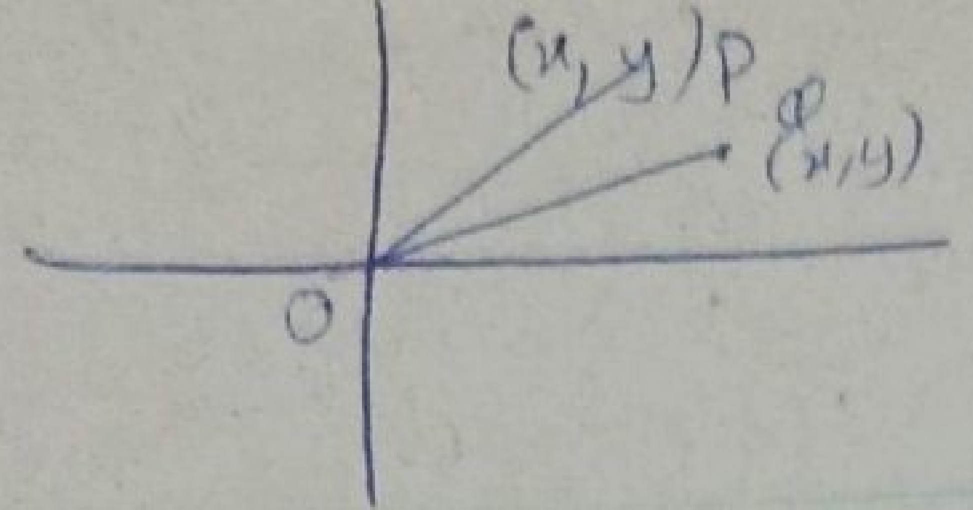
Studybeesofficial

Where Minds Pollinate

Class 11
Mathematics

Chapter 3
Exercise 3.1
Notes

National Book Foundation



vectr $\overrightarrow{OP}$ ~~vector~~

($\overrightarrow{OP}, \overrightarrow{OQ}$)

∴ No of ways when 2 Person donot sit together = Total − (sitting together)

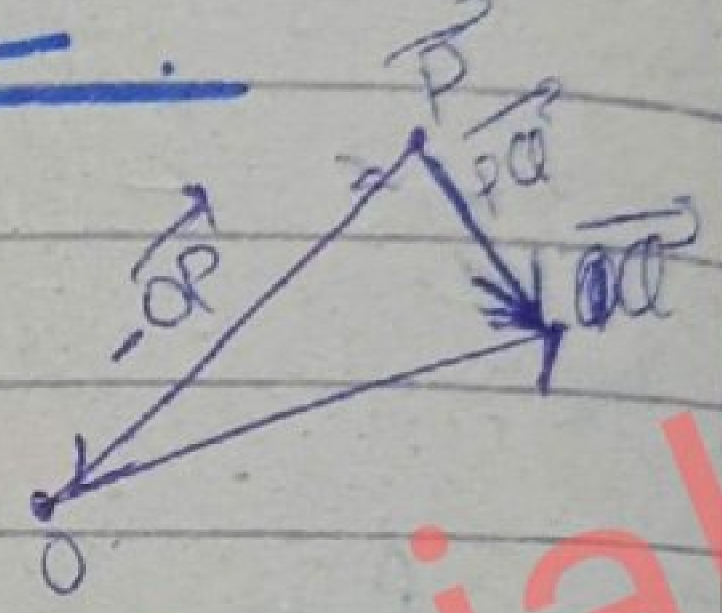
= 39916800 − 7257600

= $\boxed{32659200}$

Ch#3 Ex 3.1

Position vect is from origin

always final − Initial



Q1) (i) $\overrightarrow{PQ}$

Sol $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$\overrightarrow{PQ} = (-4, -6) - (3, -1)$

$\overrightarrow{PQ} = (-4-3, -6+1)$

$\overrightarrow{PQ} = (-7, -5)$ ∴ ordered pair form

$\boxed{\overrightarrow{PQ} = -7\hat{i}, -5\hat{j}}$ Answer.

which is required component form.

(ii) $3\overrightarrow{PQ} - \overrightarrow{RS}$

Solt $= 3[\overrightarrow{OQ} - \overrightarrow{OP}] - [\overrightarrow{OS} - \overrightarrow{OR}]$

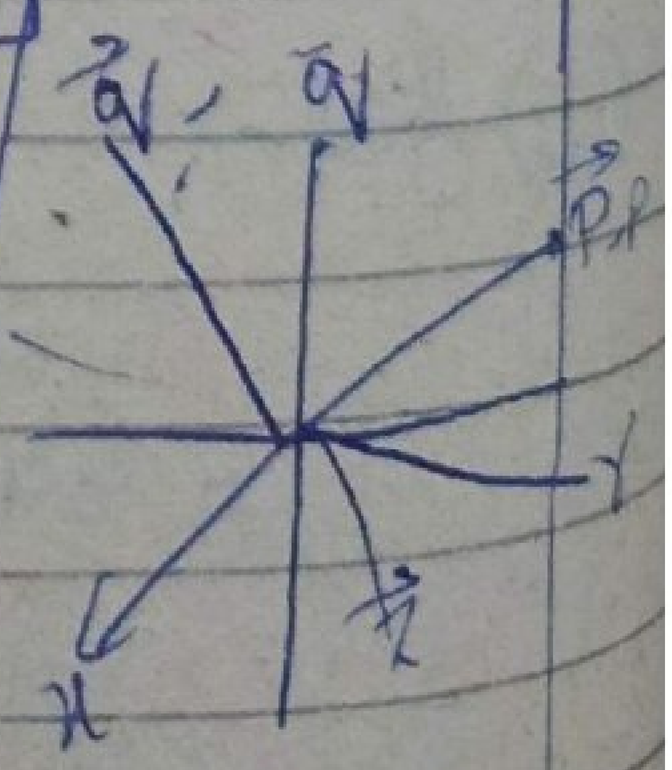
$= 3[(-4, -6) - (3, -1)] - [(2, 5) - (1, 4)]$

$= 3(-4-3, -6+1] - [2-1, 5-4)$

$= (-21, -15) - (1, 1)$

$= (-21-1, -15-1)$

$= (-22\hat{i} - 16\hat{j})$ Ans



BC AB

OC - OB OB - OA

(iii) $2\vec{PR} + 3\vec{PS}$

Sol $= 2[\vec{OR} - \vec{OP}] + 3[\vec{OS} - \vec{OP}]$

$= 2[(1,4) - (3,-1)] + 3[(2,5) - (3,-1)]$

$= 2(1-3,\ 4+1) + 3(2-3,\ 5+1)$

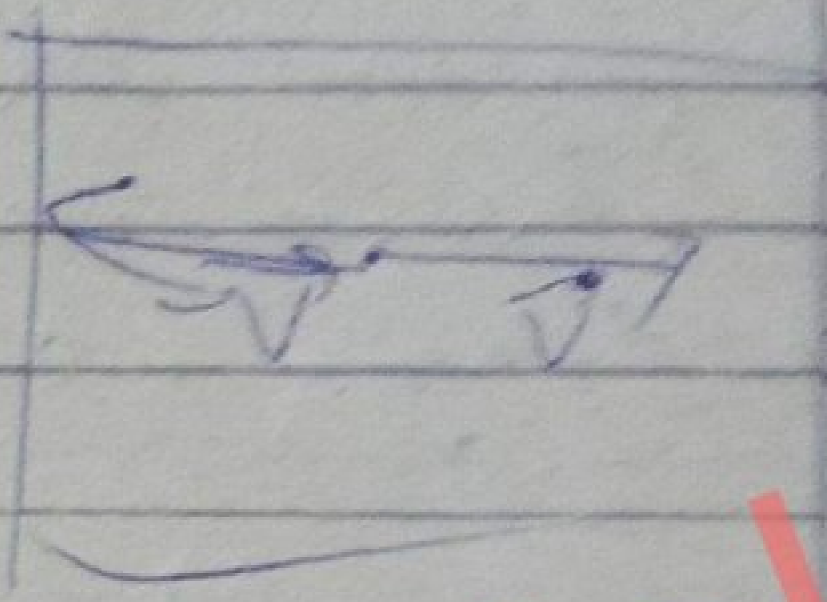
$= 2(-2,5) + 3(-1,6)$

$= (-4,10) + (-3,18)$

$= (-4-3,\ 10+18)$

$= (-7, 28)$

$= -7\hat{i} + 28\hat{j}$ Answer.



(iv) $\frac{1}{2}\vec{PQ} + \frac{5}{2}\vec{PR} - \frac{3}{2}\vec{QS}$

Sol $\frac{1}{2}[\vec{OQ} - \vec{OP}] + \frac{5}{2}[\vec{OR} - \vec{OP}] - \frac{3}{2}[\vec{OS} - \vec{OQ}]$

$= \frac{1}{2}((-4,-6) - (3,-1)) + \frac{5}{2}[(1,4) - (3,-1)] - \frac{3}{2}[(2,5) - (-4,-6)]$

$= \frac{1}{2}(-4-3,\ -6+1) + \frac{5}{2}(1-3,\ 4+1) - \frac{3}{2}(2+4,\ 5+6)$

$= \frac{1}{2}(-7,-5) + \frac{5}{2}(-2,5) - \frac{3}{2}(6,11)$

$= \left(-\frac{7}{2}, -\frac{5}{2}\right) + \left(-\frac{10}{2}, \frac{25}{2}\right) + \left(-\frac{18}{2}, -\frac{33}{2}\right)$

$= \left(-\frac{7}{2} - \frac{10}{2} - \frac{18}{2},\ -\frac{5}{2} + \frac{25}{2} - \frac{33}{2}\right)$

$= \left(-\frac{35}{2}, -\frac{13}{2}\right)$

$= -\frac{35}{2}\hat{i} - \frac{13}{2}\hat{j}$ Ans

(v) $3^2\overrightarrow{PS} - 4^2\overrightarrow{SP} + \overrightarrow{QP}$

Sol: $= 9[\overrightarrow{OS} - \overrightarrow{OP}] - 16[\overrightarrow{OP} - \overrightarrow{OS}] + [\overrightarrow{OP} - \overrightarrow{OQ}]$

$= 9[(2,5) - (3,-1)] - 16[(3,-1) - (2,5)] + [(3,-1) - (-4,-6)]$

$= 9(2-3, 5+1) - 16(3-2, -1-5) + (3+4, -1+6)$

$= 9(-1, 6) - 16(1, -6) + (7, 5)$

$= (-9, 54) + (16, 96) + (7, 5)$

$= (-9-16+7, 54+96+5)$

$= (-18, 155)$

$= -18\hat{i} + 155\hat{j}$ Ans

Q2 (i) Show $A(1,0)$, $B(6,0)$ and $C(0,0)$ are collinear.

