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Where Minds Pollinate

Class 11
Mathematics

Chapter 2
Exercise 2.6
Notes

National Book Foundation

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Exercise: 2.6

Q No. 3: Solve the system of following linear equat
by Gauss elimination method (Echeleon form)

(1) $2x + 3y + 4z = 2$
$2x + y + z = 5$
$3x - 2y + z = -3$

$$A_b \left[\begin{array}{ccc:c} 2 & 3 & 4 & 2 \\ 2 & 1 & 1 & 5 \\ 3 & -2 & 1 & -3 \end{array}\right] \rightarrow \text{Augmented Matrix (Denoted by } A_b)$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 2 & 3 & 4 & 2 \\ 2 & 1 & 1 & 5 \\ 1 & -3 & 0 & -8 \end{array}\right] \quad R_3 - R_2$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & -3 & 0 & -8 \\ 2 & 1 & 1 & 5 \\ 2 & 3 & 4 & 2 \end{array}\right] \quad R_{13}$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & -3 & 0 & -8 \\ 0 & 7 & 1 & 21 \\ 0 & 9 & 4 & 18 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 2R_1 \end{array}$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & -3 & 0 & -8 \\ 0 & 1 & 1/7 & 3 \\ 0 & 9 & 4 & 18 \end{array}\right] \quad \frac{1}{7}R_2$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & -3 & 0 & -8 \\ 0 & 1 & 1/7 & 3 \\ 0 & 0 & 19/7 & -9 \end{array}\right] \quad R_3 - 9R_2$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & -3 & 0 & -8 \\ 0 & 1 & 1/7 & 3 \\ 0 & 0 & 1 & -63/19 \end{array}\right] \quad \frac{7}{19}R_3$$

Rank A = Rank A_b = N: of
3 = 3 = 3

unique sol.

$x - 3y = -8 \rightarrow (1)$

$y + \frac{1}{7} z = 3 \rightarrow (2)$

$z = -63/19 \rightarrow (3)$

put $z = -63/19$ in eq (2)

$y + \frac{1}{7}\left(-63/19\right) = 3$

$y = -\frac{9}{19} = 3$

$y = 3 + \frac{9}{19}$

$y = \frac{66}{19}$

$y = 66/19$ put in (1)

$x - 3\left(\frac{66}{19}\right) = -8$

$x - \frac{198}{19} = -8$

$x = -8 + \frac{198}{19}$

$x = \frac{46}{19}$

$\boxed{x = \frac{46}{19}}$, $\boxed{y = \frac{66}{19}}$, $\boxed{z = -\frac{63}{19}}$

Verify:

$2\left(\frac{46}{19}\right) + 3\left(\frac{66}{19}\right) + 4\left(-\frac{63}{19}\right) = 2$

$\frac{92}{19} + \frac{198}{19} + \left(-\frac{252}{19}\right) = 2$

$2 = 2$

(2) $5x - 2y + z = 2$

$2x + 2y + 6z = 1$

$3x - 4y - 5z = 3$

$$A_b = \left[\begin{array}{ccc:c} 5 & -2 & 1 & 2 \\ 2 & 2 & 6 & 1 \\ 3 & -4 & -5 & 3 \end{array}\right]$$

$$R_{\sim} \left[\begin{array}{ccc:c} 1 & -6 & -11 & 0 \\ 2 & 2 & 6 & 1 \\ 3 & -4 & -5 & 3 \end{array}\right] \quad R_1 - 2R_2$$

$$R_{\sim} \left[\begin{array}{ccc:c} 1 & -6 & -11 & 0 \\ 0 & 14 & 28 & 1 \\ 0 & 14 & 28 & 3 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccccc} 1 & -6 & 11 & : & 0 \\ 0 & 14 & 28 & : & 1 \\ 0 & 0 & 0 & : & 2 \end{array}\right] R_3 - R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccccc} 1 & -6 & 11 & : & 0 \\ 0 & 1 & 2 & : & 1/14 \\ 0 & 0 & 0 & : & 2 \end{array}\right] \frac{1}{14} R_2$$

Rank A ≠ Rank A_b

$2 \neq 3$

No solution

(3) $2x + z = 2$

$2y - z = 3$

$x + 3y = 5$

$$A_b = \left[\begin{array}{ccccc} 2 & 0 & 1 & : & 2 \\ 0 & 2 & -1 & : & 3 \\ 1 & 3 & 0 & : & 5 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccccc} 1 & 3 & 0 & : & 5 \\ 0 & 2 & -1 & : & 3 \\ 2 & 0 & 1 & : & 2 \end{array}\right] R_{13}$$

$$\underset{\sim}{R}\left[\begin{array}{ccccc} 1 & 3 & 0 & : & 5 \\ 0 & 2 & -1 & : & 3 \\ 0 & -6 & 1 & : & -8 \end{array}\right] R_3 - 2R_1$$

$$\underset{\sim}{R}\left[\begin{array}{ccccc} 1 & 3 & 0 & : & 5 \\ 0 & 1 & -1/2 & : & 3/2 \\ 0 & -6 & 1 & : & -8 \end{array}\right] \frac{1}{2} R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccccc} 1 & 3 & 0 & : & 5 \\ 0 & 1 & 1/2 & : & 3/2 \\ 0 & 0 & 2 & : & 1 \end{array}\right] R_3 + 6R_2$$

$$\sim \begin{bmatrix} 1 & 3 & 0 & : & 5 \\ 0 & 1 & -1/2 & : & 3/2 \\ 0 & 0 & 1 & : & -1/2 \end{bmatrix} \quad -\frac{1}{2} R_3$$

unique sol.

$\text{Rank}A = \text{Rank}A_b = $ No. of unknown

$3 = 3 = 3$

$x + 3y = 5 \rightarrow (1)$

$y - \frac{1}{2} z = \frac{3}{2} \rightarrow (2)$

$z = -\frac{1}{2}$

put $z = -1/2$ in eq (2)

$y - \frac{1}{2}\left(-\frac{1}{2}\right) = \frac{3}{2}$

$y = \frac{3}{2} - \frac{1}{4}$

$y = 5/4$

put $y = 5/4$ in (1)

$x + 3y = 5$

$x + 3(5/4) = 5$

$x = 5 - \frac{15}{4} = \frac{5}{4}$

$\boxed{x = \frac{5}{4}}$ $\boxed{y = \frac{5}{4}}$ $\boxed{z = -\frac{1}{2}}$

Verify:

$2\left(\frac{5}{4}\right) + \left(-\frac{1}{2}\right) = 2$

$\frac{5}{2} - \frac{1}{2} = 2$

$2 = 2$

(4) $x + 2y + 5z = 4$

$3x - 2y + 2z = 3$

$5x - 8y - 4z = 1$

$$A_b = \begin{bmatrix} 1 & 2 & 5 & : & 4 \\ 3 & -2 & 2 & : & 3 \\ 5 & 8 & -4 & : & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 5 & : & 4 \\ 0 & -8 & -13 & : & -9 \\ 0 & -18 & -29 & : & -19 \end{bmatrix} \begin{matrix} \\ R_2 - 3R_1 \\ R_3 - 5R_1 \end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 2 & 5 & : 4 \\ 0 & 1 & 13/8 & : 9/8 \\ 0 & -18 & -29 & : -29 \end{bmatrix} \quad -\frac{1}{8}R_2$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 2 & 5 & : 4 \\ 0 & 1 & 13/8 & : 9/8 \\ 0 & 0 & 1/4 & : 5/4 \end{bmatrix} \quad R_3+18R_2$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 2 & 5 & : 4 \\ 0 & 1 & 13/8 & : 9/8 \\ 0 & 0 & 1 & : 5 \end{bmatrix} \quad 4R_3$$

Rank A = Rank A_b = No. of unknowns

3 = 3 = 3

$x+2y+5z=4 \rightarrow (1)$

$y+\frac{13}{8}z=\frac{9}{8} \rightarrow (2)$

$z=5$

put $z=5$ in eq (2)

$y+\frac{13}{8}(5)=\frac{9}{8}$

$y=\frac{9}{8}-\frac{13}{8}(5)$

$y=-7$

put value of x, y in (1)

$x+2(-7)+5(5)=4$

$x=4-11$

$x=-7$

$\boxed{x=-7}$ $\boxed{y=-7}$ $\boxed{z=5}$

Verify:

$-7+2(-7)+5(5)=4$

$-7-14+25=4$

$4=4$

System of Linear Equations

Homogeneous **Non-Homogeneous**

Homogeneous:

$$a_1x + b_1y + c_1z = k_1$$
$$a_2x + b_2y + c_2z = k_2$$
$$a_3x + b_3y + b_3z = k_3$$

If $k_1 = k_2 = k_3 = 0$

Then it represents system of homogeneous linear equations

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

This system has two solutions:

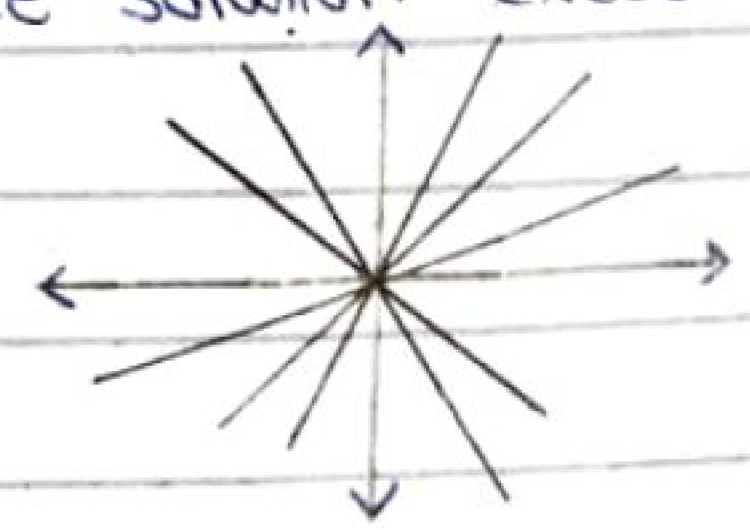
(1) Trivial Solution

(2) Non Trivial Solution

*** Trivial Solution:** If $|A| \neq 0$ then,

Only unique solution excess.

i-e: (0, 0, 0)
x y z



→ The lines passing through origion.

***Non-Trivial Solution:** If $|A| = 0$ then,

→ It's inverse doesn't exisists.

→ It has infinite many solutions.

