Studybeesofficial

Where Minds Pollinate

Class 11
Mathematics

Chapter 2
Exercise 2.5
Notes

National Book Foundation

Exercise : 2.5

Solution of System of Simultaneous Equation :-

(1) Substitution Method
(2) Elimination Method
(3) Graphical Method
(4) Inversion Method
(5) Cramer's Rule
(6) Gauss Elimination Method } → Echeleon Form
(7) Gauss Jordan Elimination Method } → Reduced Echeleon Form

Operations on Matrices: The following operations can be performed in a matrix :-

(1) We can interchange any two rows/columns of a matrix if we interchange ith row and jth row then it is denoted by R_{ij}.

Example =

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Interchange R_1 and R_3 :

$$= \begin{bmatrix} 7 & 8 & 9 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} R_{13}$$

* If columns are interchanged then C_{13}.

(2) We can multiply any row by a non-zero scalar number (K) with ith row then it is denoted by KR_i.

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \longrightarrow = \begin{bmatrix} K & 2K & 3K \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad KR_1$$

*If K is multiplied by any column the KC_1.

"IMP"

(3) We can multiply of any row to the correspon value of any other row. If we add K times of jth row to ith row. It is denoted by:- $R_i + KR_j$

Example:-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$3R_2 + R_1$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 12+1 & 15+2 & 18+3 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 13 & 17 & 21 \\ 7 & 8 & 9 \end{bmatrix}$$

Questions:

(1) $\begin{bmatrix} 2 & 5 & 7 \\ 1 & 2 & -1 \\ -3 & -6 & 3 \end{bmatrix}$

Apply Operations:

(1) R_{12}

(2) $R_2 - 2R_1$

(3) $R_3 + 3R_1$

$$\underset{\sim}{R}\begin{bmatrix}1 & 2 & -1\\ 2 & 5 & 7\\ -3 & -6 & 3\end{bmatrix} R_{12}$$

$$\begin{array}{r}\begin{bmatrix}2 & 5 & 7\end{bmatrix}\\ -2\begin{bmatrix}1 & 2 & -1\end{bmatrix}\\ \hline \begin{bmatrix}-2 & -4 & 2\end{bmatrix}\\ \hline 0 \quad 1 \quad 9\end{array}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 9\\ -3 & -6 & 3\end{bmatrix} R_2-2R_1$$

$$\begin{array}{r}\begin{bmatrix}-3 & -6 & 3\end{bmatrix}\\ 3\begin{bmatrix}1 & 2 & -1\end{bmatrix}\\ \begin{bmatrix}3 & 6 & -3\end{bmatrix}\\ \hline 0 \quad 0 \quad 0\end{array}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 9\\ 0 & 0 & 0\end{bmatrix} R_3+3R_1$$

(2) $\begin{bmatrix}1 & -1 & 2\\ -3 & -2 & 5\\ 0 & 4 & -1\end{bmatrix}$ Use operation :

(1) R_1-2R_2

(2) R_2+3R_1

(3) R_{23}

(4) $2R_1-R_3$

$$\begin{array}{r}\begin{bmatrix}1 & -1 & 2\end{bmatrix}\\ -2\begin{bmatrix}-3 & -2 & 5\end{bmatrix}\\ \begin{bmatrix}+6 & +4 & -10\end{bmatrix}\\ \hline \begin{bmatrix}7 & 3 & -8\end{bmatrix}\end{array}$$

$$\underset{\sim}{R}\begin{bmatrix}7 & 3 & -8\\ -3 & -2 & 5\\ 0 & 4 & -1\end{bmatrix} R_1-2R_2$$

$$\begin{array}{r}\begin{bmatrix}-3 & -2 & 5\end{bmatrix}\\ 3\begin{bmatrix}7 & 3 & -8\end{bmatrix}\\ \begin{bmatrix}21 & 9 & -24\end{bmatrix}\\ \hline \begin{bmatrix}18 & 7 & -19\end{bmatrix}\end{array}$$

$$\underset{\sim}{R}\begin{bmatrix}7 & 3 & -8\\ 18 & 7 & -19\\ 0 & 4 & -1\end{bmatrix} R_2+3R_1$$

$$\underset{\sim}{R}\begin{bmatrix}7 & 3 & -8\\ 0 & 4 & -1\\ 18 & 7 & -19\end{bmatrix} R_{23}$$

$$\begin{array}{r}2\begin{bmatrix}7 & 3 & 8\end{bmatrix}\\ \begin{bmatrix}-18 & -7 & 19\end{bmatrix}\\ \begin{bmatrix}14 & 6 & 16\end{bmatrix}\\ \hline -4 \quad -1 \quad 35\end{array}$$

$$\underset{\sim}{R}\begin{bmatrix}-4 & -1 & 35\\ 0 & 4 & -1\\ 18 & 7 & -19\end{bmatrix} 2R_1-R_3$$