Method (1)

Proof:- Condition:- $\vec{a} = \lambda \vec{b}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (6,0) - (1,0) = (6-1, 0-0) = 5\hat{i} + 0\hat{j}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (0,0) - (6,0) = (0-6, 0-0) = -6\hat{i} + 0\hat{j}$.

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (0,0) - (1,0) = (0-1, 0-0) = -\hat{i} + 0\hat{j}$.

$\boxed{\overrightarrow{AB} = -5\overrightarrow{AC}}$ Hence $\vec{A}, \vec{B}, \vec{C}$ are collinear.

and $\boxed{\overrightarrow{BC} = 6\overrightarrow{AC}}$

$AB + BC < AC$.

Method 2.

Condition: if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

$= \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}$

Expanding by R_3

$$= 1\begin{vmatrix} 1 & 0 \\ 6 & 0 \end{vmatrix} = 1(0-0) = 1(0) = 0$$

Hence $\vec{A}, \vec{B}$ & $\vec{C}$ are collinear.

3rd Method:-

Condition:- $\vec{AB} + \vec{BC} = \vec{AC}$

L.HS

$= \vec{AB} + \vec{BC}$

$= 5\hat{i} + 0\hat{j} - 6\hat{i} + 0\hat{j}$

$= -1\hat{i} + 0\hat{j}$

$= \vec{AC}$ Proved

Method 1

(ii) Sol:- Given $\vec{a}$ and $\vec{b}$ are collinear then parallel also

$$\vec{a} = d\vec{b}$$

putting values

$$2\hat{i} - 7\hat{j} = d\left(\frac{m}{2}\hat{i} + 11\hat{j}\right)$$

comparing $\hat{i}$ and $\hat{j}$ on both sides

$2 = \frac{md}{2}$ — (1) and $-7 = d11$ — (2)

$\boxed{d = \frac{-7}{11}}$ put in (1)

$2 = \frac{m}{2}\left(\frac{-7}{11}\right)$

$(2)(2)(11) = -7m$

$\boxed{m = \frac{44}{-7}}$ Ans

Method 2

Given two vectors are collinear so they are parallel

($<x, y>$ means $x\hat{i} + y\hat{j}$)

$$\Rightarrow \frac{2}{m/2} = \frac{-7}{11}$$

$$(2)(11) = -7\left(\frac{m}{2}\right)$$

$$\frac{(22)(2)}{-7} = m$$

$$\boxed{m = \frac{44}{-7}}$$ Ans

Q3:- If $\vec{U} = <-1, 1>$, $\vec{V} = <0, 1>$, $\vec{w} = <3, 4>$

can do without letting x

(i) Find x that satisfy $\vec{U} - 2\vec{x} = \vec{x} - \vec{w} + 3\vec{V}$

Solution:- $\vec{U} - 2\vec{x} = \vec{x} - \vec{w} + 3\vec{V}$ —(1)

let $\vec{x} = x\hat{i} + y\hat{j}$, putting values in (1) we get

$$-\hat{i} + \hat{j} - 2(x\hat{i} + y\hat{j}) = (x\hat{i} + y\hat{j}) - (3\hat{i} + 4\hat{j}) + 3(0\hat{i} + \hat{j})$$

$$-\hat{i} + \hat{j} - 2x\hat{i} - 2y\hat{j} = x\hat{i} + y\hat{j} - 3\hat{i} - 4\hat{j} + 3\hat{j}$$

$$\hat{i}(-1 - 2x) + (1 - 2y)\hat{j} = (x - 3)\hat{i} + (y - 4 + 3)\hat{j}$$

Compare $\hat{i}$ and $\hat{j}$ on b/s.

$-1 - 2x = x - 3$; $1 - 2y = y - 1$

$-1 + 3 = x + 2x$; $2 = 3y$

$2 = 3x$; $\boxed{y = \frac{2}{3}}$

$\boxed{x = \frac{2}{3}}$

$$\boxed{\vec{x} = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{i}}$$ Answer

Just ~~compare~~ eliminate $\vec{v}$

Do it without letting $\vec{v} = a + bj$

(ii) let $\vec{u} = a\hat{i} + b\hat{j}$ & $\vec{v} = c\hat{i} + d\hat{j}$.

$\vec{u} + \vec{v} = 2\hat{i} - 3\hat{j}$

$a\hat{i} + b\hat{j} + c\hat{i} + d\hat{j} = 2\hat{i} - 3\hat{j}$

$(a+c)\hat{i} + (b+d)\hat{j} = 2\hat{i} - 3\hat{j}$

Compare

$a + c = 2$ — (1)

$c + d = -3$ — (2)

Solve eq (1) & (3)

$2a + \cancel{2c} = 4 \quad : (1)\times 2$

$\pm 3a \pm \cancel{2c} = \mp 1$

$-a = 5$

$\boxed{a = -5}$ put in (1)

$-5 + c = 2 \Rightarrow \boxed{c = 7}$

Hence

$3\vec{u} + 2\vec{v} = -\hat{i} + 2\hat{j}$

$3(a\hat{i} + b\hat{j}) + 2(c\hat{i} + d\hat{j}) = -\hat{i} + 2\hat{j}$

$3a\hat{i} + 3b\hat{j} + 2c\hat{i} + 2d\hat{j} = -\hat{i} + 2\hat{j}$

Compare

$3a + 2c = -1$ — (3)

$3b + 2d = 2$ — (4)

Solve eq (2) & (4)

$2b + \cancel{2d} = -6 \quad : (2)\times 2$

$\pm 3b \pm \cancel{2d} = \pm 2$

$-b = -8$

$\boxed{b = 8}$ put in (2)

$8 + d = -3 \Rightarrow \boxed{d = -11}$

$\vec{u} = -5\hat{i} + 8\hat{j}$

$\vec{v} = 7\hat{i} - 11\hat{j}$ Answer

(iii) let $\vec{AB} = \vec{v} \Rightarrow \vec{AB} = -3\hat{i} + \hat{j} + 2\hat{k}$

$\vec{OB} - \vec{OA} = -3\hat{i} + \hat{j} + 2\hat{k}$

Suppose initial point: $OA = x\hat{i} + y\hat{j} + zk$

$OB \Rightarrow 5\hat{i} + 0\hat{j} + \hat{k} - x\hat{i} - y\hat{j} - z\hat{k} = -3\hat{i} + \hat{j} + 2\hat{k}$

$(5-x)\hat{i} - y\hat{j} + (1-z)\hat{k} = -3\hat{i} + \hat{j} + 2\hat{k}$

compare $\hat{i}, \hat{j}, \hat{k}$ then

$5 - x = -3$; $-y = 1$; $1 - z = 2$

$\boxed{x = 8}$; $\boxed{y = -1}$; $\boxed{z = -1}$

$\Rightarrow \boxed{\vec{A} = 8\hat{i} - 1\hat{j} - 1\hat{k}}$ Answer

Q4 (i) Given vectors are parallel
therefore $\vec{a} = \lambda\vec{b}$

$3\hat{i}+4\hat{j}-9\hat{k} = \lambda(\hat{i}+m\hat{j}-3\hat{k})$

$3\hat{i}+4\hat{j}-9\hat{k} = \lambda\hat{i}+m\lambda\hat{j}-3\lambda\hat{k}$

Compare

$\boxed{3=\lambda}$, $4=\lambda m$, $-9=-3\lambda$

$4=(3)m$, $\frac{-9}{-3}=\lambda$

$\boxed{m=\frac{4}{3}}$ $\boxed{\lambda=3}$

(ii) Given P, Q and R points are collinear, then

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

or

$|PQ| \neq \sqrt{\ }$

$|PR| = \sqrt{\ }$

$|QR| = \sqrt{\ }$

$|\vec{PR}| = |PQ| + |QR|$

$$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 3 & 5 \\ \lambda & 0 & -1 \end{vmatrix} = 0$$

Expanding by R_3 !

$$\lambda\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} = 0$$

$\lambda(10-9)-(3+4)=0$

$\lambda - 7 = 0$

$\boxed{\lambda = 7}$ Ans

Q5 (i) let 'P' be the direction vector

$\vec{P} = 2\vec{a} - 3\vec{b} + \vec{c}$

putting values of a, b and c

$\vec{P} = 2(\hat{i} - 2\hat{j} + \hat{k}) - 3(\hat{i} - \hat{j} - \hat{k}) + (2\hat{i} + \hat{k})$

$\vec{P} = 2\hat{i} - 4\hat{j} + 2\hat{k} - 3\hat{i} + 3\hat{j} + 3\hat{k} + 2\hat{i} + \hat{k}$