→ Take x/y or z in terms of t and $t \in \mathbb{R}$

$\downarrow$ Real Number

Q No. 1: Solve the following system of homogeneous linear equations for non-trivial solution if exists.

(1) $2x_1 - 3x_2 + 4x_3 = 0$

$x_1 - 2x_2 + 3x_3 = 0$

$4x_1 + x_2 - 6x_3 = 0$

$$\underset{A}{\begin{bmatrix} 2 & -3 & 4 \\ 1 & -2 & 3 \\ 4 & 1 & -6 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

Expanding by R_1:

$$|A| = 2\begin{vmatrix} -2 & 3 \\ 1 & -6 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 4 & -6 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix}$$

$= 2(9) + 3(-18) + 4(9)$

$= 0$

It is a non-trivial solution So,
It has infinite many solutions exists.

" NOT A PART OF SOLUTION "

$$A_b = \begin{bmatrix} 2 & -3 & 4 & : 0 \\ 1 & -2 & 3 & : 0 \\ 4 & 1 & -6 & : 0 \end{bmatrix}$$

$$R \sim \begin{bmatrix} 1 & -2 & 3 & : 0 \\ 2 & -3 & 4 & : 0 \\ 4 & 1 & -6 & : 0 \end{bmatrix} R_{21}$$

$$R \sim \begin{bmatrix} 1 & -2 & 3 & : 0 \\ 0 & 1 & 2 & : 0 \\ 0 & 9 & -18 & : 0 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - 4R_1 \end{matrix}$$

$$\overset{R}{\sim}\left[\begin{array}{ccc:c} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_1+2R_2 \\ R_3-9R_2 \end{array}$$

Reduced Echelon Form

$1x_1+0x_2-1x_3=0 \longrightarrow (1)$

$0x_1+1x_2-2x_3=0 \longrightarrow (2)$

$(1) \longrightarrow \quad x_1-x_3=0$ $\qquad$ $(2) \longrightarrow \quad x_2-2x_3=0$

$x_1=x_3$ $\qquad$ $x_2=2x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ 2x_3 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad x_3 \in \mathbb{R}$$

let $x_3=1$

$$1\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

let $x_3=-9$

$$-9\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -9 \\ -18 \\ -9 \end{bmatrix}$$

let $x_3=-1$

$$-1\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$$

(2) $\quad 2x_1-3x_2+4x_3=0$

$x_1+x_2+x_3=0$

$x_1-4x_2+3x_3=0$

$$\underset{A}{\begin{bmatrix} 2 & -3 & 4 \\ 1 & 1 & 1 \\ 1 & -4 & 3 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

Expanding by R_1:

$$= 2\begin{vmatrix}1 & 1\\ -4 & 3\end{vmatrix} + 3\begin{vmatrix}1 & 1\\ 1 & 3\end{vmatrix} + 4\begin{vmatrix}1 & 1\\ 1 & -4\end{vmatrix}$$

$= 2(7)+3(3)+4(-5)$

$= 0$

$|A|=0$ It is a non-trivial solution. So, it has infinite many solutions.

$<$ $>$

$$A_b = \begin{bmatrix}1 & 1 & 1 & : 0\\ 2 & -3 & 4 & : 0\\ 1 & -4 & 3 & : 0\end{bmatrix}$$

$$\overset{R}{\sim}\begin{bmatrix}1 & 1 & 1\\ 2 & -3 & 4\\ 1 & -4 & 3\end{bmatrix} \quad R_{21}$$

$$\overset{R}{\sim}\begin{bmatrix}1 & 1 & 1\\ 0 & -5 & 2\\ 0 & -5 & 2\end{bmatrix} \quad \begin{matrix}R_2-2R_1\\ R_3-R_1\end{matrix}$$

$$\overset{R}{\sim}\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & -2/5\\ 0 & -5 & 2\end{bmatrix} \quad -\frac{1}{5}R_2$$

$$\overset{R}{\sim}\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & -2/5\\ 0 & 0 & 0\end{bmatrix} \quad R_3+5R_2$$

$$\overset{R}{\sim}\begin{bmatrix}1 & 1 & 1 & : 0\\ 0 & 1 & -2/5 & : 0\\ 0 & 0 & 0 & : 0\end{bmatrix}$$ Reduced Echeleon

$x_1 + x_2 + x_3 = 0$ —— (1)

$x_2 - 2/5\, x_3 = 0$ —— (2)

From (2)

$$x_2 = \frac{2}{5}x_3$$

put in (3)

$$x_1 + \frac{2}{5}x_3 + x_3 = 0$$

$$x_1 + \frac{7}{5}x_3 = 0$$

$$x_1 = -\frac{7}{5}x_3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7/5 x^3 \\ 2/5\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -7/5 \\ 2/5 \\ 1 \end{bmatrix}$$

let $x_3 = 1$

$$1 \begin{bmatrix} -7/5 \\ 2/5 \\ 1 \end{bmatrix} = \begin{bmatrix} -7/5 \\ 2/5 \\ 1 \end{bmatrix}$$

let $x_3 = -2$

$$-2 \begin{bmatrix} -7/5 \\ 2/5 \\ 1 \end{bmatrix} = \begin{bmatrix} 14/5 \\ -4/5 \\ -2 \end{bmatrix}$$

let $x_3 : 3$

$$3 \begin{bmatrix} -7/5 \\ 2/5 \\ 1 \end{bmatrix} : \begin{bmatrix} -21/5 \\ 6/5 \\ 3 \end{bmatrix}$$

(3) $x_1 + x_2 - 3x_3 = 0$

$3x_1 - 2x_2 + x_3 = 0$

$4x_1 - x_2 - 2x_3 = 0$

$$\underset{A}{\begin{bmatrix} 1 & 1 & -3 \\ 3 & -2 & 1 \\ 4 & -1 & -2 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$= 1 \begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix} - 1 \begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix} - 3 \begin{vmatrix} 3 & -2 \\ 4 & -1 \end{vmatrix}$$

$= 5 + 10 - 15 = 0$ Non-Trivial, Infinite many sol. exists.

< >

$$A_b : \begin{bmatrix} 1 & 1 & -3 & : 0 \\ 3 & -2 & 1 & : 0 \\ 4 & -1 & -2 & : 0 \end{bmatrix}$$

[illegible]

Reduced Echelon

$x_1 + x_2 - 3x_3 = 0$ — (1)

$x_2 - 2x_3 = 0$ — (2)

From (2) put $x_2 = 2x_3$ in (1)

$x_2 = 2x_3$ $x_1 + 2x_3 - 3x_3 = 0$

$x_1 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ 2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

let $x_3 = 5$

$$5\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix}$$

let $x_3 = -4$

$$-4\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \\ -4 \end{bmatrix}$$

(4) $5x_1 + 6x_2 - 7x_3 = 0$

$2x_1 - x_2 + x_3 = 0$

$x_1 + 2x_2 + 2x_3 = 0$

$$\underset{A}{\begin{bmatrix} 5 & 6 & -7 \\ 2 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$: 5\begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} - 6\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 7\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$: 5(-4) - 6(3) - 7(5)$

$= -20 - 18 - 35$

$= -73 \neq 0$ Non-Trivial Sol. doesn't exists.

Q No.2: Find the value of λ for which following system of homogenieous linear equations may have non-trivial solution. Also solve the system for value of λ.

(1) $2x_1 - \lambda x_2 + x_3 = 0$

$2x_1 + 3x_2 - x_3 = 0$

$3x_1 - 2x_2 + 4x_3 = 0$

$$\underset{A}{\begin{bmatrix} 2 & -\lambda & 1 \\ 2 & 3 & -1 \\ 3 & -2 & 4 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

As, given non-trivial sol. : $|A| = 0$

$$\begin{vmatrix} 2 & -\lambda & 1 \\ 2 & 3 & -1 \\ 3 & 2 & 4 \end{vmatrix} = 0 \quad = 2\begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix} + \lambda\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$$

$= 2(10) + \lambda(11) + 1(-13) = 0$

$20 + 11\lambda - 13 = 0$

$11\lambda = -7$

$$\boxed{\lambda = -7/11}$$

Put value of λ in A:

$$\begin{vmatrix} 2 & -7/11 & 1 \\ 2 & 3 & -1 \\ 3 & 2 & 4 \end{vmatrix} = \quad A_b = \begin{bmatrix} 2 & -7/11 & 1 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 2 & 4 & : 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 7/11 & 2 \\ -1 & 3 & 2 \\ 4 & 2 & 3 \end{bmatrix} C_{13}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 7/11 & 2 \\ 0 & 40/11 & 4 \\ 0 & -50/11 & -5 \end{bmatrix} \begin{matrix} R_2 + R_1 \\ R_3 \; \cancel{4}R_1 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 7/11 & 2 \\ 0 & 1 & 11/10 \\ 0 & -50/11 & -5 \end{bmatrix} \frac{11}{40} R_2$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 7/11 & 2 \\ 0 & 1 & 11/10 \\ 0 & 0 & 0 \end{bmatrix} R_3 + \frac{50}{11} R_2$$

$$x_1 + \frac{7}{11} x_2 + 2x_3 = 0 \quad \text{---} (1)$$

$$x_2 + \frac{11}{10} x_3 = 0 \quad \text{---} (2)$$

From (2)

$$x_2 = -\frac{11}{10} x_3$$

Put in (1)

$$x_1 - \frac{7}{10} x_3 + 2x_3 = 0$$

$$x_1 = \frac{13}{10} x_3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 13/10\, x_3 \\ -11/10\, x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 13/10 \\ -11/10 \\ 1 \end{bmatrix}$$

let $x_3 = 1$

$$1 \begin{bmatrix} 13/10 \\ -11/10 \\ 1 \end{bmatrix} = \begin{bmatrix} 13/10 \\ -11/10 \\ 1 \end{bmatrix}$$

let $x_3 = 2$

$$2 \begin{bmatrix} 13/10 \\ [illegible] \\ 1 \end{bmatrix} = \begin{bmatrix} [illegible] \\ [illegible] \\ 2 \end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 7/11 & 2\\ -1 & 3 & 2\\ 4 & 2 & 3\end{bmatrix} \quad C_{13}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 7/11 & 2\\ 0 & 40/11 & 4\\ 0 & -50/11 & -5\end{bmatrix} \quad \begin{matrix}R_2+R_1\\ R_3-4R_1\end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 7/11 & 2\\ 0 & 1 & 11/10\\ 0 & -50/11 & -5\end{bmatrix} \quad \frac{11}{40}R_2$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 7/11 & 2\\ 0 & 1 & 11/10\\ 0 & 0 & 0\end{bmatrix} \quad R_3+\frac{50}{11}R_2$$

$$x_1+\frac{7}{11}x_2+2x_3=0 \text{ —— (1)}$$

$$x_2+\frac{11}{10}x_3=0 \text{ —— (2)}$$

From (2)

$$x_2 = -\frac{11}{10}x_3$$

put in (1)

$$x_1-\frac{7}{10}x_3+2x_3=0$$

$$x_1 = 13/10\,x_3$$

$$\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}13/10\,x_3\\ -11/10\,x_3\\ x_3\end{bmatrix} = x_3\begin{bmatrix}13/10\\ -11/10\\ 1\end{bmatrix}$$

let $x_3=1$

$$1\begin{bmatrix}13/10\\ -11/10\\ 1\end{bmatrix} = \begin{bmatrix}13/10\\ -11/10\\ 1\end{bmatrix}$$

let $x_3=2$

$$2\begin{bmatrix}13/10\\ -11/10\\ 1\end{bmatrix} =$$

(2) $x_1 - 4x_2 + 3x_3 = 0$

$2x_1 + \lambda x_2 + x_3 = 0$

$x_1 - 2x_2 + \lambda x_3 = 0$

$$\underbrace{\begin{bmatrix} 1 & -4 & 3 \\ 2 & \lambda & 1 \\ 1 & -2 & \lambda \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{B}$$

