + \underline{b}) \cdot \{-(\underline{a} + \underline{b})\}$ ∴ using value of $\underline{c} = -(\underline{a} + \underline{b})$

$\underline{c}^2 = +(\underline{a} + \underline{b})(\underline{a} + \underline{b})$

$\underline{c}^2 = |\underline{a}|^2 + |\underline{b}|^2 + 2\underline{a} \cdot \underline{b}$

$\underline{c}^2 = \underline{a}^2 + \underline{b}^2 + 2|a||b| \cos(\pi - \gamma)$

$\boxed{\underline{c}^2 = a^2 + b^2 - 2ab\cos\gamma}$ proved

Q13) (i) Proof Given $(\vec{a}+\vec{b})\cdot\vec{a}=0$

$|\vec{a}|^2+\vec{a}\cdot\vec{b}=0$ —— (1)

And

$=(7\vec{a}+\vec{b})\cdot\vec{b}$

$=7\vec{a}\cdot\vec{b}+|\vec{b}|^2$ $\quad\therefore \vec{b}\cdot\vec{b}=|\vec{b}|^2$

$=7\vec{a}\cdot\vec{b}+(\sqrt{7}\,|\vec{a}|)^2$ $\quad\therefore$ using given $|\vec{a}|=\frac{1}{\sqrt{7}}|\vec{b}|$

$|\vec{b}|=\sqrt{7}|\vec{a}|$

$=7\vec{a}\cdot\vec{b}+7|\vec{a}|^2$

$=7\left[|\vec{a}|^2+\vec{a}\cdot\vec{b}\right]$

$=7(0)$ $\therefore$ using (1)

$=0$

Hence $(7\vec{a}+\vec{b})\cdot\vec{b}=0$

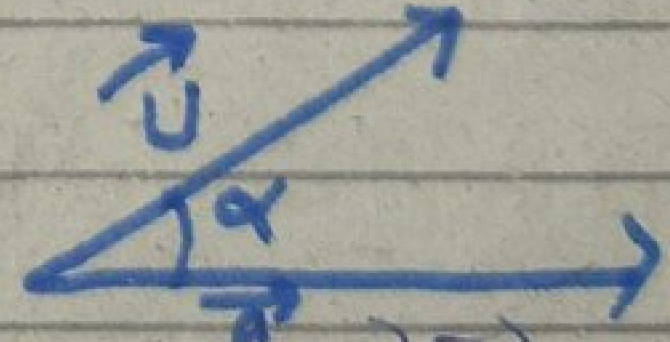
$=(7\vec{a}+\vec{b})\perp\vec{b}$ proved

(ii) let $\vec{U}=\hat{a}+\hat{b}$ or $\vec{U}=\frac{\vec{a}}{|a|}+\frac{\vec{b}}{|b|}$

And α be the angle b/w $\vec{a}$ and $\vec{b}$

1st $\vec{U}$ is inclined at $\vec{a}$ vector

$\Rightarrow$ Projection $\vec{U}$ along $\vec{a}=\frac{\vec{U}\cdot\vec{a}}{|a|}=\left(\frac{\vec{a}}{|a|}+\frac{\vec{b}}{|b|}\right)\cdot\frac{\vec{a}}{|a|}$



$=\frac{\vec{a}}{|\vec{a}|}\cdot\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}\cdot\vec{a}}{|\vec{b}||\vec{a}|}$

$=\frac{\vec{a}\cdot\vec{a}}{|\vec{a}|^2}+\frac{\cancel{|\vec{b}||\vec{a}|}\cos\alpha}{\cancel{|\vec{b}||\vec{a}|}}$ $\quad\therefore \vec{a}\,\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\therefore \vec{a}\cdot\vec{a}=|\vec{a}|^2$

$=\frac{|\vec{a}|^2}{|\vec{b}|^2}+\cos\alpha$ $\Rightarrow 1+\cos\alpha$ —— (1)

2nd Projection $\vec{u}$ along $\vec{b}$.

$$= \frac{\vec{u}\cdot\vec{b}}{|\vec{b}|} = \left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right)\cdot\frac{\vec{b}}{|\vec{b}|}.$$

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} + \frac{\vec{b}\cdot\vec{b}}{|\vec{b}|^2} = \frac{\cancel{|\vec{a}||\vec{b}|}\cos\alpha}{\cancel{|\vec{a}||\vec{b}|}} + \frac{|\vec{b}|^2}{|\vec{b}|^2} = \cos\alpha + 1$$

From (1) & (2) hence proved that

$\hat{a}+\hat{b}$ is equally inclined with $\vec{a}$ & $\vec{b}$.

Q14) Sol:- $\vec{F} = 3\hat{i} - 5\hat{j} + 7\hat{k}$, $|d| = 14\,m$

$\vec{d} = \hat{i} - 3\hat{j} + \hat{k}$.

~~we know that~~ $\hat{d} = \frac{\vec{d}}{|d|}$

$$\Rightarrow \boxed{\vec{d} = |d|\cdot\hat{d}} \quad -(1)$$

Now find $\hat{d} = \frac{\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{(1)^2+(-3)^2+(1)^2}}$.

$\hat{d} = \frac{\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{11}}$ put in (1).

$\vec{d} = 14\cdot\left(\frac{\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{11}}\right)$

Hence $w = \vec{F}\cdot\vec{d} = (3\hat{i} - 5\hat{j} + 7\hat{k}) \times \frac{14}{\sqrt{11}}(\hat{i} - 3\hat{j} + \hat{k})$

$w = \frac{14}{\sqrt{11}}\left[3(1) - 5(-3) + 7(1)\right]$

$w = \frac{14}{\sqrt{11}} \cdot [3+15+7] = \frac{14}{\sqrt{11}}(25)$

$\boxed{w = \frac{350}{\sqrt{11}}}$ joules.

Q15) $F = F_1 + F_2 = 5\hat{i} + 10\hat{j} + 3K$

$d = \vec{PQ} = \vec{OQ} - \vec{OP}$

$d = 2\hat{i} + \hat{j} + \frac{5}{2}\hat{k} - \hat{i} + 3\hat{j} - \frac{3}{2}\hat{k}$

$\boxed{d = \hat{i} + 4\hat{j} + \hat{k}}$

$\Rightarrow w = \vec{F} \cdot \vec{d} = (5\hat{i} + 10\hat{j} + 3\hat{k}) \cdot (\hat{i} + 4\hat{j} + \hat{k})$

$= 5(1) + 10(4) + 3(1)$

$w = 5 + 40 + 3 \Rightarrow \boxed{48J}$ Ans

Q16) Sol: $F = 30N$, $\theta = 30°$, $w = ?$, $d = 10m$

we know that

$w = F \cdot d \cos\theta$

$w = (30)(10)\cos(30)$

$w = 300\left(\frac{\sqrt{3}}{2}\right)$

$w = 150\sqrt{3}\,J$ Ans