$\vec{P} = \hat{i} - \hat{j} + 6\hat{k}$

$\vec{P} = \hat{i} - \hat{j} + 6\hat{k}$

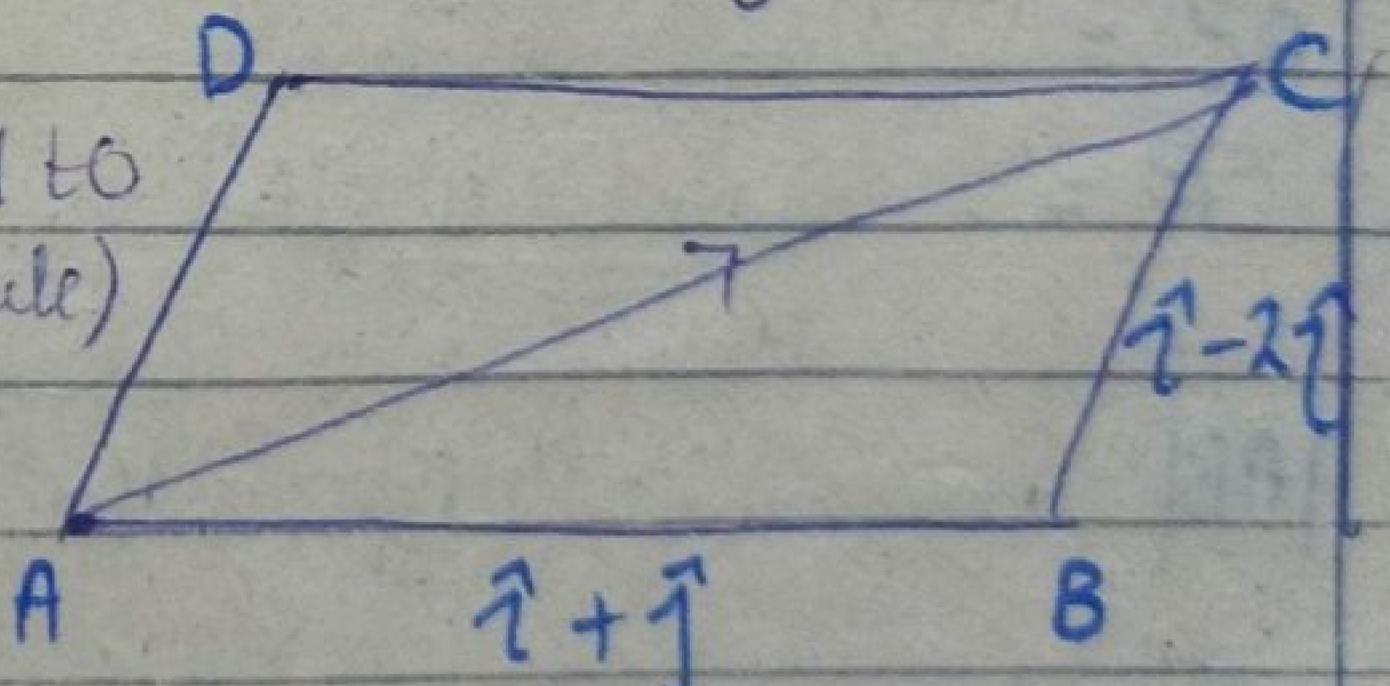
For unit vector

$|\vec{P}| = \sqrt{(1)^2 + (-1)^2 + (6)^2} = \sqrt{38}$

$\hat{P} = \dfrac{\vec{P}}{|\vec{P}|} = \dfrac{\hat{i} - \hat{j} + 6\hat{k}}{\sqrt{38}}$

$\boxed{\hat{P} = \dfrac{1}{\sqrt{38}}\hat{i} - \dfrac{1}{\sqrt{38}}\hat{j} + \dfrac{6}{\sqrt{38}}\hat{k}}$ Ans

(ii) By using Parallelogram law of Addition



$\vec{AB} + \vec{BC} = \vec{AC}$ (Head to tail rule)

putting values

$(\hat{i} + \hat{j}) + (\hat{i} - 2\hat{j}) = \vec{AC}$

$2\hat{i} - \hat{j} = \vec{AC}$

$|\vec{AC}| = \sqrt{(2)^2 + (-1)^2}$

$\boxed{|\vec{AC}| = \sqrt{5}}$ Ans

Take $|PQ|^2 + |QR|^2 = |PR|^2$

Q6) Proof) (i) Let $\vec{A} = \hat{i} - \hat{j}$; $\vec{B} = 4\hat{i} - 3\hat{j} + \hat{k}$; $\vec{C} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA} = 4\hat{i} - 3\hat{j} + \hat{k} - \hat{i} + \hat{j}$

$\boxed{\vec{AB} = 3\hat{i} - 2\hat{j} + \hat{k}}$

$\vec{BC} = \vec{OC} - \vec{OB} = 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} + 3\hat{j} - \hat{k}$

$\boxed{\vec{BC} = -2\hat{i} - \hat{j} + 4\hat{k}}$

$\vec{AC} = \vec{OC} - \vec{OA} = 2\hat{i} - 4\hat{j} + 5\hat{k} - \hat{i} + \hat{j}$

$\boxed{\vec{AC} = \hat{i} - 3\hat{j} + 5\hat{k}}$ or then take $|AB|^2 + |BC|^2 = |AC|^2$

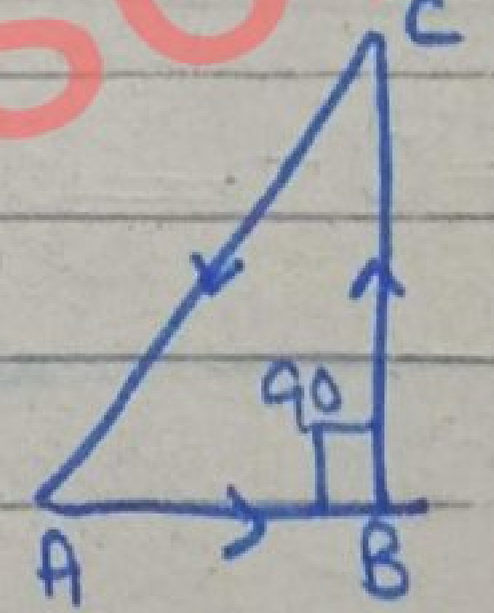
Now

$\vec{AB} \cdot \vec{BC} = (3\hat{i} - 2\hat{j} + \hat{k}) \cdot (-2\hat{i} - \hat{j} + 4\hat{k})$

$= -6 + 2 + 4$

$\boxed{\vec{AB} \cdot \vec{BC} = 0}$

Proved



(ii) Let $\vec{AB} = 2\hat{i} + 3\hat{j} + \sqrt{3}\hat{k}$

$\vec{BC} = \sqrt{10}\hat{i} - \hat{j} + \sqrt{5}\hat{k}$

$\vec{CA} = -3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$

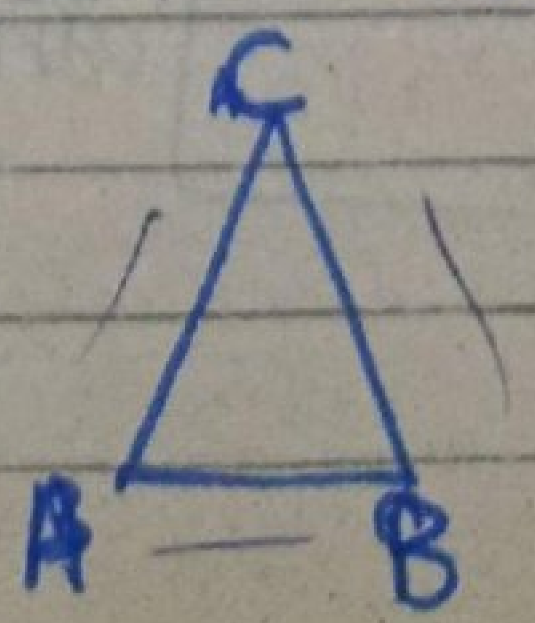
$|\vec{AB}| = \sqrt{(2)^2 + (3)^2 + (\sqrt{3})^2} = \sqrt{4+9+3} = \sqrt{16} \Rightarrow |\vec{AB}| = \boxed{4}$

$|\vec{BC}| = \sqrt{(\sqrt{10})^2 + (-1)^2 + (\sqrt{5})^2} = \sqrt{10+1+5} = \sqrt{16} \Rightarrow |\vec{BC}| = 4$

$|\vec{CA}| = \sqrt{(-3)^2 + (\sqrt{3})^2 + (2)^2} = \sqrt{9+3+4} = \sqrt{16} \Rightarrow |\vec{CA}| = 4$

$|\vec{AB}| = |\vec{BC}| = |\vec{CA}|$

Hence proved



Q7 (i) $|\vec{a}| = |3\vec{b}|$

$\sqrt{(1)^2+(-3)^2+(d)^2} = \sqrt{(3)^2+(6)^2+(-3)^2}$

$(\sqrt{1+9+d^2})^2 = (\sqrt{9+36+9})^2$

$10+d^2 = 54$

$d^2 = 44$

$\sqrt{d^2} = \sqrt{44}$

$\boxed{d = \pm 2\sqrt{11}}$

(ii) Sol:- $|\vec{a}| = \sqrt{(2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13}$

$|\vec{b}| = \sqrt{(-3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13}$

Hence $|\vec{a}| = |\vec{b}|$

Ans $\vec{a} \neq \vec{b}$ Answer

Q8 (i) let $\vec{P} = \vec{a} - 2\vec{b}$

$\vec{P} = 2\hat{i} - 3\hat{j} + \hat{k} - 2(\hat{i} - 3\hat{j} + 5\hat{k})$

$= \cancel{2\hat{i}} - 3\hat{j} + \hat{k} - \cancel{2\hat{i}} + 6\hat{j} - 10\hat{k}$

$\vec{P} = 3\hat{j} - 9k$

$|\vec{P}| = \sqrt{(3)^2+(-9)^2} = \sqrt{90}$

$\hat{P} = \frac{\vec{P}}{|\vec{P}|} = \frac{3\hat{i}-9\hat{k}}{\sqrt{90}} = \frac{\cancel{3}(\hat{i}-3\hat{k})}{3\sqrt{10}}$

As

$\Rightarrow$ $5\hat{P} = \frac{5(\hat{i}-3\hat{k})}{\sqrt{10}}$

$5\hat{P} = 5\hat{i} - 15\hat{k}$ Ans

(ii) let $\vec{P} = 3\vec{a}+\vec{b} = 3(2\hat{i}-3\hat{j}+\hat{k})+(\hat{i}-3\hat{j}+5\hat{k})$

$\vec{P} = 6\hat{i}-9\hat{j}+3\hat{k}+\hat{i}-3\hat{j}+5\hat{k}$

$\vec{P} = 7\hat{i}-12\hat{j}+8\hat{k}$

$|\vec{P}| = \sqrt{(7)^2+(-12)^2+(8)^2}$

$|\vec{P}| = \sqrt{49+144+64}$

$|\vec{P}| = \sqrt{257}$

$\hat{P} = \dfrac{\vec{P}}{|\vec{P}|} = \dfrac{7\hat{i}-12\hat{j}+8\hat{k}}{\sqrt{257}}$