As given, non-trivial sol. :

$$= 1\begin{vmatrix} \lambda & 1 \\ -2 & \lambda \end{vmatrix} + 4\begin{vmatrix} 2 & 1 \\ 1 & \lambda \end{vmatrix} + 3\begin{vmatrix} 2 & \lambda \\ 1 & -2 \end{vmatrix}$$

$= \lambda^2 - 2) + 4(2\lambda - 1) + 3(-4-\lambda) = 0$

$\lambda^2 - 2 + 8\lambda - 4 - 12 - 3\lambda = 0$

$\lambda^2 + 5\lambda - 14 = 0$

$\lambda^2 + 7\lambda - 2\lambda - 14 = 0$

$\lambda(\lambda+7) - 2(\lambda+7) = 0$

$(\lambda+7)(\lambda-2) = 0$

$\lambda = -7 \qquad \lambda = 2$

Put value of λ in A :

$$\begin{bmatrix} 1 & -4 & 3 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -4 & 3 \\ 0 & 10 & -5 \\ 0 & 2 & -1 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -1/2 \\ 0 & 2 & -1 \end{bmatrix} \quad \frac{1}{10}R_2$$

$$\sim \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 - 2R_2$$

When $\lambda = -7$

$$\begin{bmatrix} 1 & -4 & 3 \\ 2 & -7 & 1 \\ 1 & -2 & -7 \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -5 \\ 0 & 2 & -10 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 - 2R_2$$

(2) $x_1 - 4x_2 + 3x_3 = 0$

$2x_1 + \lambda x_2 + x_3 = 0$

$x_1 - 2x_2 + \lambda x_3 = 0$

$$\underset{A}{\begin{bmatrix} 1 & -4 & 3 \\ 2 & \lambda & 1 \\ 1 & -2 & \lambda \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

As given, non-trivial sol. :

$$= 1 \begin{vmatrix} \lambda & 1 \\ -2 & \lambda \end{vmatrix} + 4 \begin{vmatrix} 2 & 1 \\ 1 & \lambda \end{vmatrix} + 3 \begin{vmatrix} 2 & \lambda \\ 1 & -2 \end{vmatrix}$$

$(\lambda^2+2) + 4(2\lambda-1) + 3(-4-\lambda) = 0$

$\lambda^2 + 2 + 8\lambda - 4 - 12 - 3\lambda = 0$

$\lambda^2 + 5\lambda - 14 = 0$

$\lambda^2 + 7\lambda - 2\lambda - 14 = 0$

$\lambda(\lambda+7) - 2(\lambda+7) = 0$

$(\lambda+7)(\lambda-2) = 0$

$\lambda = -7 \qquad \lambda = 2$

Put value of λ in A:

$$\begin{bmatrix} 1 & -4 & 3 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -4 & 3 \\ 0 & 10 & -5 \\ 0 & 2 & -1 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -1/2 \\ 0 & 2 & -1 \end{bmatrix} \quad \frac{1}{10} R_2$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 - 2R_2$$

When $\lambda = 7$

$$\begin{bmatrix} 1 & -4 & 3 \\ 2 & -7 & 1 \\ 1 & -2 & -7 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -5 \\ 0 & 2 & -10 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 - 2R_2$$

$x_1 + 4x_2 + 3x_3 = 0$ — (1)

$x_2 - \frac{1}{2}x_3 = 0$ — (2)

$x_2 = \frac{1}{2}x_3$

From (2)

$x_1 + 2x_3 + 3x_3 = 0$

$x_1 = -5x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5x_3 \\ 1/2\, x_3 \\ x_3 \end{bmatrix}$$

$$'' = x_3 \begin{bmatrix} -5 \\ 1/2 \\ 1 \end{bmatrix}$$

$x_1 - 4x_2 + 3x_3 = 0$ — (1)

$x_2 - 5x_3 = 0$ — (2)

From(2)

$x_2 = 5x_3$

$x_1 - 20x_3 + 3x_3 = 0$

$x_1 = 17x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17x_3 \\ 5x_3 \\ x_3 \end{bmatrix}$$

$$'' = x_3 \begin{bmatrix} 17 \\ 5 \\ 1 \end{bmatrix}$$

Q No. 5: Solve the following system of linear equation by using cramer's Rule.

(1) $x_1 + x_2 + 2x_3 = 8$

$-x_1 - 2x_2 + 3x_3 = 1$

$3x_1 - 7x_2 + 4x_3 = 10$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 2 \\ -1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 8 \\ 1 \\ 10 \end{bmatrix}}$$

$$|A| = 1 \begin{vmatrix} -2 & 3 \\ -7 & 4 \end{vmatrix} - 1 \begin{vmatrix} -1 & 3 \\ 3 & 4 \end{vmatrix} + 2 \begin{vmatrix} -1 & -2 \\ 3 & -7 \end{vmatrix}$$

$= 1(+13) - 1(-13) + 2(13)$

$= 13 + 13 + 26$

$= 52 \neq 0$

$$|Ax_1| = \begin{vmatrix} 8 & 1 & 2 \\ 1 & -2 & 3 \\ 10 & -7 & 4 \end{vmatrix} = 8\begin{vmatrix} -2 & 3 \\ -7 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 10 & 4 \end{vmatrix} + 2\begin{vmatrix} 1 & -2 \\ 10 & -7 \end{vmatrix}$$

$$= 8(13) - 1(-26) + 2(13)$$

$$= 104 + 26 + 26 = 156$$

$$x_1 = \frac{|Ax_1|}{|A|} = \frac{156}{52} = 3$$

$$x_1 = 3$$

$$|\Delta x_2| = \begin{vmatrix} 1 & 8 & 2 \\ -1 & +1 & 3 \\ 3 & 10 & 4 \end{vmatrix} = 1\begin{vmatrix} 1 & 3 \\ 10 & 4 \end{vmatrix} - 8\begin{vmatrix} -1 & 3 \\ 3 & 4 \end{vmatrix} + 2\begin{vmatrix} -1 & 1 \\ 3 & 10 \end{vmatrix}$$

$$= 1(-26) - 8(-13) + 2(-13)$$

$$= -26 + 104 - 26 = 52$$

$$x_2 = \frac{|Ax_2|}{|A|} = \frac{52}{52} = 1$$

$$x_2 = 1$$

$$|Ax_3| = \begin{vmatrix} 1 & 1 & 8 \\ -1 & -2 & 1 \\ 3 & -7 & 10 \end{vmatrix} = 1\begin{vmatrix} -2 & 1 \\ -7 & 10 \end{vmatrix} - 1\begin{vmatrix} -1 & 1 \\ 3 & 10 \end{vmatrix} + 8\begin{vmatrix} -1 & -2 \\ 3 & -7 \end{vmatrix}$$

$$= -13 + 13 + 104 = 104$$

$$x_3 = \frac{|Ax_3|}{|A|} = \frac{104}{52} = 2$$

$$x_3 = 2$$

$x_1 = 3 \qquad x_2 = 1 \qquad x_3 = 2$

(2) $2x_1 + 2x_2 + x_3 = 0$

$-2x_1 + 5x_2 + 2x_3 = 1$

$8x_1 + x_2 + 4x_3 = -1$

$$\underbrace{\begin{bmatrix} 2 & 2 & 1 \\ -2 & 5 & 2 \\ 8 & 1 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}}_{B}$$

$$|A| = 2\begin{vmatrix} 5 & 2 \\ 1 & 4 \end{vmatrix} - 2\begin{vmatrix} -2 & 2 \\ 8 & 4 \end{vmatrix} + 1\begin{vmatrix} -2 & 5 \\ 8 & 1 \end{vmatrix}$$

$= 2(-18) - 2(-8) + 1(-42)$

$= -42$

$$|Ax_1| = \begin{vmatrix} 0 & 2 & 1 \\ 1 & 5 & 2 \\ -1 & 1 & 4 \end{vmatrix} = -2\begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 5 \\ -1 & 1 \end{vmatrix}$$

$= -2(6) + 1(+6) = -6$

$x_1 = \frac{|Ax_1|}{|A|} = -\frac{6}{42} = -\frac{1}{7}$

$$|Ax_2| = \begin{vmatrix} 2 & 0 & 1 \\ -2 & 1 & 2 \\ 8 & -1 & 4 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} + 1\begin{vmatrix} -2 & 1 \\ 8 & -1 \end{vmatrix}$$

$= 2(6) + 1(-6)$

$= 12 - 6 = 6$

$x_2 = \frac{|Ax_2|}{|A|} = \frac{6}{42} = \frac{1}{7}$

$$|Ax_3| = \begin{vmatrix} 2 & 2 & 0 \\ -2 & 5 & 1 \\ 8 & 1 & -1 \end{vmatrix} = 2\begin{vmatrix} 5 & 1 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} -2 & 1 \\ 8 & -1 \end{vmatrix}$$

$= 2(-6) - 2(-6) = 0$ $x_3 = 0$

$x_1 = -\frac{1}{7}$ $x_2 = \frac{1}{7}$ $x_3 = 0$

(3) $-2x_2 + 3x_3 = 1$

$3x_1 + 6x_2 - 3x_3 = -2$

$6x_1 + 6x_2 + 3x_3 = 5$

$$\underset{A}{\begin{bmatrix} 0 & -2 & 3 \\ 3 & 6 & -3 \\ 6 & 6 & 3 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}}$$

$$|A| = 2\begin{vmatrix} 3 & -3 \\ 6 & 3 \end{vmatrix} + 3\begin{vmatrix} 3 & 6 \\ 6 & 6 \end{vmatrix} = 2(27) + 3(-18) = 54 - 54 = 0$$

A is singular matrix so, solution doesn't exists.