(3) $\begin{bmatrix} 1 & 2 & 3 & 7 \\ 4 & 5 & 6 & 8 \\ 9 & 1 & 2 & 3 \end{bmatrix}$

Use operation:

(1) $R_1 - 2R_2$

$$\begin{array}{r} \begin{bmatrix} 1 & 2 & 3 & 7 \end{bmatrix} \\ 2\begin{bmatrix} 4 & 5 & 6 & 8 \end{bmatrix} \\ \begin{bmatrix} -8 & -10 & -12 & -16 \end{bmatrix} \\ \hline \begin{bmatrix} -7 & -8 & -9 & -9 \end{bmatrix} \end{array}$$

$\underset{\sim}{R} \begin{bmatrix} -7 & -8 & -9 & -9 \\ 4 & 5 & 6 & 8 \\ 9 & 1 & 2 & 3 \end{bmatrix}$

Echeleon Form Rules:

(1) Zero rows are at the bottom.

It means, row with all zeros must be at the bottom.

(2) First non-zero number in each row should be move to the more then the first non-zero number in the above row.

(3) In each row all numbers before the leading (non-zero number) entry must be zero.

(4) In each column with a leading entry everything below must be zero.

Reduce Echeleon Form Rule:

(1) It must already be in echelon form.

(2) The first non-zero number (leading entry) must be 1

(3) Every other entery in column of leading entry (above and below) must be zero.

Examples:

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Echeleon, Reduced Echeleon

$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Echeleon

$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

Not- Echeleon

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

Echeleon

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Echeleon, Reduced Echeleon

$$\begin{bmatrix} 1 & -2 & 3/2 \\ 0 & 1 & 2/9 \\ 0 & 0 & 1 \end{bmatrix}$$

Echeleon

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Echeleon, Reduced Echeleon

$$\begin{bmatrix} 1 \end{bmatrix}$$

Echeleon, Reduced Echeleon

$$\begin{bmatrix} 3 \end{bmatrix}$$

Echeleon

M.CQs

(1) Which one is echeleon form

(a) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \end{bmatrix}$ (d) All ✓

Q.No 1: First reduce each matrix into echeleon and then into reduced echeleon.

(i)
$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 3 & 5 \\ 0 & 26 & 33 \\ -4 & 6 & 5 \end{bmatrix} \quad R_2+6R_1$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 3 & 5 \\ 0 & 26 & 33 \\ 0 & 18 & 25 \end{bmatrix} \quad R_3+4R_1$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 33/26 \\ 0 & 18 & 25 \end{bmatrix} \quad \frac{1}{26}R_2$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 33/26 \\ 0 & 0 & 28/13 \end{bmatrix} \quad R_3-18R_2$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 33/26 \\ 0 & 0 & 1 \end{bmatrix} \quad \frac{13}{28}R_3$$

Echelon Form

$$\overset{R}{\sim}\begin{bmatrix} 1 & 0 & 31/26 \\ 0 & 1 & 33/26 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1-\frac{31}{26}R_3 \quad R_2-\frac{33}{26}R_3$$

Reduced Echeleon

(2) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 9 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1/2 \\ 3 & 2 \\ 1 & 9 \end{bmatrix} \quad \frac{1}{2}R_1$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1/2 \\ 0 & 1/2 \\ 0 & 17/2 \end{bmatrix} \quad \begin{matrix} R_2 - 3R_1 \\ R_3 - R_1 \end{matrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \\ 0 & 17/2 \end{bmatrix} \quad 2R_2$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad R_3 - \frac{17}{2}R_2$

Echeleon Form

$\underset{\sim}{R} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad R_1 - \frac{1}{2}R_2$

Reduced Echeleon Form

Q no 2 part (6)

(3) $\begin{bmatrix} 2 & -1 & 0 \\ 4 & 7 & 8 \\ -3 & 1 & 3 \end{bmatrix}$

$\underset{R}{\sim} \begin{bmatrix} 1 & -1/2 & 0 \\ 4 & 7 & 8 \\ -3 & 1 & 3 \end{bmatrix} \quad \frac{1}{2}R_1$

$\underset{R}{\sim} \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 9 & 8 \\ 0 & -1/2 & 3 \end{bmatrix} \quad \begin{matrix} R_2 - 4R_1 \\ R_3 + 3R_1 \end{matrix}$