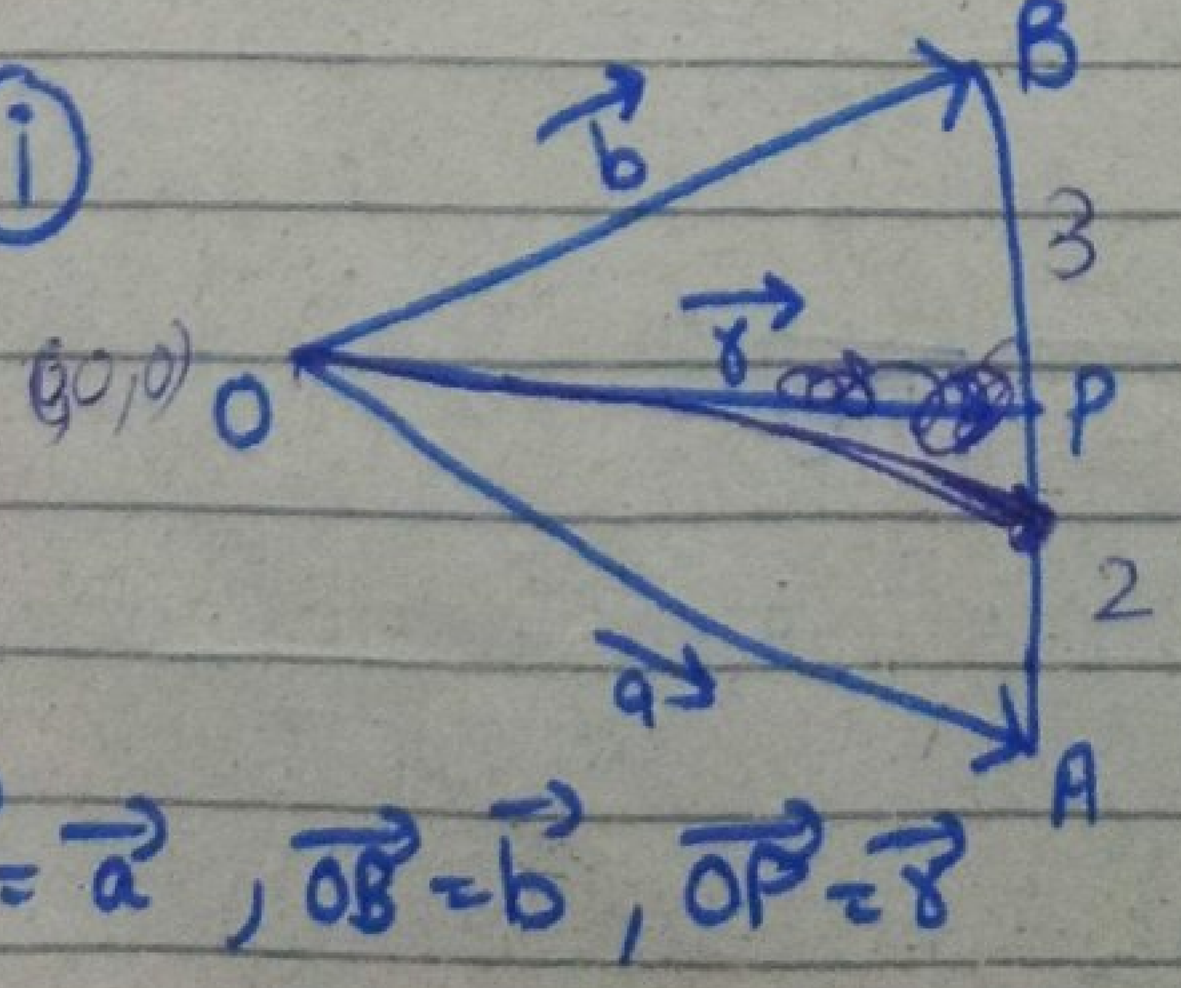
And opposite direction

$= \dfrac{-3}{7}\hat{P}$

$= \dfrac{-3}{7}\left(\dfrac{7\hat{i}-12\hat{j}+8\hat{k}}{\sqrt{257}}\right)$

$= \dfrac{-3\hat{i}}{\sqrt{257}} + \dfrac{36\hat{j}}{7\sqrt{257}} - \dfrac{24}{7\sqrt{257}}\hat{k}$

Q 9) (i)



$\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$, $\vec{OP} = \vec{r}$

let Position vector 'r' dividing the line segment internally

$$\overline{OP} = \vec{r} = \frac{m_2\vec{b} + m_1\vec{a}}{m_2 + m_1}$$

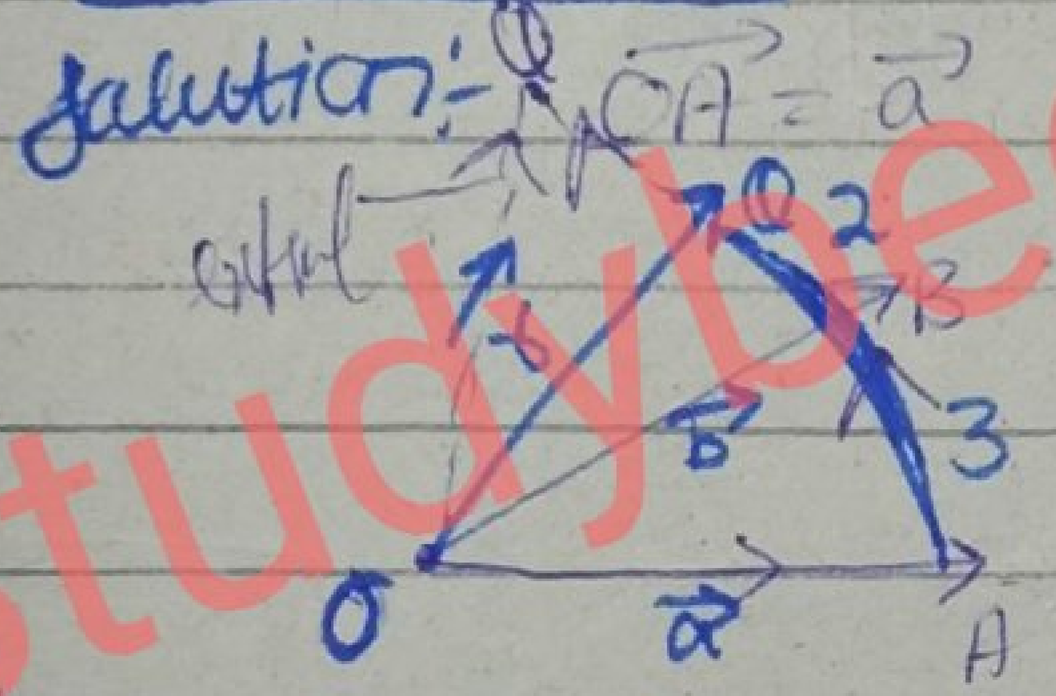
let $m_1 = 2$, $m_2 = 3$

$$\vec{r} = \frac{2(2\hat{i} + 3\hat{j} - \hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})}{2+3}$$

$$\overline{OP} = \vec{r} = \frac{4\hat{i} + 6\hat{j} - 2\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}}{5}$$

$\boxed{\vec{r} = \frac{7\hat{i} + \hat{k}}{5}}$ Answer or $\overline{OP} = \frac{7\hat{i} + \hat{k}}{5}$

(ii) Solution:- $\overline{OA} = \vec{a}$, $\overline{OB} = \vec{b}$, $\overline{OQ} = \vec{r}$



$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n}$$ for externally

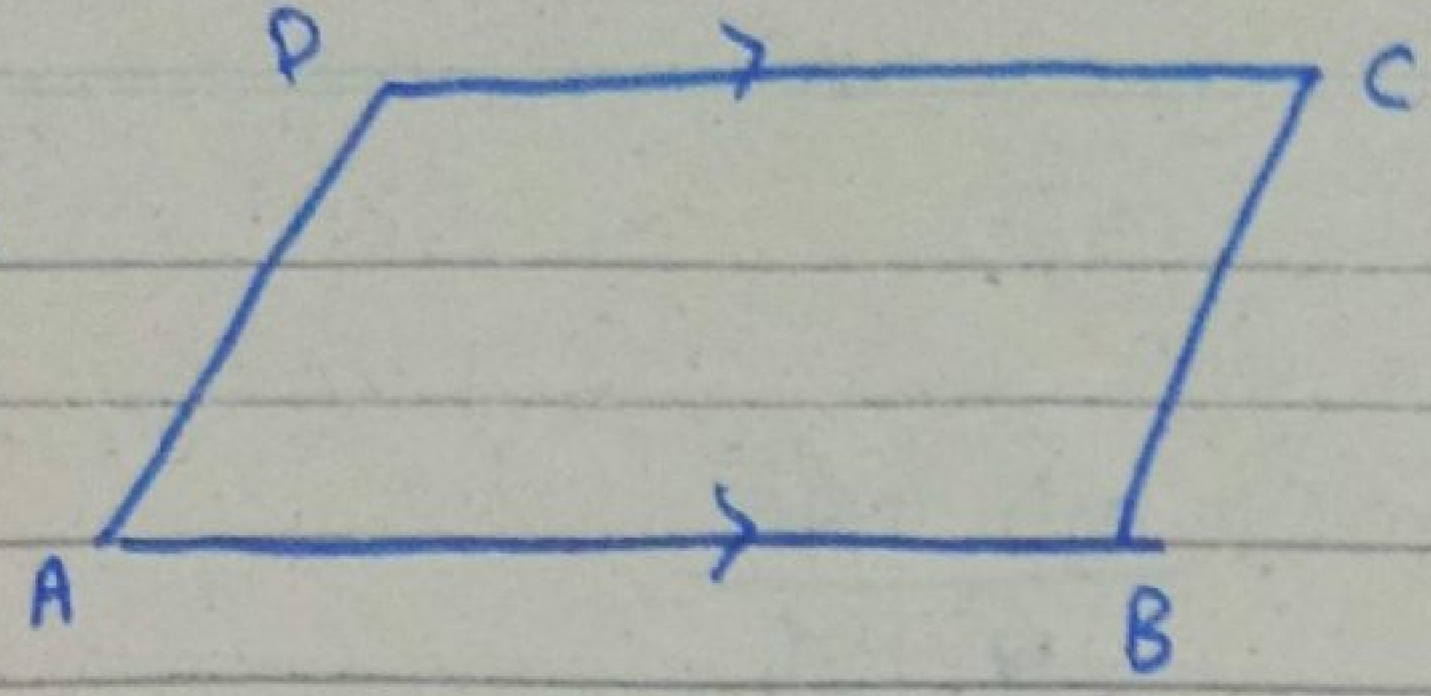
let position vector 'r' dividing the segment $\overline{AB}$ externally.

$$\Rightarrow \vec{r} = \frac{2(2\hat{i} + 3\hat{j} - \hat{k}) - 3(\hat{i} - 2\hat{j} + \hat{k})}{2-3}$$

$$\vec{r} = \frac{\hat{i} + 12\hat{j} - 5\hat{k}}{-1}$$

$\vec{r} = -\hat{i} - 12\hat{j} + 5\hat{k}$ Answer.