(4) $2x_1 + x_2 + 3x_3 = 1$

$x_1 - 2x_2 + x_3 = 2$

$3x_1 - 4x_2 - x_3 = 4$

$$\underset{A}{\begin{bmatrix} 2 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 4 & -1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}}$$

$$|A| = 2\begin{vmatrix} -2 & 1 \\ -4 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} + 3\begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix} = 2(6) - 1(-4) + 3(2$$

$$= 22 \neq 0$$

$$|Ax_1| = \begin{vmatrix} 1 & 1 & 3 \\ 2 & -2 & 1 \\ 4 & -4 & -1 \end{vmatrix} = 1\begin{vmatrix} -2 & 1 \\ -4 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} + 3\begin{vmatrix} 2 & -2 \\ 4 & -4 \end{vmatrix}$$

$$= 1(6) - 1(-6) + 3(0) = 12$$

$$x_1 = \frac{|Ax_1|}{|A|} = \frac{12}{22} = 6/11$$

$$|Ax_2| = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 4 & -1 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$= 2(-6) - 1(-4) + 3(-2) = -14$$

$$x_2 = -14/22 = -7/11$$

$$|A_{x3}| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & 2 \\ 3 & -4 & 4 \end{vmatrix} = 2\begin{vmatrix} -2 & 2 \\ -4 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix}$$

$$= 0 + 2 + 2 = 4$$

$$x_3 = 4/22 = 2/11$$

$$x_1 = 6/11 \qquad x_2 = -7/11 \qquad x_3 = 2/11$$

Q No.6: Solve the following system of linear equation by Matrix Inversion Method.

(i)
$$5x + 3y + z = 6$$
$$2x + y + 3z = 19$$
$$x + 2y + 4z = 25$$

$$\underset{A}{\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 6 \\ 19 \\ 25 \end{bmatrix}}$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{\text{Adj of } A}{|A|}$$

$$|A| = 5\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 5(-2) - 3(5) + 1(3) = -22 \neq 0$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A_{11} = (-1)^2 \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 1(4-6) = 1(-2) = -2$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = -1(8-3) = -(5) = -5$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 1(4-1) = 1(3) = 3$$

$$A_{21} = (-1)^3 \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = -1(12-2) = -1(10) = -10$$

$$A_{22} = (-1)^4 \begin{vmatrix} 5 & 1 \\ 1 & 4 \end{vmatrix} = 1(20-1) = 19$$

$$A_{23} = (-1)^5 \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -1(10-3) = -(7) = -7$$

$$A_{31} = (-1)^4 \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 1(9-1) = 8$$

$$A_{32} = (-1)^5 \begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = -1(15-2) = -(13) \; -13$$

$$A_{33} = (-1)^6 \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} = 1(5-6) = -1$$

$$\operatorname{adj} A = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}, \quad A^{-1} = -\frac{1}{22} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 19 \\ 25 \end{bmatrix}$$

$$= -\frac{1}{22} \begin{bmatrix} -12 - 190 + 200 \\ -30 + 361 - 325 \\ 18 - 133 \;\; -25 \end{bmatrix} = -\frac{1}{22} \begin{bmatrix} -2 \\ 6 \\ -140 \end{bmatrix} = \begin{bmatrix} 1/11 \\ -3/11 \\ 70/11 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/11 \\ -3/11 \\ 70/11 \end{bmatrix}$$

$x = 1/11$

$y = -3/11$

$z = 70/11$

(2) $x+2y-3z=5$

$2x-3y+2z=1$

$-x+2y-5z=-3$

$$\underset{A}{\begin{bmatrix}1 & 2 & -3\\ 2 & -3 & 2\\ -1 & 2 & -5\end{bmatrix}} \underset{X}{\begin{bmatrix}x\\ y\\ z\end{bmatrix}} = \underset{B}{\begin{bmatrix}5\\ 1\\ -3\end{bmatrix}}$$

$X = A^{-1}B$

$$|A| = 1\begin{vmatrix}-3 & 2\\ 2 & 5\end{vmatrix} -2\begin{vmatrix}2 & 2\\ -1 & -5\end{vmatrix} -3\begin{vmatrix}2 & -3\\ -1 & 2\end{vmatrix}$$

$= 1(11)-2(-8)-3(1)$

$= 11+6-3 \quad = 24 \neq 0$

$$A_{11} = (-1)^2\begin{vmatrix}-3 & 2\\ 2 & -5\end{vmatrix} = (-1)^2(15-4) = 11$$

$$A_{12} = (-1)^3\begin{vmatrix}2 & 2\\ -1 & -5\end{vmatrix} = (-1)(-10+2) = -(8) = 8$$

$$A_{13} = (-1)^4\begin{vmatrix}2 & -3\\ -1 & 2\end{vmatrix} = 1(4-3) = 1$$

$$A_{21} = (-1)^3\begin{vmatrix}2 & -3\\ 2 & -5\end{vmatrix} = -1(-10+6) = -(-4) = 4$$

$$A_{22} = (-1)^4\begin{vmatrix}1 & -3\\ -1 & -5\end{vmatrix} = 1(-5-3) = -8$$

$$A_{23} = (-1)^5\begin{vmatrix}1 & 2\\ -1 & 2\end{vmatrix} = -1(2+2) = -4$$

$$A_{31} = (-1)^4\begin{vmatrix}2 & -3\\ -3 & 2\end{vmatrix} = +1(4-9) = -5$$

$$A_{32} = (-1)^5\begin{vmatrix}1 & -3\\ 2 & 2\end{vmatrix} = -1(6+2) = -8$$

$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 1(-3-4) = -7$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 11 & 4 & -5 \\ 8 & 8 & -8 \\ 1 & 4 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ -3 \end{bmatrix}$$

$$= \frac{1}{24} \begin{bmatrix} 55+4+15 \\ 40-8+24 \\ 5-4+21 \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 74 \\ 56 \\ 22 \end{bmatrix} = \begin{bmatrix} 74/24 \\ 56/24 \\ 22/24 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37/12 \\ 7/3 \\ 11/12 \end{bmatrix}$$

$x = 37/12$

$y = 7/3$

$z = 11/12$

---

(1) $-x+3y-5z=0$

$2x+4y-6z=1$

$x-2y+3z=3$

$$\begin{bmatrix} -1 & 3 & -5 \\ 2 & 4 & -6 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$|A| = -1 \begin{vmatrix} 4 & -6 \\ -2 & 3 \end{vmatrix} - 3 \begin{vmatrix} 2 & -6 \\ 1 & 3 \end{vmatrix} - 5 \begin{vmatrix} 2 & 4 \\ 1 & -2 \end{vmatrix}$

$= -1(0) - 3(12) - 5(-8)$

$= -36 + 40 = 4 \neq 0$

$A_{11} = (-1)^2 \begin{vmatrix} 4 & 6 \\ -2 & 3 \end{vmatrix} = 1(0) = 0$ $\qquad A_{21} = (-1)^3 \begin{vmatrix} 3 & -5 \\ -2 & 3 \end{vmatrix} = -1(-1) = 1$

$A_{12} = (-1)^3 \begin{vmatrix} 2 & -6 \\ 1 & 3 \end{vmatrix} = -1(12) = -12$ $\qquad A_{22} = (-1)^4 \begin{vmatrix} -1 & -5 \\ 1 & 3 \end{vmatrix} = 1(= 2$

$A_{13} = (-1)^4 \begin{vmatrix} 2 & 4 \\ 1 & -2 \end{vmatrix} = 1(-8) = -8$ $\qquad A_{23} = (-1)^5 \begin{vmatrix} -1 & 3 \\ 1 & -2 \end{vmatrix} = -(-1) = 1$

$$A_{31} = (-1)^4 \begin{vmatrix} 3 & -5 \\ 4 & -6 \end{vmatrix} = 1(2) = 2$$

$$A_{32} = (-1)^5 \begin{vmatrix} -1 & -5 \\ 2 & -6 \end{vmatrix} = -(16) = -16$$

$$A_{33} = (-1)^6 \begin{vmatrix} -1 & 3 \\ 2 & 4 \end{vmatrix} = +(-10) = -10$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 0 & 1 & 2 \\ -12 & 2 & -16 \\ -8 & 1 & -10 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 0+1+6 \\ 0+2-48 \\ 0+1-30 \end{bmatrix}$$

$$〃 = \frac{1}{4} \begin{bmatrix} 7 \\ -46 \\ -29 \end{bmatrix} = \begin{bmatrix} 7/4 \\ -23/2 \\ -29/4 \end{bmatrix}$$

$x = 7/4$ $\quad y = -23/2$ $\quad z = -29/4$

(4)
$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$
$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$
$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$|A| = 2 \begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} - 3 \begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} + 10 \begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix}$$

$$= 2(75) - 3(-110) + 10(72)$$
$$= 150 + 330 + 720$$
$$= 1200 \neq 0$$

$A_{12} = (-1)^3 \begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} = -(-110) = 110$ $\quad$ $A_{21} = (-1)^3 \begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix} = -(-150) = 150$

$A_{11} = (-1)^2 \begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} = +(75) = 75$ $\quad$ $A_{22} = (-1)^4 \begin{vmatrix} 2 & 10 \\ 6 & -20 \end{vmatrix} = +(-100) = 100$

$A_{13} = (-1)^4 \begin{vmatrix} 4 & 6 \\ -6 & 9 \end{vmatrix} = +(72) = 72$ $\quad$ $A_{23} = (-1)^5 \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = -(0) = 0$

$A_{31} = (-1)^4 \begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix} = 72$ $\quad$ $A_{32} = (-1)^5 \begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix} = -(-30) = 30$

$A_{33} = (-1)^6 \begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix} = +(-24) = -24$

$$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$" = \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$" = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$x = 2 \qquad y = 3 \qquad z = 5$

Non-Homogeneous Linear Equation:

$a_1x + b_1y + c_1z = k_1$

$a_2x + b_2y + c_2z = k_2$

$a_3x + b_3y + c_3z = k_3$

If atleast one k_1, k_2 and k_3 are non-zero. Then the system becomes non-homogenous.