$\underset{R}{\sim} \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1 & 8/9 \\ 0 & -1/2 & 3 \end{bmatrix} \quad \frac{1}{9}R_2$

$\underset{R}{\sim} \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1 & 8/9 \\ 0 & 0 & 31/9 \end{bmatrix} \quad R_3 + \frac{1}{2}R_2$

$\underset{R}{\sim} \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1 & 8/9 \\ 0 & 0 & 1 \end{bmatrix} \quad 9/31\, R_3$

Echeleon Form

$\underset{R}{\sim} \begin{bmatrix} 1 & 0 & 4/9 \\ 0 & 1 & 8/9 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1 + \frac{1}{2}R_2$

$\underset{R}{\sim} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} R_1 - \frac{4}{9}R_3 \\ R_2 - \frac{8}{9}R_3 \end{matrix}$

Reduced Echeleon

(4) $\begin{bmatrix} 2 & -4 & 3 \\ 4 & 1 & 8 \\ 7 & 3 & 0 \end{bmatrix}$

$\underset{R}{\sim} \begin{bmatrix} 1 & -2 & 3/2 \\ 4 & 1 & 8 \\ 7 & 3 & 0 \end{bmatrix} \quad \frac{1}{2}R_1$

$\underset{R}{\sim} \begin{bmatrix} 1 & -2 & 3/2 \\ 0 & 9 & 2 \\ 0 & 17 & -21/2 \end{bmatrix} \quad \begin{matrix} R_2 - 4R_1 \\ R_3 - 7R_1 \end{matrix}$

$\underset{R}{\sim} \begin{bmatrix} 1 & -2 & 3/2 \\ 0 & 1 & 2/9 \\ 0 & 17 & -21/2 \end{bmatrix} \quad \frac{1}{9}R_2$

$\underset{R}{\sim} \begin{bmatrix} 1 & -2 & 3/2 \\ 0 & 1 & 2/9 \\ 0 & 0 & -257/18 \end{bmatrix} \quad R_3 - 17R_2$

$\underset{R}{\sim} \begin{bmatrix} 1 & -2 & 3/2 \\ 0 & 1 & 2/9 \\ 0 & 0 & 1 \end{bmatrix} \quad -\frac{18}{257}R_3$ **Echelcon**

$\underset{R}{\sim} \begin{bmatrix} 1 & 0 & 35/18 \\ 0 & 1 & 2/9 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1 + 2R_2$

$\underset{R}{\sim} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} R_1 - \frac{35}{18}R_3 \\ R_2 - \frac{2}{9}R_3 \end{matrix}$

(5) $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 9 & 8 \end{bmatrix}$

$\underset{R}{\sim} \begin{bmatrix} 1 & 1/3 & 2/3 \\ 2 & 9 & 8 \end{bmatrix} \quad \frac{1}{3}R_1$

$\underset{R}{\sim} \begin{bmatrix} 1 & 1/3 & 2/3 \\ 0 & 25/3 & 20/3 \end{bmatrix} \quad R_2 - 2R_1$

$\underset{R}{\sim} \begin{bmatrix} 1 & 1/3 & 2/3 \\ 0 & 1 & 4/5 \end{bmatrix} \quad \frac{3}{25}R_2$

Echeleon Form

$\underset{R}{\sim} \begin{bmatrix} 1 & 0 & 2/5 \\ 0 & 1 & 4/5 \end{bmatrix} \quad R_1 - \frac{1}{3}R_2$ Reduced Echeleon

(6) $\begin{bmatrix} 0 & 2 & 4 \\ 0 & 3 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

$\underset{R}{\sim} \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 6 \\ 0 & 1 & 2 \end{bmatrix} \quad \frac{1}{2}R_1$

$\underset{R}{\sim} \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_2 - 3R_1 \\ R_3 - R_1 \end{matrix}$

$\underset{R}{\sim} \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Echeleon, Reduced Echeleon

Q No. 2: Find the rank of following:

(1) $\begin{bmatrix} 5 & 9 & 3 \\ 3 & -5 & 6 \\ 2 & 10 & 6 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 5 & 9 & 3 \\ 1 & -15 & 0 \\ 2 & 10 & 6 \end{bmatrix} \quad R_2 - R_3$

$\underset{\sim}{R} \begin{bmatrix} 1 & -15 & 0 \\ 5 & 9 & 3 \\ 2 & 10 & 6 \end{bmatrix} \quad R_{12}$