1) 10 (i)



Let D(x, y)

$\Rightarrow$ $\vec{AB} \parallel \vec{DC}$ then $\vec{AB} = \vec{DC}$

$\vec{OB} - \vec{OA} = \vec{OC} - \vec{OD}$

$(-\hat{i}-3\hat{j})-(3\hat{i}-4\hat{j}) = (-6\hat{i}+2\hat{j})-(x\hat{i}+y\hat{j})$

$-4\hat{i}+\hat{j} = (-6-x)\hat{i}+(2-y)\hat{j}$

Compare $\hat{i}$ & $\hat{j}$

$-4 = -6 - x$

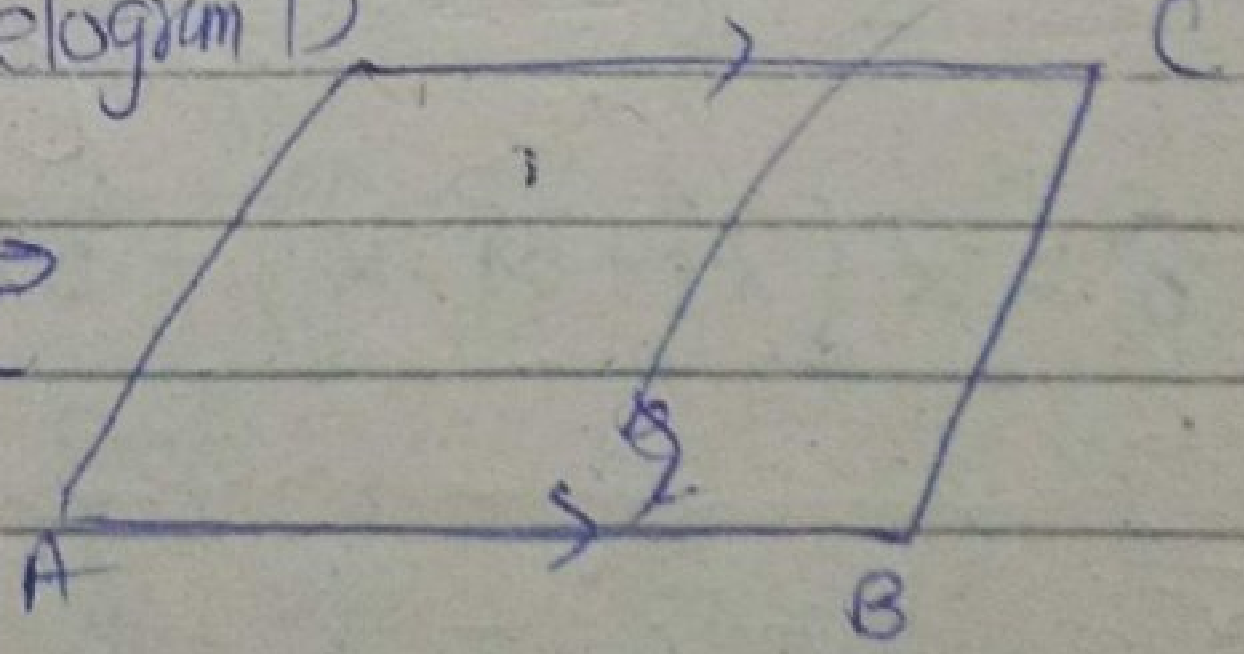
$x = -6 + 4$

$\boxed{x = -2}$ & $\boxed{y = 1}$

Hence D(-2, 1)

Answer

(ii) Sol:- Given parallelogram



$\vec{AB} \parallel \vec{DC}$ & $\vec{AD} \parallel \vec{BC}$

then

$\vec{AB} = \vec{DC}$

$\vec{OB} - \vec{OA} = \vec{OC} - \vec{OD}$

$(4\hat{i}+y\hat{j})-(\hat{i}+2\hat{j}) = (x\hat{i}+6\hat{j})-(3\hat{i}+5\hat{j})$

$3\hat{i}+(y-2)\hat{i} = (x-3)\hat{i}+1\hat{j}$

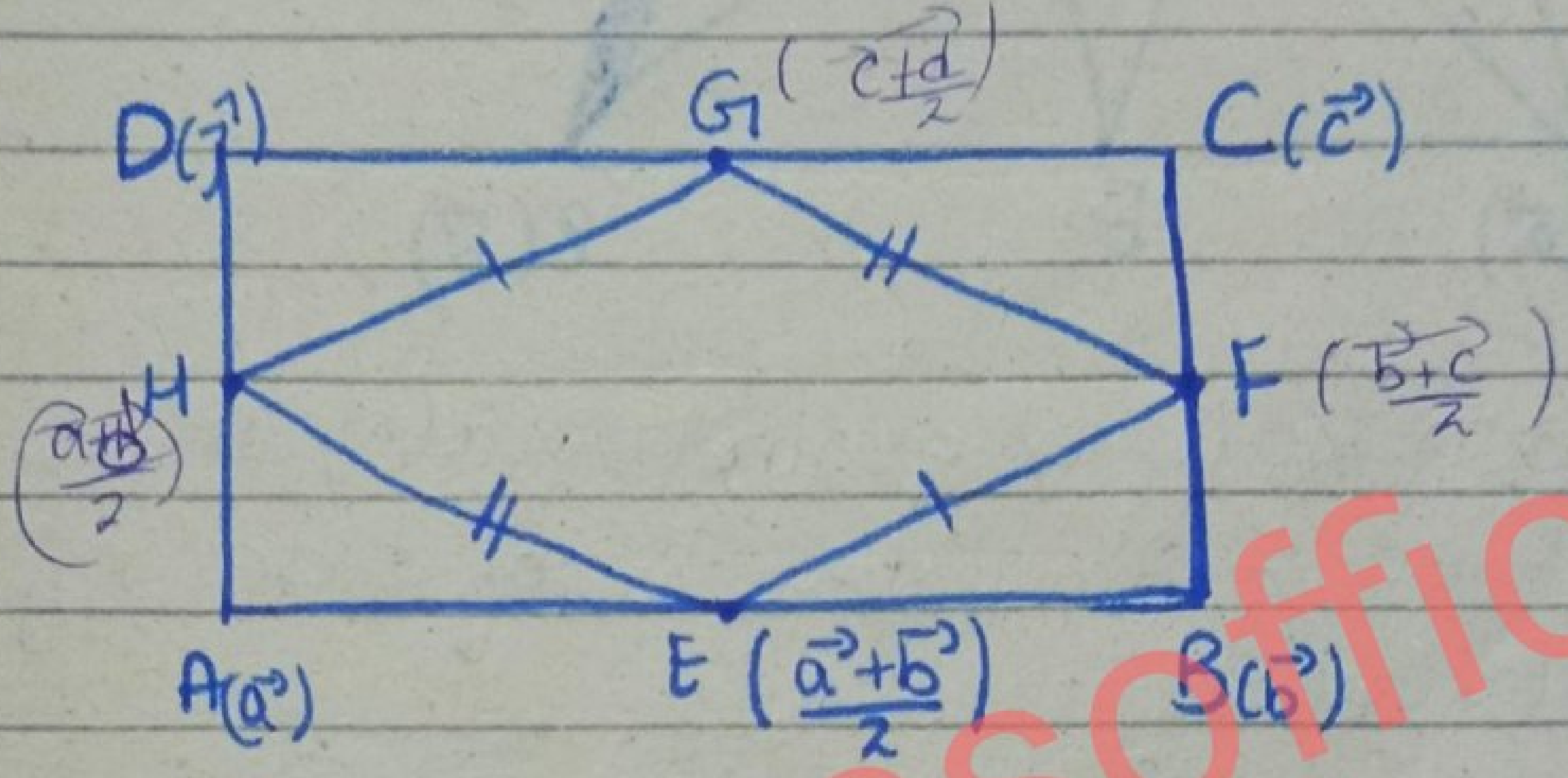
compare $\vec{i}$ & $\vec{j}$

$3 = x - 3$; $y - 2 = 1$

$3 + 3 = x$; $y = 1 + 2$

$\boxed{6 = x}$; $\boxed{y = 3}$ solution

Q11)



Let ABCD be quadrilateral and EFGH be the midpoints of sides of quadrilateral therefore position vectors of E, F, G, H are $\frac{\vec{a}+\vec{b}}{2}$, $\frac{\vec{b}+\vec{c}}{2}$, $\frac{\vec{c}+\vec{d}}{2}$ and $\frac{\vec{a}+\vec{d}}{2}$

then $\vec{EF} = \vec{OF} - \vec{OE} = \frac{\vec{b}+\vec{c}}{2} - \frac{\vec{a}+\vec{b}}{2} = \frac{\vec{b}+\vec{c}-\vec{a}-\vec{b}}{2}$

$= \frac{\vec{c}-\vec{a}}{2}$

now $\vec{HG} = \vec{OG} - \vec{OH} = \frac{\vec{c}+\vec{d}}{2} - \frac{\vec{a}+\vec{d}}{2} = \frac{\vec{c}+\vec{d}-\vec{a}-\vec{d}}{2} = \frac{\vec{c}-\vec{a}}{2}$