Example: $2x+3y-4=5$

$x-y+z=0$

$4x+5y-3z=9$

↳ It has (2) solutions:

(1) Consistant

(2) Non- Consistant

Consistant: It has two cases

unique solution | infinite many solutions

*** unique:** The unique solution has following properties:

→ $|A| \neq 0$

→ Rank of A = Rank of A_b (Augmented)

example:

$$\underbrace{\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{A} \begin{matrix} : 0 \\ : 5 \\ : 4 \end{matrix}}_{A_b} \rightarrow \text{constants}$$

Rank A = Rank A_b

3 = 3

$$\begin{bmatrix} x+2y-3z=-3 \\ 2x-5y+4z=13 \\ 5x+4y-z=5 \end{bmatrix} = \left[\begin{array}{ccc:c} 1 & 2 & -3 & -3 \\ 2 & -5 & 4 & 13 \\ 5 & 4 & -1 & 5 \end{array}\right]$$

Rank A = Rank A_b

3 = 3

*** infinite many solutions:**

properties:

→ $|A|=0$

→ Rank $A <$ Total number of number of variable.

Example: $\begin{bmatrix} 1 & 0 & 0 & : & 0 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$

Rank $A = 2$: Rank $A_b = 2$

$2 = 2$

No. of variable in xyz $2 > 3$

$x - y - z = 2$
$2x - y - 3z = 6$ $=$ $\begin{bmatrix} 1 & -1 & -1 & : & 2 \\ 2 & -1 & -3 & : & 6 \\ 1 & 0 & -2 & : & 4 \end{bmatrix}$
$x + 0y - 2z = 4$

$\underset{\sim}{R} \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ $R_2 - 2R_1$
$R_3 - R_1$

$R_2 = R_3$ so,

$= [0, 1, -1, 2] - [0, 1, -1, 2]$

$= [0, 0, 0, 0]$

$\underset{\sim}{R} \begin{bmatrix} 1 & -1 & -1 & : & 2 \\ 0 & 1 & -1 & : & 6 \\ 0 & 0 & 0 & & 4 \end{bmatrix}$ $R_3 - R_2$

$x - y - z = 2$ — (1)

$y - z = 2$ — (2)

$\cdot\ y = z$

$x - 2z = 2$

$\cdot\ x = 2 + 2z$

$z \in \mathbb{R}$

Inconsistant:

No. of solutions exists.

properties: $|A| = 0$

$\text{Rank } A \neq \text{Rank } A_b$

example:

$$x - y + 2z = 2$$
$$2x - 6y + 5z = 1$$
$$3x + 5y + 6z = 4$$

$$A_b = \begin{bmatrix} 1 & -1 & 2 & : & 2 \\ 2 & -6 & 5 & : & 1 \\ 3 & 5 & 6 & : & 4 \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -1 & 2 & : & 2 \\ 0 & -4 & 1 & : & -3 \\ 0 & 8 & 0 & : & -2 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 \end{matrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -1 & 2 & : & 2 \\ 0 & -4 & 1 & : & -3 \\ 0 & 0 & 2 & : & -8 \end{bmatrix} R_3 + 2R_2 \leftarrow \text{Wrong}$$

$[0, 0, 2, -8]$

QNo.4: Solve the following system of linear equ by Gauss - Jordan Method.

(1)

$$2x_1 - x_2 - x_3 = 2$$
$$3x_1 - 4x_2 + 3x_3 = 7$$
$$4x_1 + 2x_2 - 5x_3 = 10$$

$$A_b = \begin{bmatrix} 2 & -1 & -1 & : & 2 \\ 3 & -4 & 3 & : & 7 \\ 4 & 2 & -5 & : & 10 \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 2 & -1 & -1 & : & 2 \\ 1 & -3 & 4 & : & 5 \\ 0 & 4 & -3 & : & 6 \end{bmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - 2R_1 \end{matrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -3 & 4 & : & 5 \\ 2 & -1 & -1 & : & 2 \\ 0 & 4 & -3 & : & 6 \end{bmatrix} R_{12}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & -3 & 4 & : & 5 \\ 0 & 5 & -9 & : & -8 \\ 0 & 4 & -3 & : & 6 \end{bmatrix} \quad R_2 - 2R_1$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & -3 & 4 & : & 5 \\ 0 & 1 & -6 & : & -14 \\ 0 & 4 & -3 & : & 6 \end{bmatrix} \quad R_2 - R_3$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & -14 & : & -37 \\ 0 & 1 & -6 & : & -14 \\ 0 & 0 & 21 & : & 62 \end{bmatrix} \quad \begin{matrix} R_1 + 3R_2 \\ R_3 - 4R_2 \end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & -14 & : & -37 \\ 0 & 1 & -6 & : & -14 \\ 0 & 0 & 1 & : & 62/21 \end{bmatrix} \quad \frac{1}{21}R_3$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 0 & : & 13/3 \\ 0 & 1 & 0 & : & 26/7 \\ 0 & 0 & 1 & : & 62/21 \end{bmatrix} \quad \begin{matrix} R_1 + 14R_3 \\ R_2 + 6R_3 \end{matrix}$$

$\therefore$ unique sol.

$x = 13/13$ $\quad x_2 = 26/7$ $\quad x_3 = 62/21$

(2) $2x_1 - 3x_2 + 7x_3 = 1$

$4x_1 + 5x_2 - 3x_3 = 4$

$10x_1 - 4x_2 + 18x_3 = 7$

$$A_b : \begin{bmatrix} 2 & -3 & 7 & : & 1 \\ 4 & 5 & -3 & : & 4 \\ 10 & -4 & 18 & : & 7 \end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & -3/2 & 7/2 & : & 1/2 \\ 4 & 5 & -3 & : & 4 \\ 10 & -4 & 18 & : & 7 \end{bmatrix} \quad \frac{1}{2}R_1$$

$$\overset{R}{\sim}\left[\begin{array}{ccc:c} 1 & -3/2 & 7/2 & 1/2 \\ 0 & 11 & -17 & 2 \\ 0 & 11 & -17 & 2 \end{array}\right] \quad \begin{array}{l} R_2-4R_1 \\ R_3-10R_1 \end{array}$$

$$\overset{R}{\sim}\left[\begin{array}{ccc:c} 1 & -3/2 & 7/2 & 1/2 \\ 0 & 11 & -17 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 - R_2$$

$$\overset{R}{\sim}\left[\begin{array}{ccc:c} 1 & -3/2 & 7/2 & 1/2 \\ 0 & 1 & -17/11 & 2/11 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \frac{1}{11}R_2$$

$$\overset{R}{\sim}\left[\begin{array}{ccc:c} 1 & 0 & 13/11 & 17/22 \\ 0 & 1 & -17/11 & 2/11 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_1+\frac{3}{2}R_2$$

Rank A = Rank A_b < No. of unknowns

2   2   3   ∴ Infinite many sol.

From last row:

$0x_1 + x_2(0) + 0x_3 = 0$

let $x_3 = t$

If $t=0$

$x_3 = 0$

From second row:

$x_2 - \frac{17}{11}x_3 = \frac{2}{11}$

$x_2 - \frac{17}{11}t = \frac{2}{11}$

$x_2 = \frac{2}{11} + \frac{17}{11}t$

if $t=0$

$x_2 = 2/11$

From row one:

$x_1 + \frac{13}{11}x_3 = 17/22$

$x_1 + \frac{13}{11}t = \frac{17}{22}$

$x_1 = \frac{17}{22} - \frac{13}{11}t$

if $t=0$

$x_1 = \frac{17}{22}$

(3) $x_1 + x_2 + x_3 = 3$

$2x_1 - 3x_2 + 2x_3 = 7$

$4x_1 + 2x_2 - 5x_3 = 10$

$$A_b = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 2 & -3 & 2 & 7 \\ 4 & 2 & -5 & 10 \end{array}\right]$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & -5 & 0 & 1 \\ 0 & -2 & 9 & -2 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 4R_1 \end{array}$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & -1/5 \\ 0 & -2 & -9 & -2 \end{array}\right] \quad -\frac{1}{5} R_2$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 0 & 1 & 16/5 \\ 0 & 1 & 0 & -1/5 \\ 0 & 0 & -9 & -12/5 \end{array}\right] \quad \begin{array}{l} R_1 - R_2 \\ R_3 + 2R_2 \end{array}$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 0 & 1 & 16/5 \\ 0 & 1 & 0 & -1/5 \\ 0 & 0 & 1 & 4/15 \end{array}\right] \quad -\frac{1}{9} R_3$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 0 & 0 & 44/15 \\ 0 & 1 & 0 & -1/5 \\ 0 & 0 & 1 & 4/15 \end{array}\right] \quad R_1 - R_3$$

Rank-A = Rank Ab = No. of unknowns

3 = 3 = 3 $\quad \therefore$ unique sol.

$x = \frac{44}{15} \qquad x_2 = -1/5 \qquad x_3 = 4/15$

(9) $2x_1 - 7x_2 + 10x_3 = 1$

$x_1 + 2x_2 - 4x_3 = 8$

$2x_1 - 11x_2 + 13x_3 = 7$

$$A_b = \left[\begin{array}{cccc} 2 & -7 & 10 & : 1 \\ 1 & 2 & -4 & : 8 \\ 2 & -11 & 13 & : 7 \end{array}\right]$$

$$\overset{R}{\sim} \left[\begin{array}{cccc} 1 & 2 & -4 & : 8 \\ 2 & -7 & 10 & : 1 \\ 2 & -11 & 13 & : 7 \end{array}\right] \quad R_{12}$$

$$\overset{R}{\sim} \left[\begin{array}{cccc} 1 & 2 & -4 & : 8 \\ 0 & -11 & 18 & : 15 \\ 0 & -15 & 21 & : -9 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 2R_1 \end{array}$$

$$\overset{R}{\sim} \left[\begin{array}{cccc} 1 & 2 & 4 & : 8 \\ 0 & 1 & -8/11 & : 15/11 \\ 0 & -15 & 21 & : -9 \end{array}\right] \quad -\frac{1}{11} R_2$$

$$\overset{R}{\sim} \left[\begin{array}{cccc} 1 & 0 & -8/11 & : 58/11 \\ 0 & 1 & -18/11 & : 15/11 \\ 0 & 0 & -39/11 & : 126/11 \end{array}\right] \quad \begin{array}{l} R_1 - 2R_2 \\ R_3 + 15R_2 \end{array}$$

$$\overset{R}{\sim} \left[\begin{array}{cccc} 1 & 0 & -8/11 & : 58/11 \\ 0 & 1 & -18/11 & : 15/11 \\ 0 & 0 & 1 & : -42/13 \end{array}\right] \quad \frac{-11}{39} R_3$$

$$\overset{R}{\sim} \left[\begin{array}{cccc} 1 & 0 & 0 & : 38/13 \\ 0 & 1 & 0 & : -51/13 \\ 0 & 0 & 1 & : -4/13 \end{array}\right] \quad \begin{array}{l} R_1 + 8/11\, R_3 \\ R_2 + 18/11\, R_3 \end{array}$$

Rank A = Rank A_b = No. of unknowns

3 = 3 = 3

∴ unique sol.