$\underset{\sim}{R} \begin{bmatrix} 1 & -15 & 0 \\ 0 & 84 & 3 \\ 0 & 40 & 6 \end{bmatrix} \quad \begin{matrix} R_2 - 5R_1 \\ R_3 - 2R_1 \end{matrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & -15 & 0 \\ 0 & 1 & 1/28 \\ 0 & 40 & 6 \end{bmatrix} \quad \frac{1}{84} R_2$

$\underset{\sim}{R} \begin{bmatrix} 1 & -15 & 0 \\ 0 & 1 & 1/28 \\ 0 & 0 & 32/7 \end{bmatrix} \quad R_3 - 40R_2$

$\underset{\sim}{R} \begin{bmatrix} 1 & -15 & 0 \\ 0 & 1 & 1/28 \\ 0 & 0 & 1 \end{bmatrix} \quad \frac{1}{32} R_3$

Echelon

Rank: 3

(2) $\begin{bmatrix} -1 & -2 & 3 \\ -1 & 2 & -1 \\ -5 & 2 & 3 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 2 & -3 \\ -1 & 2 & -1 \\ -5 & 2 & 3 \end{bmatrix}$ $-R_1$

$\underset{\sim}{R} \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & -4 \\ 0 & 12 & -12 \end{bmatrix}$ R_2+R_1 R_3+5R_1

$\underset{\sim}{R} \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 12 & -12 \end{bmatrix}$ $\frac{1}{4}R_2$

$\underset{\sim}{R} \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ R_3-12R_2

Echeleon Form.

Rank: 2

(3) $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 6 \\ 4 & -1 & 0 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1 & -2 \\ 2 & +1 & 6 \\ 4 & -1 & 0 \end{bmatrix}$ R_1-R_2

$\underset{\sim}{R} \begin{bmatrix} 1 & 1 & -2 \\ 0 & -1 & 10 \\ 0 & -5 & 8 \end{bmatrix}$ R_2-2R_1 R_3-4R_1

$$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -10 \\ 0 & -5 & 8 \end{bmatrix} \quad -R_2$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -10 \\ 0 & 0 & -42 \end{bmatrix} \quad R_3+5R_2$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix} \quad -\frac{1}{42}R_3$$

Rank: 3

(4) $\begin{bmatrix} 1 & 3 \\ 2 & 9 \\ 1 & 6 \end{bmatrix}$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 3 \\ 0 & 3 \\ 0 & 3 \end{bmatrix} \quad \begin{matrix} R_2-2R_1 \\ R_3-R_1 \end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 3 \end{bmatrix} \quad \frac{1}{3}R_1$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad R_3-3R_2$$

Rank: 2

* If we have to find inverse of a matrix using row operation First we have to convert it into reduced echeleon form.

* "First we have to find determinant to check whether inverse exisists or not."

Q3: With help of row operation find inverse of a matrix if it exists. Also verify $AA^{-1} = A^{-1}A = I$

(1) $\begin{bmatrix} 5 & 2 & 3 \\ -1 & -2 & 3 \\ 1 & -2 & 3 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & -2 & 3 \\ -1 & -2 & 3 \\ -5 & 2 & 3 \end{bmatrix} R_{13}$

$\underset{\sim}{R} \begin{bmatrix} 1 & -2 & 3 \\ 0 & -4 & 6 \\ 0 & -8 & 18 \end{bmatrix} \begin{matrix} R_2+R_1 \\ R_3+5R_1 \end{matrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -3/2 \\ 0 & -8 & 18 \end{bmatrix} -\frac{1}{4}R_2$

$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/2 \\ 0 & 0 & 6 \end{bmatrix} \begin{matrix} R_1+2R_2 \\ R_3+8R_2 \end{matrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/2 \\ 0 & 0 & 1 \end{bmatrix} \frac{1}{6}R_3$

$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R_2+\frac{3}{2}R_3$

Consider, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Apply all operations on identity matrix to get A^{-1}

$\underset{\sim}{R} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} R_{13}$

$\underset{\sim}{R} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 5 \end{bmatrix} \begin{matrix} R_2+R_1 \\ R_3+5R_1 \end{matrix}$

$\underset{\sim}{R} \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1/4 & -1/4 \\ 1 & 0 & 5 \end{bmatrix} -\frac{1}{4}R_2$

$\underset{\sim}{R} \begin{bmatrix} 0 & -1/2 & 1/2 \\ 0 & -1/4 & -1/4 \\ 1 & -2 & 3 \end{bmatrix} \begin{matrix} R_1+2R_2 \\ R_3+8R_2 \end{matrix}$

$\underset{\sim}{R} \begin{bmatrix} 0 & -1/2 & 1/2 \\ 0 & -1/4 & -1/4 \\ 1/6 & -\frac{1}{6} & 1/2 \end{bmatrix} \frac{1}{6}R_3$