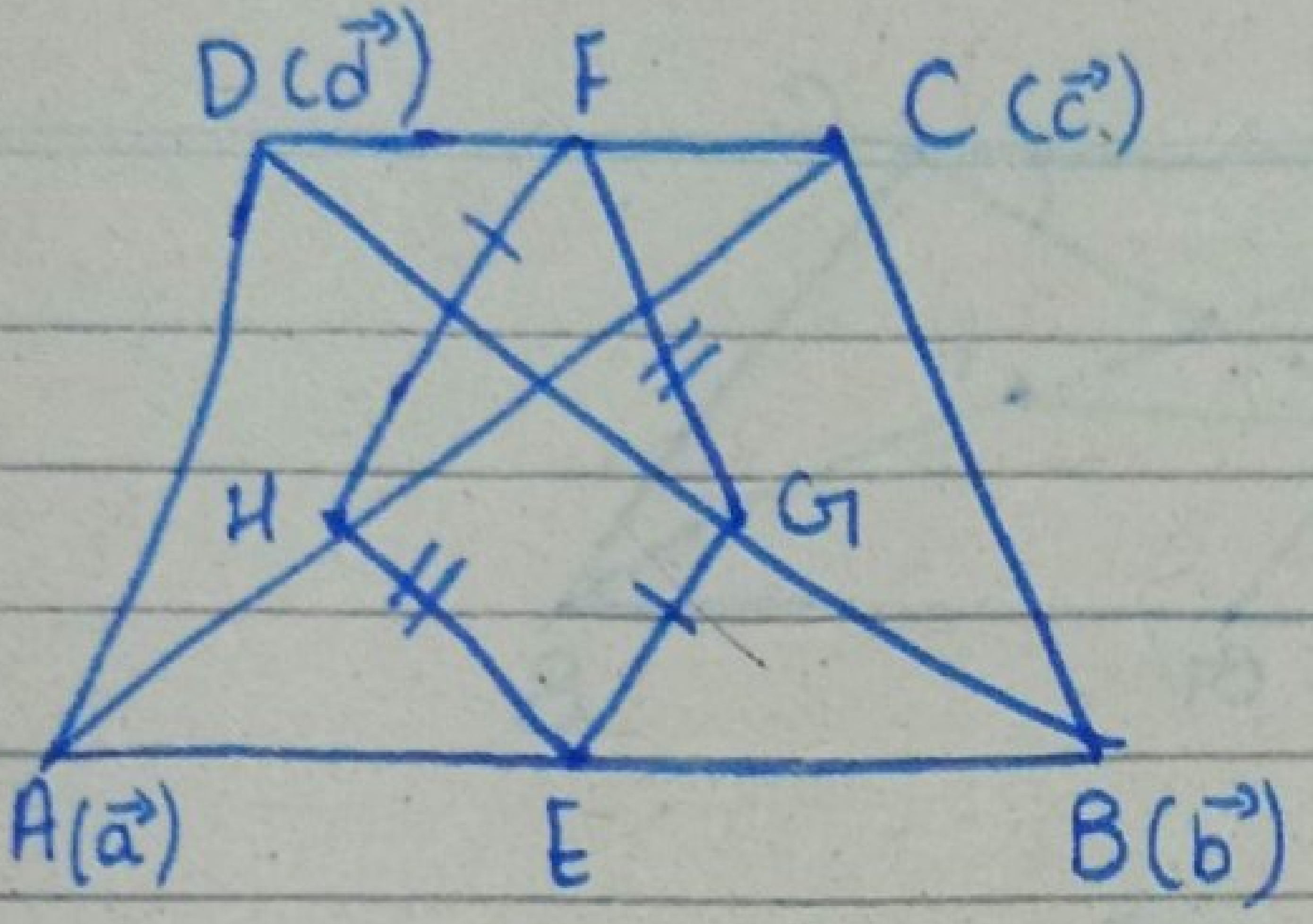
As $\vec{EF} = \vec{HG} \Rightarrow \vec{EF} \parallel HG$

Similarly $\vec{EH} = \vec{FG} \Rightarrow \vec{EH} \parallel \vec{FG}$

Hence EFGH is a parallelogram Proved

also proved other two sides $EH = FG$

Q12.



let ABCD be a quadrilateral and H, G be the midpoints of the diagonal and E, F be the midpoints of $\vec{AB}$ and $\vec{DC}$ sides respt

Position vectors of E, F, G and H are:

$\frac{\vec{a}+\vec{b}}{2}$, $\frac{\vec{c}+\vec{d}}{2}$, $\frac{\vec{b}+\vec{d}}{2}$, $\frac{\vec{a}+\vec{c}}{2}$ respectively.

$\vec{EG} = \vec{OG} - \vec{OE} = \frac{\vec{b}+\vec{d}}{2} - \frac{\vec{a}+\vec{b}}{2} = \frac{\vec{b}+\vec{d}-\vec{a}-\vec{b}}{2} = \frac{\vec{d}-\vec{a}}{2}$

$\vec{HF} = \vec{OF} - \vec{OH} = \frac{\vec{c}+\vec{d}}{2} - \frac{\vec{a}+\vec{c}}{2} = \frac{\vec{c}+\vec{d}-\vec{a}-\vec{c}}{2} = \frac{\vec{d}-\vec{a}}{2}$

$\vec{EH} = \vec{OH} - \vec{OE} = \frac{\vec{a}+\vec{c}}{2} - \frac{\vec{a}+\vec{b}}{2} = \frac{\vec{a}+\vec{c}-\vec{a}-\vec{b}}{2} = \frac{\vec{c}-\vec{b}}{2}$

$\vec{GF} = \vec{OF} - \vec{OG} = \frac{\vec{c}+\vec{d}}{2} - \frac{\vec{b}+\vec{d}}{2} = \frac{\vec{c}+\vec{d}-\vec{b}-\vec{d}}{2} = \frac{\vec{c}-\vec{b}}{2}$

therefore

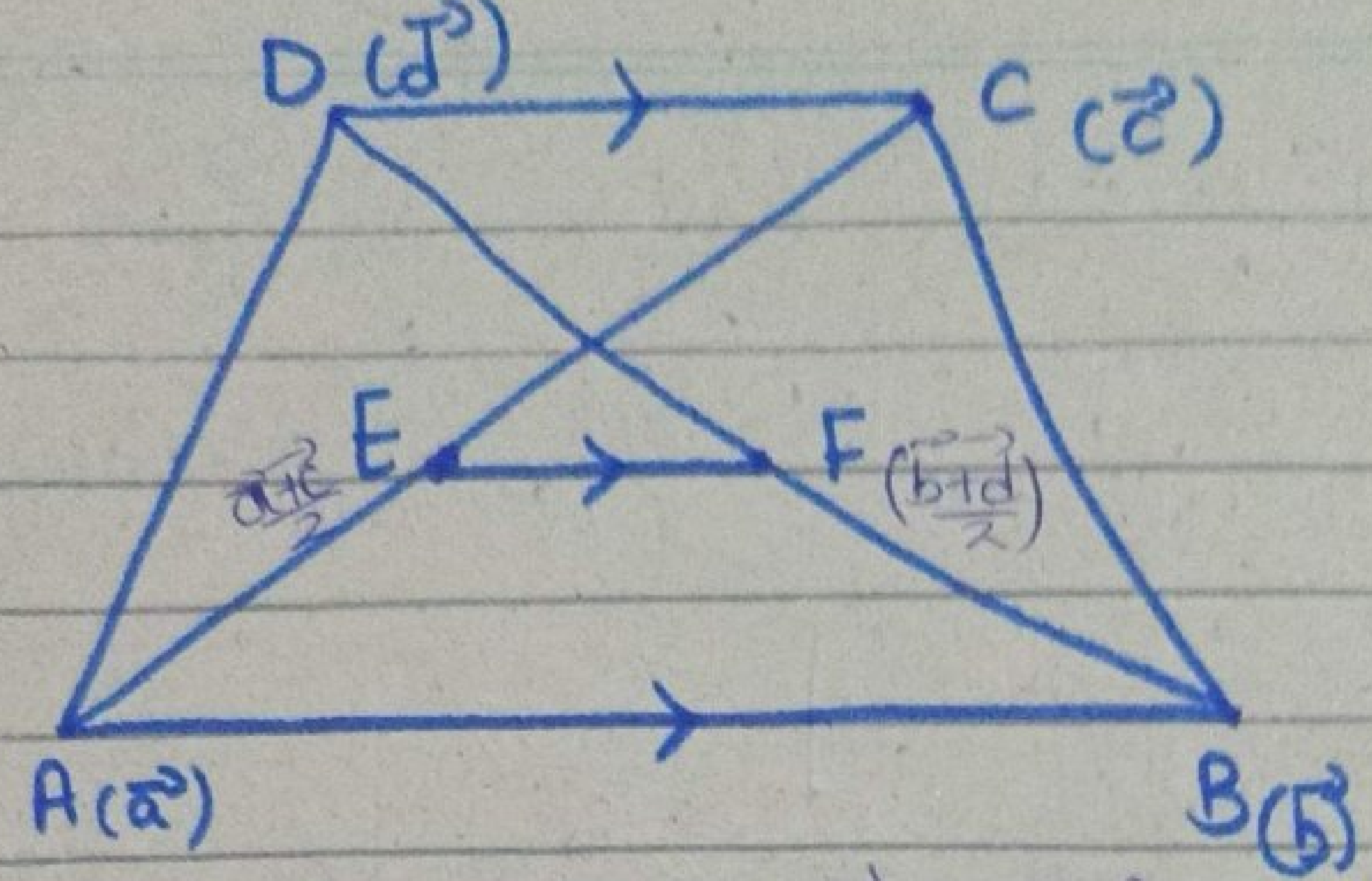
$\vec{EG} = \vec{HF} \Rightarrow \vec{EG} \parallel \vec{HF}$ — (1)

& $\vec{EH} = \vec{GF} \Rightarrow \vec{EH} \parallel \vec{GF}$ — (2)

from (1) and (2) EGFH is parallelogram

Ex 3.1

Q13



(i) $\overline{EF}$ is parallel to $\overrightarrow{AB}$ & $\overrightarrow{DC}$.