$x_1 = \frac{38}{13}$ $\quad x_2 = \frac{-51}{13}$ $\quad x_3 = \frac{-42}{13}$

QNo.7: If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$; find A^{-1} and hence solve the system of equations.

$|A| = 3\begin{vmatrix} -1 & 2 \\ 3 & -3 \end{vmatrix} - 2\begin{vmatrix} 4 & 2 \\ 7 & -3 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ 7 & 3 \end{vmatrix}$

$= 3(-3) - 2(-26) + 1(19)$

$= -9 + 52 + 19 = 62 \neq 0$

$A_{11} = (-1)^2\begin{vmatrix} -1 & 2 \\ 3 & -3 \end{vmatrix} = (1)(-3) = -3$

$A_{21} = (-1)^3\begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix} = -1(-9) = 9$

$A_{12} = (-1)^3\begin{vmatrix} 4 & 2 \\ 7 & -3 \end{vmatrix} = -1(-26) = 26$

$A_{22} = (-1)^4\begin{vmatrix} 3 & 1 \\ 7 & -3 \end{vmatrix} = +1(-16) = 16$

$A_{13} = (-1)^4\begin{vmatrix} 4 & -1 \\ 7 & 3 \end{vmatrix} = 1(19) = 19$

$A_{23} = (-1)^5\begin{vmatrix} 3 & 2 \\ 7 & 3 \end{vmatrix} = -1(-5) = 5$

$A_{31} = (-1)^4\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 1(5) = 5$

$A_{32} = (-1)^5\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = -1(2) = 2$

$A_{33} = (-1)^6\begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} = 1(-11) = -11$

$A^{-1} = \begin{bmatrix} -3/62 & 9/62 & 5/62 \\ 13/31 & -8/31 & -1/31 \\ 19/62 & 5/62 & -11/62 \end{bmatrix}$

Now, solve system for equation,

$3x + 4y + 7z = 14$

$2x - y + 3z = 4$

$x + 2y - 3z = 0$

$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$

$A \quad X = B$

$X = A^{-1}B$

$$|A| = 3\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} + 7\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$$

$$= 3(-3) - 4(-9) + 7(5)$$

$$= -9+36+35 \quad = 62 \neq 0$$

$A_{11} = (-1)^2\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} = 1(-3) = -3$ $\quad A_{21} = (-1)^3\begin{vmatrix} 4 & 7 \\ 2 & -3 \end{vmatrix} = -1(-26) = 2$

$A_{12} = (-1)^3\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} = -1(-9) = +9$ $\quad A_{22} = (-1)^4\begin{vmatrix} 3 & 7 \\ 1 & -3 \end{vmatrix} = 1(-16) = -1$

$A_{13} = (-1)^4\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 1(5) = 5$ $\quad A_{23} = (-1)^5\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = -1(2) = -2$

$A_{31} = (-1)^4\begin{vmatrix} 4 & 7 \\ -1 & 3 \end{vmatrix} = 1(19) = 19$ $\quad A_{32} = (-1)^5\begin{vmatrix} 3 & 7 \\ 2 & 3 \end{vmatrix} = -1(-5) = 5$

$A_{33} = (-1)^6\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1(-11) = -11$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$= \frac{1}{62}\begin{bmatrix} -42+104+0 \\ 126-64+0 \\ 70-8+0 \end{bmatrix}$$

$$= \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x = 1 \quad y = 1 \quad z = 1$

Before Q. 8:-

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

∴ consistant system

Rank-A = 3 Rank A_b = 3

Rank A = Rank A_b = No. of variables

unique solution

case I

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

∴ consistant system

Rank A = 2 Rank A_b = 2

No. of variables = 3

Rank A < No. of variables

infinite solution

case II

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 5 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 5 \end{array}\right] \rightarrow 5 \neq 0$$

∴ Inconsistant system

Rank A = 2 Rank A_b = 3

Rank A ≠ Rank A_3

no solution

case III

QNo. 8: Determine the value of λ for which the following system has no solution, unique solution or infinite many solutions.

(i) $x + 2y - 3z = 4$

$3x - y + 5z = 2$

$4x + y + (\lambda^2 - 14)z = \lambda + 2$

$$\underset{A}{\begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 5 \\ 4 & 1 & \lambda^2-14 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 2 \\ \lambda+2 \end{bmatrix}}$$

$$A_b = \left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & \lambda^2-14 & \lambda+2 \end{array}\right]$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & \lambda^2-2 & \lambda-14 \end{array}\right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 4R_1 \end{array}$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & \lambda^2 16 & \lambda-4 \end{array}\right] \; R_3 - R_2$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & \lambda^2-16 & \lambda-4 \end{array}\right] \; -\frac{1}{7}R_2$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & \lambda^2-16 & \lambda-4 \end{array}\right]$$

For no solution:

Rank $A \neq$ Rank A_b

$\lambda^2 - 16 = 0$ and $\lambda - 4 \neq 0$

$\lambda^2 = 16$

$\boxed{\lambda = \pm 4}$ → no sol.

For unique sol.: doesn't

Rank A = Rank A_b exists.

$\lambda^2 - 16 \neq 0$ No included.

$\lambda^2 \neq 16$. $\boxed{\lambda \neq \pm 4}$

For infinte many sol:

Rank = A = Rank A_b < No. of unknown

$\lambda^2 - 16 = \lambda - 4$

*** "If infinite and no solution exists than unique sol. doesn't exists."**

Q No. 9: Show that the system of equation:

$2x - y + 3z = \alpha$ is inconsistent if $\gamma \neq 2\alpha - 3\beta$

$3x + y - 5z = \beta$

$-5x - 5y + 21z = \gamma$

$$A_b = \left[\begin{array}{ccc:c} 2 & -1 & 3 & \alpha \\ 3 & 1 & -5 & \beta \\ -5 & -5 & 21 & \gamma \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 2 & -1 & 3 & \alpha \\ 1 & 2 & -8 & \beta-\alpha \\ -5 & -5 & 21 & \gamma \end{array}\right] \quad R_2 - R_1$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & 2 & -8 & \beta-\alpha \\ 2 & -1 & 3 & \alpha \\ -5 & -5 & 21 & \gamma \end{array}\right] \quad R_{12}$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & 2 & -8 & \beta-\alpha \\ 0 & -5 & +19 & 3\alpha-2\beta \\ 0 & 5 & -19 & \gamma+5\beta-5\alpha \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 + 5R_1 \end{array}$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & 2 & -8 & \beta-\alpha \\ 0 & -5 & 19 & 3\alpha-2\beta \\ 0 & 0 & 0 & \gamma+3\beta-2\alpha \end{array}\right] \quad R_3 + R_2$$

$$\underset{\sim}{R} \left[\begin{array}{ccc:c} 1 & 2 & -8 & \beta-\alpha \\ 0 & 1 & -19/5 & 3\alpha-2\beta/5 \\ 0 & 0 & 0 & \gamma+3\beta-2\alpha \end{array}\right] \quad -\frac{1}{5}R_2$$

System is inconsistant if:

Rank A ≠ Ran A_b

$\gamma + 3\beta - 2\alpha \neq 0$

$$\boxed{\gamma = 2\alpha - 3\beta}$$

Q8- Determine the value of λ for which the following system has no solution and infinite many solution.

$$\begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 4 \\ 0 & 0 & \lambda+1 & : & \lambda+7 \end{bmatrix}$$

$$A_b = \begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 4 \\ 0 & 0 & \lambda+1 & : & \lambda+7 \end{bmatrix}$$

For no solution:

$\lambda+1 = 0$ $\quad$ $\lambda+7 \neq 0$

$\lambda = -1$ $\quad$ $\lambda \neq -7$

unique solution:.

$\lambda \neq -1$

For infinite solution:

$\lambda = -7$ $\quad$ Doesn't exists.

Translations

- When the graph moves c unit up.

Take : $y' = y + c$

- When the graph moves c unit down.

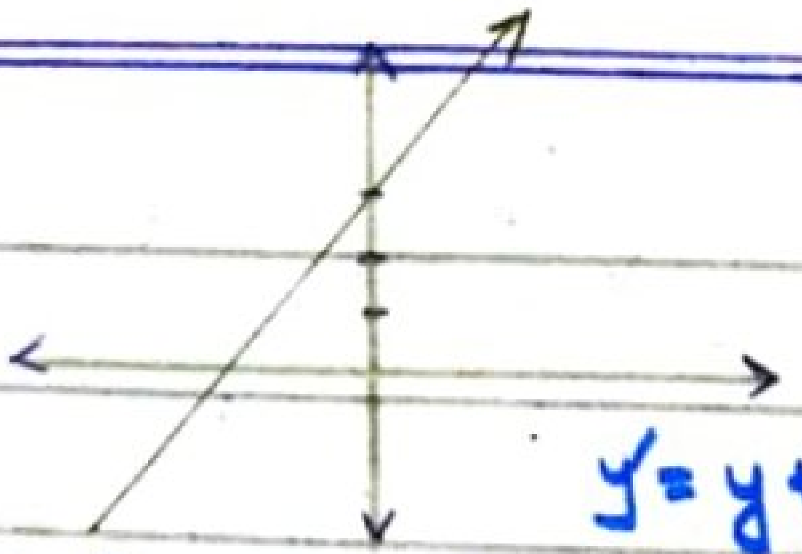
Take : $y' = y - c$

- When the graph moves c unit left.

Take : $x' = x - c$

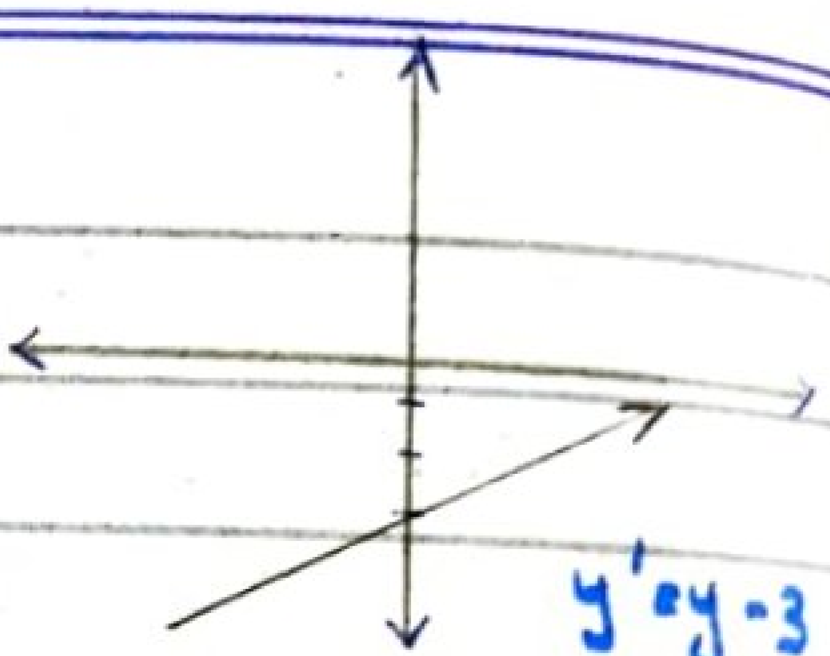
- When the graph moves c unit right.

Take : $x' = x + c$



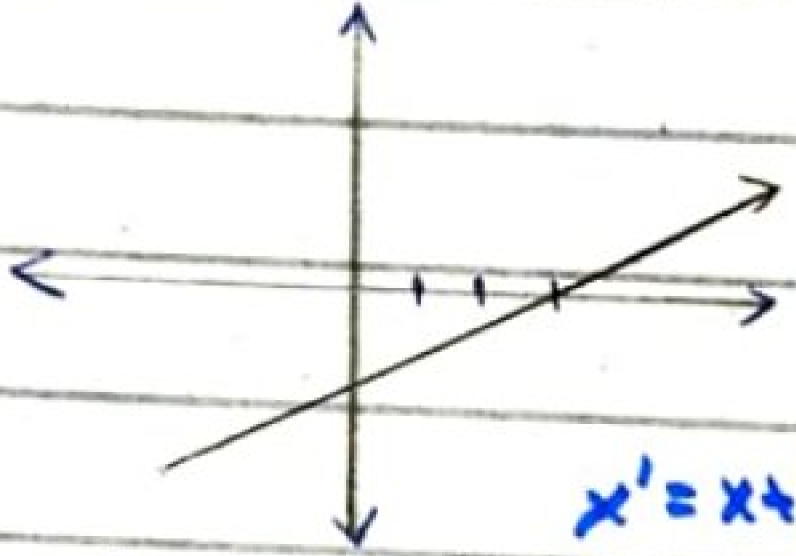
$y' = y+3$

Shift graph 3 unit up



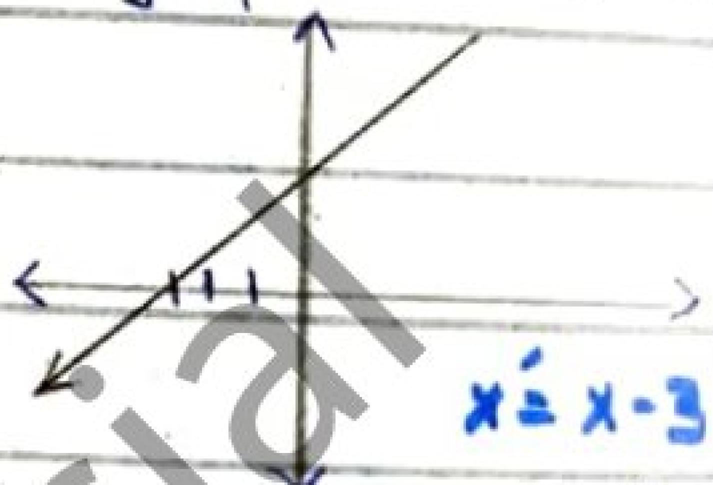
$y' = y-3$

Shift graph 3 unit down



$x' = x+3$

Shift graph 3 unit on right axis



$x' = x-3$

3 unit on left axis

Q No.13: Write the transformation matrix and transform equation of line when it moves 2 units above and 3 unit to the right.

$$2x = 7y-3$$

when it moves 2 unit above:

$$y' = y+2$$

when it moves 3 unit right:

$$x' = x+3$$

Transformation Matrix is:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x+3 \\ y+2 \end{bmatrix} = \begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Given: $y' = y+2$ $\qquad$ $x' = x+3$

$y = y'-2$ $\qquad$ $x = x'-3$

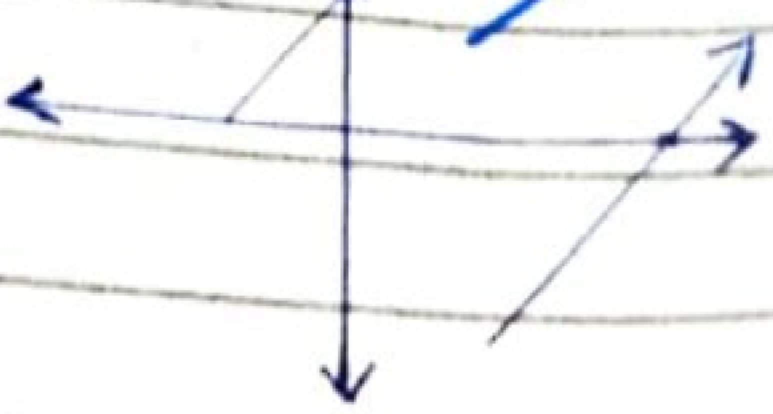
put x, y in eq:

$$2(x'-3) = 7(y'-2)-3$$

$$2x'-6 = 7y'-14-3$$

$$2x'-6 = 7y'-17$$

$$2x'-7y'+11 = 0$$



Q No.11: $x^2+4x-3y=8$ when line moves 2 units on left.

$x' = x-2$

$x = x'+2$ $\qquad y' = y+0$

Translation Matrix is:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x-2 \\ y+0 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} -1 \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$(x'+2)^2 + 4(x'+2) - 3(y') = 8$

$x'^2 + 4x' + 4 + 4x' + 8 - 3y' = 8$

$x'^2 + 8x' - 3y' + 4 = 0$

QNo.10: By making use of matrix of order 2-by-2 and 3-by-3 encode and decode the following words.

(1) PAKISTAN

Let encoding by 2-by-2 Matrix:

$$A = \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix}$$

Cut Word "Pakistan" into pieces each of length 2.

(PA), (KI), (ST), (AN)

Matrix Form:

$$\begin{bmatrix} 16 & 1 \end{bmatrix}, \begin{bmatrix} 11 & 9 \end{bmatrix}, \begin{bmatrix} 9 & 20 \end{bmatrix}, \begin{bmatrix} 1 & 14 \end{bmatrix}$$

Now Coded as:

$$\begin{bmatrix} 16 & 1 & 11 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 27 & 13 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 14 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 28 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 19 & 20 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 29 & 58 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 27 & 13 & 16 \end{bmatrix}, \begin{bmatrix} 29 & 58 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 28 & 1 \end{bmatrix}$$

encrypted

The receiver decodes the message by reverse key, so, first we find A^{-1}

$$|A| = 1\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = -2$$

$A_{11} = (-1)^2 \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0$ $\quad A_{21} = 1 \quad A_{31} = -2$

$A_{12} = (-1)^3 \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$ $\quad A_{22} = -1 \quad A_{32} = 0$

$A_{13} = (-1)^4 \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = 1(2) = -2$ $\quad A_{23} = -1 \quad A_{33} = 2$

$$A^{-1} = -\frac{1}{2}\begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -1/2 & 1 \\ 0 & 1/2 & 0 \\ 1 & 1/2 & -1 \end{bmatrix}$$

Decoded as,

$$\begin{bmatrix} 27 & 13 & 16 \end{bmatrix}\begin{bmatrix} 0 & -1/2 & 1 \\ 0 & 1/2 & 0 \\ 1 & 1/2 & -1 \end{bmatrix} = \begin{bmatrix} 0+0+16 & -\frac{27}{2}+\frac{13}{2}+8 & 27-16 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 1 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 29 & 58 & 9 \end{bmatrix}\begin{bmatrix} 0 & -1/2 & 1 \\ 0 & 1/2 & 0 \\ 1 & 1/2 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 19 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 28 & 1 \end{bmatrix}\begin{bmatrix} & & \\ & " & \\ & & \end{bmatrix} = \begin{bmatrix} 1 & 14 & 0 \end{bmatrix}$$

Sequence:

$$\begin{bmatrix} 16 & 1 & 11 \end{bmatrix}, \begin{bmatrix} 9 & 19 & 20 \end{bmatrix}, \begin{bmatrix} 1 & 14 & 0 \end{bmatrix}$$

P A K I S T A N

(2) ISLAMABAD:

Let encoding 2-by-2 matrix is: $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(IS), (LA), (MA), (BA), (D)

$\begin{bmatrix} 9 & 19 \end{bmatrix}, \begin{bmatrix} 12 & 1 \end{bmatrix}, \begin{bmatrix} 13 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \end{bmatrix}, \begin{bmatrix} 4 & 0 \end{bmatrix}$

Coded Matrix	Encoding Matrix	Code row Matrix
$\begin{bmatrix} 9 & 19 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 9 & 28 \end{bmatrix}$
$\begin{bmatrix} 12 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 12 & 13 \end{bmatrix}$
$\begin{bmatrix} 13 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 13 & 14 \end{bmatrix}$
$\begin{bmatrix} 2 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & 3 \end{bmatrix}$
$\begin{bmatrix} 4 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 4 & 4 \end{bmatrix}$