(ii) $\overline{EF} = \frac{1}{2}(AB - DC)$

Sol: $\overrightarrow{EF} = OF - OE = \frac{\vec{b}+\vec{d}}{2} - \frac{\vec{a}-\vec{c}}{2}$

$= \frac{1}{2}[\vec{b}+\vec{d}-\vec{a}-\vec{c}]$

$= \frac{1}{2}[(\vec{b}-\vec{a}) - (\vec{c}-\vec{d})]$

$= \frac{1}{2}[\overrightarrow{AB} - \overrightarrow{DC}]$ — (1)

$\because AB = OB - OA = \vec{b} - \vec{a}$

$\because \overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = \vec{c} - \vec{d}$

It is clear that length of EF is half the difference of lengths of parallel sides ($\overrightarrow{AB}$ & $\overrightarrow{DC}$)

As **AB** is parallel to **DC**

$$\overrightarrow{DC} = \lambda \overrightarrow{AB}$$

put in (1)

$\overrightarrow{EF} = \frac{1}{2}[\overrightarrow{AB} - \lambda\overrightarrow{AB}]$

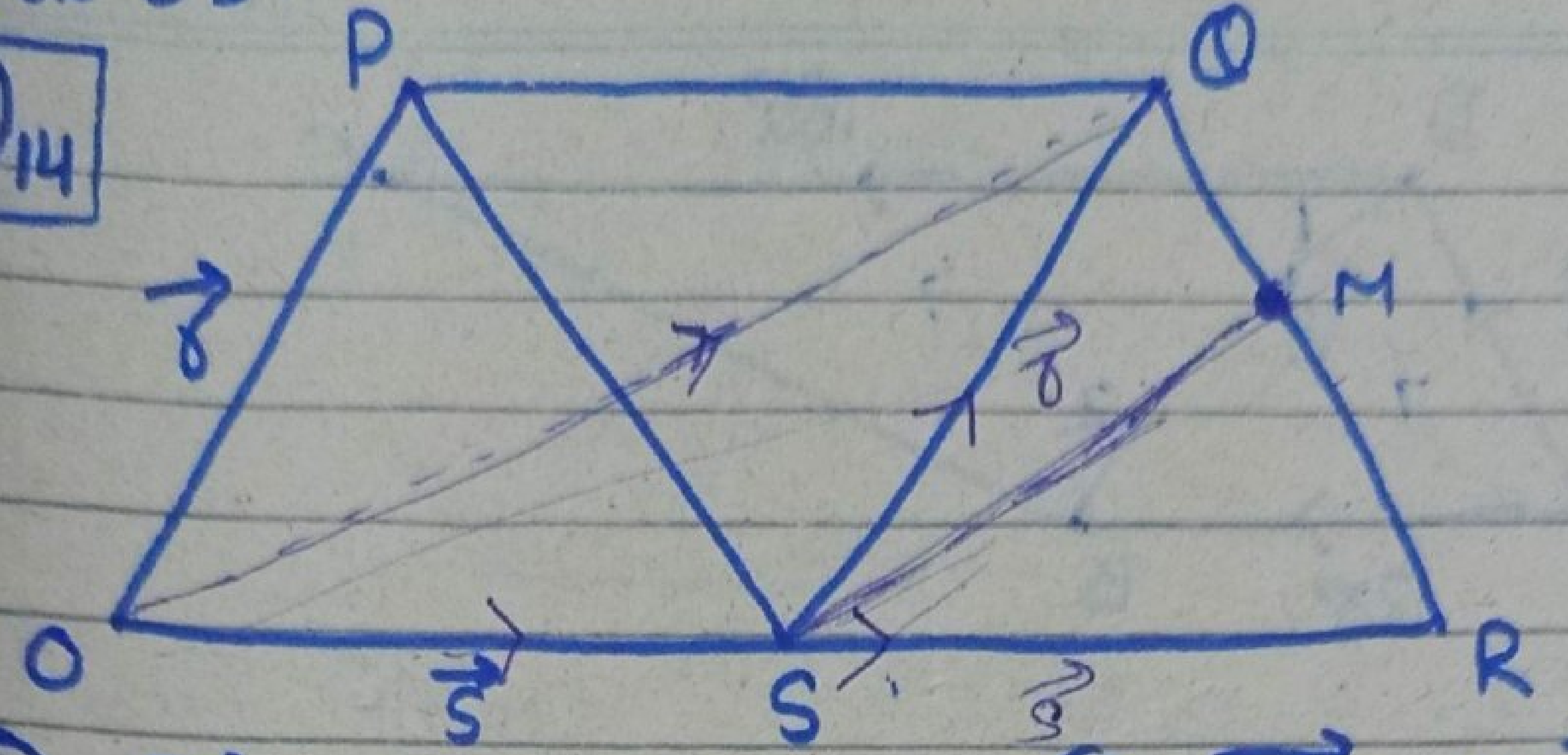
$= \frac{1}{2}(1-\lambda)\overline{AB}$

let $\therefore \lambda_1 = \frac{1}{2}(1-\lambda)$

$\overrightarrow{EF} = \lambda_1 \overline{AB}$

This shows that EF is parallel to $\overrightarrow{AB}$. As $\overrightarrow{AB} \parallel \overrightarrow{DC}$ so EF is also paralel to DC ex 3.1.

Q14



(i) $\vec{PS} = \vec{OS} - \vec{OP}$

$\vec{PS} = \vec{s} - \vec{r}$.

(ii) $\vec{OQ} = \lambda \vec{SM}$?

(ii) $\vec{OQ} = \vec{OS} + \vec{SQ}$ (By head to tail rule)

$\vec{OQ} = \vec{s} + \vec{r}$ $\therefore OP \parallel SQ$

$\vec{SM} = OM - OS$

$= \frac{\vec{s}+\vec{r}+2s}{2} - \vec{s}$

As given that M is the mid point of $\vec{QR}$

$\vec{SM} = \frac{s+r+2s-2s}{2}$

$\vec{QR} = OR + OQ$

$= 2s -$

$\vec{SM} = \frac{\vec{s}+\vec{r}}{2}$

$M = \frac{(\vec{s}+\vec{r})+2s}{2}$ —(1)

$\vec{OR} = \vec{OS} + \vec{SR}$

$= \vec{s} + \vec{s}$

$\vec{OR} = 2\vec{s}$

$\vec{SM} = \frac{1}{2}(\vec{s}+\vec{r})$

$\vec{SM} = \frac{1}{2}(\vec{OQ})$ $\therefore \vec{s}+\vec{r} = OQ$

$2SM = \vec{OQ}$

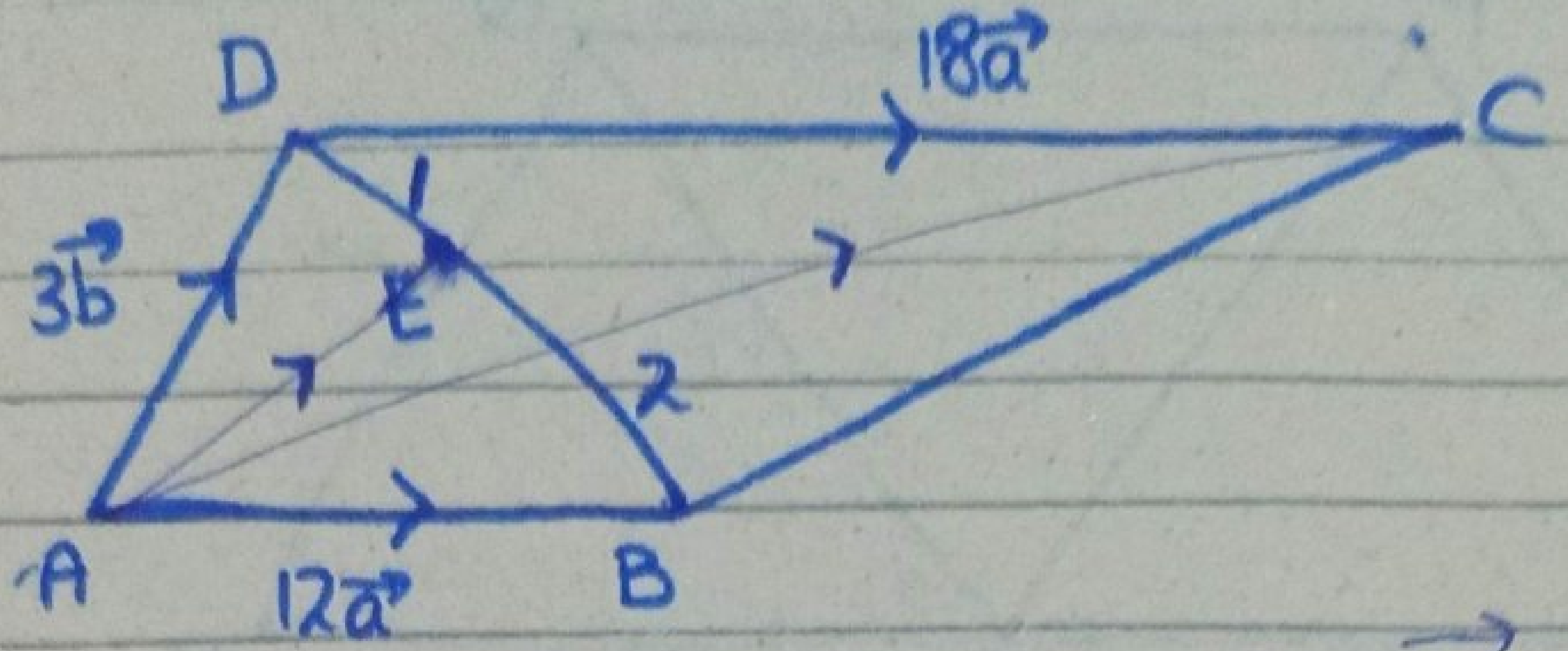
$OQ = 2SM$

let $\lambda = 2$

$OQ = \lambda SM$

This shows that OQ is parallel to SM.

(15)



BC = α AE = proved ? let $\vec{AE} = \vec{r}$

four $n = 1$; $m = 2$

$\vec{AE} = \vec{r} = \dfrac{n\vec{a} + m\vec{b}}{n+m}$

$\vec{AE} = \vec{r} = \dfrac{1(12\vec{a}) + 2(3\vec{b})}{1+2}$

$\vec{AE} = \vec{r} = \dfrac{12a + 6\vec{b}}{3}$

$\vec{AE} = \vec{r} = \dfrac{\cancel{6}^{2}(2a + \vec{b})}{\cancel{3}}$

$\vec{AE} = \vec{r} = 2(2\vec{a} + \vec{b})$ — (1)

now $\vec{AC} = \vec{AB} + \vec{BC}$

$\vec{BC} = \vec{AC} - \vec{AB}$

$\vec{AC} = \vec{AD} + \vec{DC}$

$\vec{AC} = 3\vec{b} + 18\vec{a}$

$\vec{BC} = 18a + 3b - 12a$

$\vec{BC} = 6a + 3b$

$\vec{BC} = 3(2a + b)$

× & ÷ by 2 to make eq (1).

$\vec{BC} = \dfrac{3}{2} \times 2(2a + b)$

$\vec{BC} = \dfrac{3}{2} \vec{AE}$

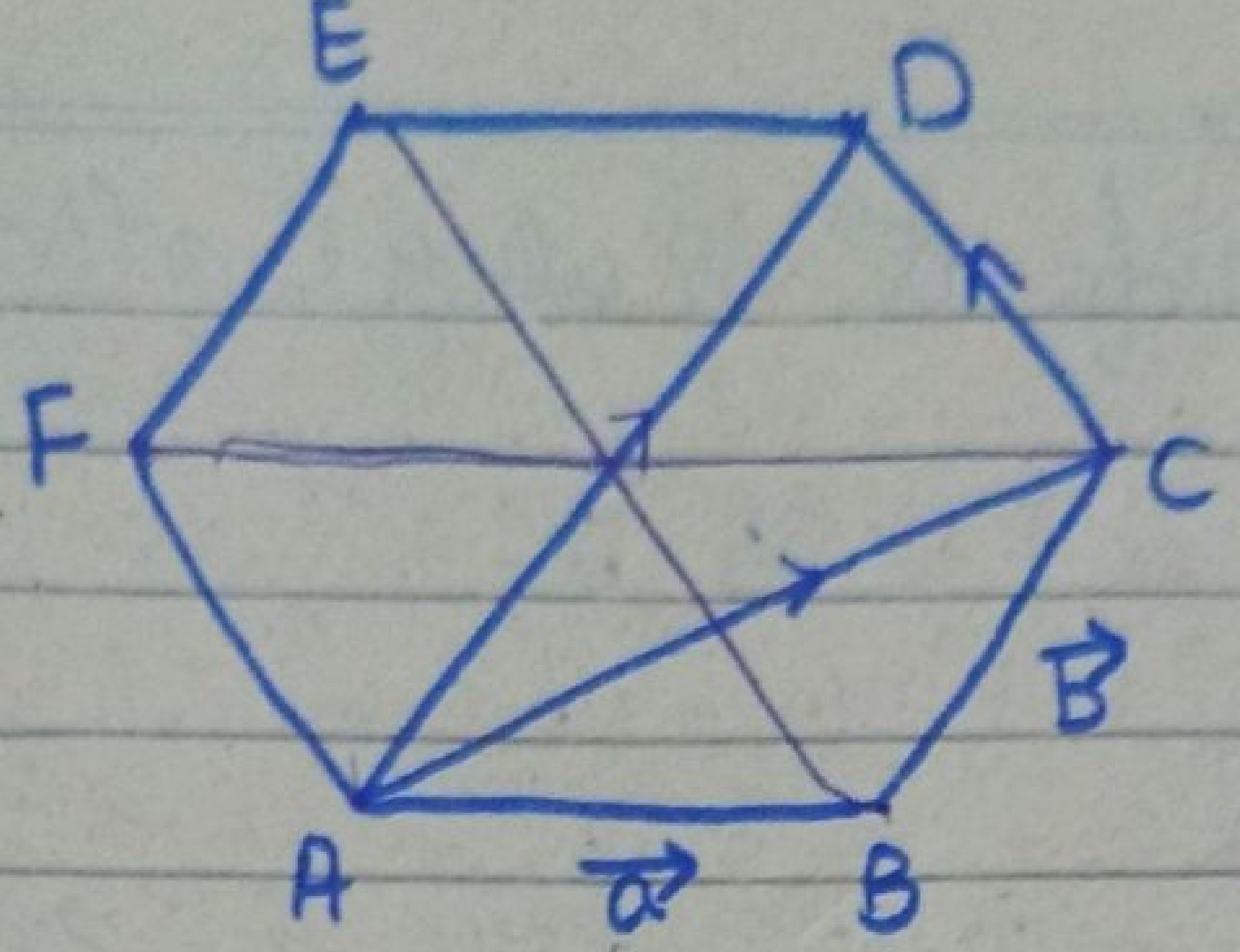
let $\alpha = 3/2$

$\vec{BC} = \alpha \vec{AE}$

Hence BC ∥ AE.

$\therefore$ In regular hexagon diagonal is 2 times sides

16



① $\vec{AC} = ?$

$\vec{AC} = \vec{AB} + \vec{BC}$

$\boxed{\vec{AC} = \vec{a} + \vec{b}}$

② $\vec{CD} = ?$

$\vec{AD} = \vec{AC} + \vec{CD}$

$\vec{CD} = \vec{AD} - \vec{AC}$

$= 2b - (\vec{a} + \vec{b})$

$= 2b - a - b \Rightarrow \boxed{\vec{CD} = \vec{b} - \vec{a}}$

$DA = ?$

③ As AD ~~(diagonal)~~ $= 2 \times BC$

$\vec{AD} = 2\vec{b}$

$\boxed{\vec{DA} = -2\vec{b}}$

④ $EF = ?$

$\vec{FE} = \vec{BC}$ (In hexagon opposite sides equal)

$-\vec{EF} = b$

$\boxed{\vec{EF} = -\vec{b}}$

⑦ $\vec{FC} = 2\vec{AB}$

$\boxed{FC = 2a}$

⑤ $\vec{EB} = ?$

$\vec{BE} = 2\vec{CD}$

$\vec{BE} = 2(\vec{b} - \vec{a})$

$\vec{EB} = -2(\vec{b} - \vec{a})$

$\boxed{\vec{EB} = 2(\vec{a} - \vec{b})}$

⑥ $FA = ?$

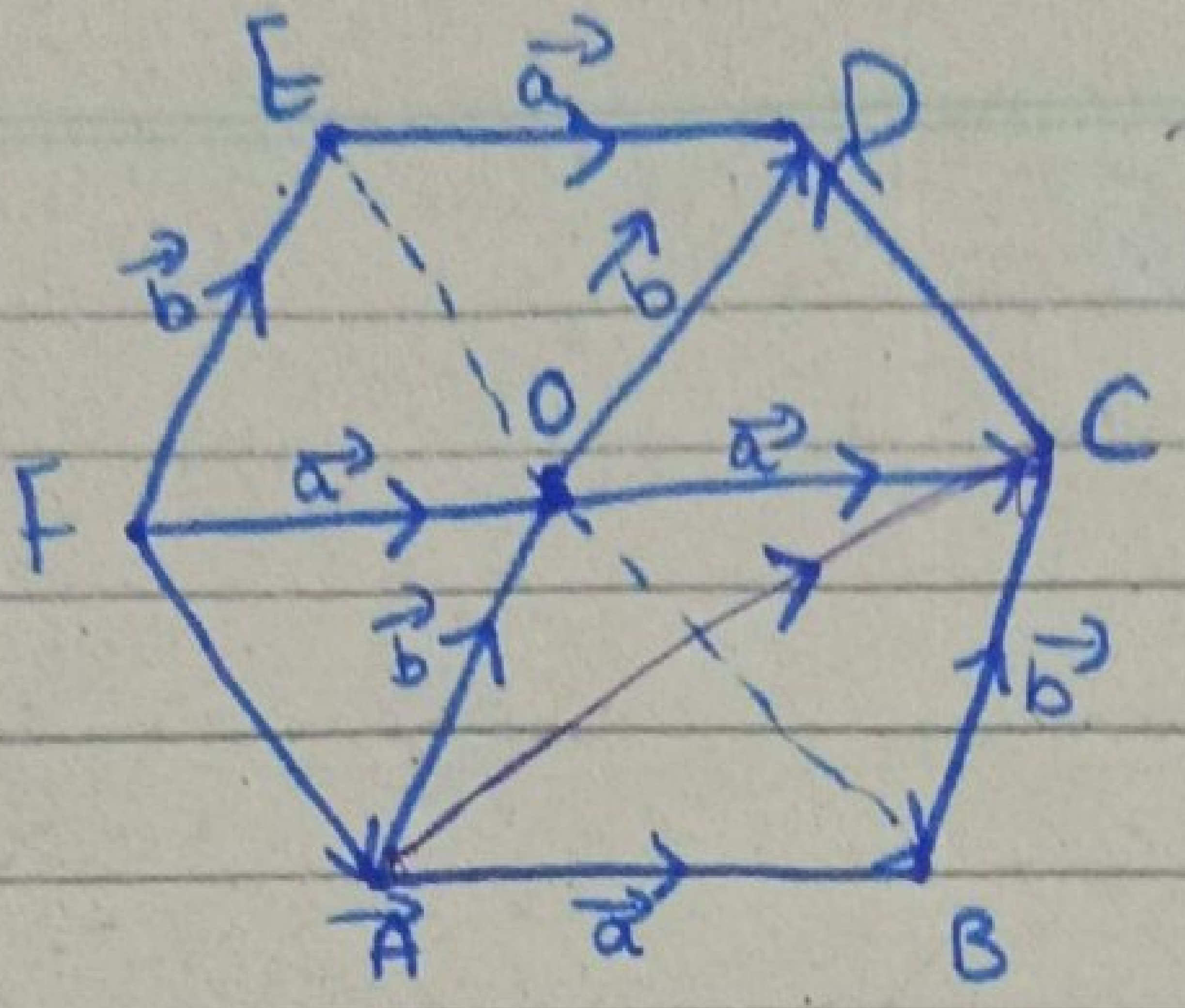
$\vec{AF} = \vec{CD}$

$\vec{AF} = \vec{b} - \vec{a}$

$\boxed{\vec{FA} = \vec{a} - \vec{b}}$

• Diagonal = 2 × sides (In hexagon),

Q16



figure

Sol Given regular hexagon so all vectors have same length

$\vec{AC} = \vec{AB} + \vec{BC}$ (By head to tail rule)

$$\boxed{\vec{AC} = \vec{a} + \vec{b}}$$

Consider OABC, OBCD, OCDE, ODEF, OEFA and OFAB are parallelograms

$\Rightarrow \vec{AB} = \vec{OC} = \vec{ED} = \vec{FO} = \vec{a}$

And $\vec{BC} = \vec{OD} = \vec{FE} = \vec{AO} = \vec{b}$

Now find $\vec{CD} = \vec{CO} + \vec{OD}$

$$\boxed{\vec{CD} = -\vec{a} + \vec{b}}$$

and $\boxed{\vec{EF} = -\vec{b}}$

and $\vec{DA} = \vec{DO} + \vec{OA}$

$\vec{DA} = -\vec{b} - \vec{b}$

$$\boxed{\vec{DA} = -2\vec{b}}$$

$\vec{EB} = \vec{EO} + \vec{OB}$

$\vec{EB} = (\vec{ED} + \vec{DO}) + (\vec{OC} + \vec{CB})$

$\vec{EB} = (\vec{a} - \vec{b}) + (\vec{a} - \vec{b})$

$\boxed{\vec{EB} = 2\vec{a} - 2\vec{b}}$

$\vec{FA} = \vec{FO} + \vec{OA}$

$\boxed{\vec{FA} = \vec{a} - \vec{b}}$

$\vec{FC} = \vec{FO} + \vec{OC}$

$\vec{FC} = \vec{a} + \vec{a}$

$\boxed{\vec{FC} = \vec{2a}}$ Answer.