↳ Encrypted

$A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 9 & 28 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 9 & 19 \end{bmatrix} = \begin{bmatrix} I & S \end{bmatrix}$
$\begin{bmatrix} 12 & 13 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 12 & 1 \end{bmatrix} = \begin{bmatrix} L & A \end{bmatrix}$
$\begin{bmatrix} 13 & 14 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 13 & 1 \end{bmatrix} = \begin{bmatrix} M & A \end{bmatrix}$
$\begin{bmatrix} 2 & 3 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & 1 \end{bmatrix} = \begin{bmatrix} B & A \end{bmatrix}$
$\begin{bmatrix} 4 & 4 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 4 & 0 \end{bmatrix} = \begin{bmatrix} D \end{bmatrix}$

ISLAMABAD

↳ Decrypted

(2) by 3-by-3 Matrix:

let the Matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

(ISL), (AMA), (BAD)

$\begin{bmatrix} 9 & 19 & 12 \end{bmatrix}$, $\begin{bmatrix} 1 & 13 & 1 \end{bmatrix}$, $\begin{bmatrix} 2 & 1 & 4 \end{bmatrix}$

Uncoded Matrix	Encoding Matrix	Coded row matrix
$\begin{bmatrix} 9 & 19 & 12 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 28 & 12 & -19 \end{bmatrix}$
$\begin{bmatrix} 1 & 13 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 14 & 1 & -13 \end{bmatrix}$
$\begin{bmatrix} 2 & 1 & 4 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 3 & 4 & -1 \end{bmatrix}$

Encrpted

Decrypt:-

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 28 & 12 & -19 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 9 & 19 & 12 \end{bmatrix}$
$\begin{bmatrix} 4 & 1 & -13 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 13 & 1 \end{bmatrix}$
$\begin{bmatrix} 3 & 4 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 2 & 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 9 & 19 & 12 \end{bmatrix}$ $\begin{bmatrix} 1 & 13 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 & 4 \end{bmatrix}$

I S L A M A B A D

QNo. 10:- (c) COLLEGE

let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(CO), (LL), [EG], (E)

$\begin{bmatrix} 3 & 15 \end{bmatrix}, \begin{bmatrix} 12 & 12 \end{bmatrix}, \begin{bmatrix} 5 & 7 \end{bmatrix}, \begin{bmatrix} 5 & 0 \end{bmatrix}$

Uncoded Matrix	Encoded Matrix	Coded row Matrix
$\begin{bmatrix} 3 & 15 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 3 & 18 \end{bmatrix}$
$\begin{bmatrix} 12 & 12 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 12 & 24 \end{bmatrix}$
$\begin{bmatrix} 5 & 7 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 5 & 12 \end{bmatrix}$
$\begin{bmatrix} 5 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 5 & 5 \end{bmatrix}$

Coded / Encrypted

$A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3 & 18 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 3 & 15 \end{bmatrix}$ (C O)
$\begin{bmatrix} 12 & 24 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 12 & 12 \end{bmatrix}$ (L L)
$\begin{bmatrix} 5 & 12 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 5 & 7 \end{bmatrix}$ (E G)
$\begin{bmatrix} 5 & 5 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 5 & 0 \end{bmatrix}$ (E)

By 3-by-3 Matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

(COL) (LEG) (E)

$[3 \ 15 \ 12]$ $[12 \ 5 \ 7]$ $[5 \ 0 \ 0]$

$\begin{bmatrix} 3 & 15 & 12 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 18 & 12 & -15 \end{bmatrix}$
$\begin{bmatrix} 12 & 5 & 7 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 17 & 7 & -5 \end{bmatrix}$
$\begin{bmatrix} 5 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 5 & 0 & 0 \end{bmatrix}$

Coded

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$\begin{bmatrix} 18 & 12 & -15 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 3 & 15 & 12 \end{bmatrix}$ = COL
$\begin{bmatrix} 17 & 7 & -5 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 12 & 5 & 7 \end{bmatrix}$ = LEG
$\begin{bmatrix} 5 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 5 & 0 & 0 \end{bmatrix}$ = E

Important MCQ's

(1) $|\text{adj } A| = |A|^{\overset{\text{order}}{n-1}}$

(2) $|KA| = K^{\overset{\text{order}}{n}} |A|$

Scalar Multiplication

(3) $|KA| = 0$ (If A is singular)

(4) $\text{adj}(KA) = K^{n-1}(\text{adj} A)$

(5) $(KA)^{-1} = \frac{1}{K} A^{-1}$

(6) A is square matrix If $|A| = I$ then matrix is orthogonal.

(1) If A is square matrix of order 3x3 with $C|A| = 3$ then value of $|\text{adj } A| = ?$

$|\text{adj } A| = |A|^{n-1}$ $n=3$

$|\text{adj } A| = (3)^{3-1}$

$= (3)^2 = 9$

(2) If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}_{3\times 3}$ then $|\text{adj} A| = ?$

$|\text{adj} A| = |A|^{n-1}$

$= (a^3)^{3-1}$

$= (a^3)^2 = a^6$

(3) Product of $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}_{3\times 1} \times \begin{bmatrix} 4 & 5 & 2 \end{bmatrix}_{1\times 3} \times \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}_{3\times 1}$

$$\begin{bmatrix} 4 & 5 & 2 \\ -8 & -10 & -4 \\ 12 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 27 \\ 26 \\ 9 \end{bmatrix}$$

(4) If A has order 3x3 and $|A|=2$ the $|2A|=?$

$|A|=2$

$|KA| = K^n(A)$

$|2A| = 2^3(2) = 2^4$

$= 16$

(5) If A has order 4x4 , $|A|=6$ find $|1-A|=?$

$|1-A| = (-1)^4|A|$

$= 1(6)$

$= 6$

* Order of matrix must be same for this property.

(6) If A is 2x2 then $adj(2A) = ?$, $adj\ A = 4$

$adj(k^n) = k^{n-1}(adj\ A)$

$k=2$ $n=2$ $adj\ A=4$

$adj(2A) = (2)^{2-1}(4)$

$= 2(4) = 8$

(7) If A is invertable (inverse exists) then $(5A)^{-1}=?$

$A^{-1}=2$

$(KA)^{-1} = \frac{1}{K}A^{-1}$

$= \frac{1}{5}(2) = \frac{2}{5}$

(8) If A is a matrix of order 2x3 and B^t is a matrix of order 3x3 then order of $AB=?$

$A^t_{2\times3}$ $B^t_{3\times3}$

$(A^t)^t_{3\times2}$ $(B^t)^t_{3\times3}$

$A_{3\times2}$ $B_{3\times3}$

Order doesn't exists.

(9) A is a singular matrix then $|-3A| = ?$

zero (0)

Test Chapter: 2 2nd half

Section: A

(1) If A is a square matrix of $\wedge$ order and $|A| = 3$ then value of $|adj A| =$ is:

(a) $1/3$ (b) 3 [✓ (c) 9] (d) 6

(2) If the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ then it is a:

[✓ (a) echeleon] (b) reduced echeleon (c) none (d) both

(3) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ it is a

(a) echeleon (b) R. echeleon (c) both (d) none

(4) If order of A is $M \times N$ and order of B^t is $n \times p$ then order of BA is:

(a) $n \times p$ (b) $p \times n$ (c) $m \times p$ [✓ (d) doesn't exists]

(5) In row echeleon form the first non-zero no. in row is called.

[✓ (a) pivot (leading)] (b) Determinant eleme. (c) Minor (d) Cofactor

(6) The rank of $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ is

[✓ (a) 1] (b) 2 (c) 0 (d) 3

Section: B

(1)

$$\begin{matrix} 3a & 1 & 2a+1 \\ 2a+1 & 1 & a+2 \\ 3 & 1 & 2 \end{matrix} = (a-1)(a-2)$$

L.H.S: $R_2 - R_1$, $R_3 - R_1$

$$= \begin{vmatrix} 3a & 1 & 2a+1 \\ -a+1 & 0 & -a+1 \\ 3-3a & 0 & 1-2a \end{vmatrix}$$

Expanding by C_2:

$$= -1 \begin{vmatrix} -a+1 & -a+1 \\ 3-3a & 1-2a \end{vmatrix} = (a-1) \begin{vmatrix} 1 & 1 \\ 3-3a & 2-2a \end{vmatrix}$$

$$= (a-1) \; 2-2a-3+3a = (a-1)(a-2)$$

(2) Find Rank of $\begin{bmatrix} 1 & 3 \\ 2 & 9 \\ 1 & 6 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 3 \\ 0 & 3 \\ 0 & 3 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - R_1 \end{matrix} \quad \underset{\sim}{R} \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 3 \end{bmatrix} \frac{1}{3} R_1$

$\underset{\sim}{R} \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} R_3 - 3R_2$

Rank = 2

(3) Show that $B = \begin{bmatrix} 1 & 2 & 0 \\ -3 & 4 & 9 \\ 2 & 1 & 6 \end{bmatrix}$, $B_{13} + B_{23} + B_{31} = 4$

$B_{13} = (-1)^4 \begin{vmatrix} -3 & 4 \\ 2 & 1 \end{vmatrix} = 1(-3-8) = -11$

$B_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -1(1-4) = 3$

$B_{31} = (-1)^6 \begin{vmatrix} 2 & 0 \\ 4 & 9 \end{vmatrix} = 1(18) = 18$

$-11 + 3 + 18 = 4$

$10 \neq 4$

(4) Convert into echeleon form and find rank

$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 9 & 9 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1/3 & 1/3 \\ 2 & 9 & 9 \end{bmatrix} \frac{1}{3} R_1 \quad \underset{\sim}{R} \begin{bmatrix} 1 & 1/3 & 1/3 \\ 0 & 1 & 1 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1/3 & 1/3 \\ 0 & 25/3 & 25/3 \end{bmatrix} R_2 - 2R_1$

Rank = 2

Section : C

Convert into reduced echeleon and find rank

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 6 \\ 4 & -1 & 0 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 2/3 & 4/3 \\ 2 & 1 & 6 \\ 4 & -1 & 0 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 2/3 & 4/3 \\ 0 & 2/3 & 2/3 \\ 0 & 2/3 & 4/3 \end{bmatrix} \quad \begin{matrix} R_2 - 4R_1 \\ R_3 + R_1 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 2/3 & 4/3 \\ 0 & 1 & 1 \\ 0 & 2/3 & 4/3 \end{bmatrix} \quad \frac{3}{2}R_2$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & 1 \\ 0 & 0 & 2/3 \end{bmatrix} \quad \begin{matrix} R_1 - \frac{2}{3}R_2 \\ R_3 - \frac{2}{3}R_2 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 1 & 2/3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \frac{3}{2}R3$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} R_1 - \frac{2}{3}R_3 \\ R_2 - R_1 \end{matrix}$$

Rank = 3