$\underset{\sim}{R} \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1/4 & -3/4 & 1/2 \\ 1/6 & -1/3 & 1/2 \end{bmatrix} R_2+3R_3$

A^{-1}

$$A^{-1} = \frac{1}{12}\begin{bmatrix} 0 & -6 & 6 \\ 3 & -9 & 6 \\ 2 & -4 & 6 \end{bmatrix}$$

2	2, 4, 6, 3
2	1, 2, 3, 3
3	1, 1, 3, 3
	1, 1, 1, 1

$$AA^{-1} : \begin{bmatrix} -5 & 2 & 3 \\ -1 & -2 & 3 \\ 1 & -2 & 3 \end{bmatrix} \frac{1}{12}\begin{bmatrix} 0 & -6 & 6 \\ 3 & -9 & 6 \\ 2 & -4 & 6 \end{bmatrix}$$

$$= \frac{1}{12}\begin{bmatrix} 0+6+6 & 30-18-12 & -30+12+18 \\ 0-6+6 & 6+18-12 & -6-12+18 \\ 0-6+6 & -6+18-12 & 6-12+18 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \rightarrow (1)$$

$$A^{-1}A : \frac{1}{12}\begin{bmatrix} 0 & -6 & 6 \\ 3 & -9 & 6 \\ 2 & -4 & 6 \end{bmatrix}\begin{bmatrix} -5 & 2 & 3 \\ -1 & -2 & 3 \\ 1 & -2 & 3 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \rightarrow (2)$$

Proved, (1), (2) $\therefore AA^{-1} = A^{-1}A = I$

(1) $$\begin{bmatrix} 0 & -1 & -1 \\ -1 & 3 & 0 \\ 1 & -1 & 4 \end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & -1 & 4 \\ -1 & 3 & 0 \\ 0 & -1 & -1 \end{bmatrix} \quad R_{13}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & -1 & 4 \\ 0 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix} \quad R_2+R_1$$

$$\sim \begin{bmatrix}1&-1&4\\0&1&2\\0&-1&-1\end{bmatrix} \frac{1}{2}R_2 \qquad \sim \begin{bmatrix}0&0&1\\0&1&1\\1&0&0\end{bmatrix} R_2+R_1$$

$$\sim \begin{bmatrix}1&0&6\\0&1&2\\0&0&1\end{bmatrix} \begin{matrix}R_1+R_2\\R_3+R_2\end{matrix} \qquad \sim \begin{bmatrix}0&0&0\\0&1/2&1/2\\1&0&0\end{bmatrix} \frac{1}{2}R_2$$

$$\sim \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \begin{matrix}R_2-2R_3\\R_1-6R_3\end{matrix} \qquad \sim \begin{bmatrix}0&1/2&3/2\\0&1/2&1/2\\1&1/2&1/2\end{bmatrix} \begin{matrix}R_1+R_2\\R_3+R_2\end{matrix}$$

Consider,

$$A = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \qquad \sim \begin{bmatrix}-6&-5/2&-3/2\\2&-1/2&-1/2\\1&1/2&1/2\end{bmatrix} \begin{matrix}R_2-2R_3\\R_1-6R_3\end{matrix}$$

Apply all operations on identity matrix to get A^{-1}

$$\sim \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix} R_{13}$$

A^{-1}

$$A^{-1} = \frac{1}{2}\begin{bmatrix}-12&-5&-3\\-4&-1&-1\\2&1&1\end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix}0&-1&-1\\1&3&0\\1&-1&4\end{bmatrix} \frac{1}{2}\begin{bmatrix}-12&-5&-3\\-4&-1&-1\\2&1&1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = I \rightarrow (1)$$

$$A^{-1}A = \frac{1}{2}\begin{bmatrix}-12&-5&-3\\-4&-1&-1\\2&1&1\end{bmatrix}\begin{bmatrix}0&-1&-1\\1&3&0\\1&-1&4\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = I \rightarrow (2)$$

From (1) and (2), proved that $AA^{-1} = A^{-1}A = I$

(2) $\begin{bmatrix} 1 & 2 & 5 \\ -3 & 0 & 1 \\ 4 & 2 & 5 \end{bmatrix}$ Sol. with second method.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 & 1 & 0 \\ 4 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\overset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 6 & 16 & 3 & 1 & 0 \\ 0 & -6 & -15 & -4 & 0 & 1 \end{array}\right] \begin{matrix} R_2+3R_1 \\ R_3-4R_1 \end{matrix}$$

$$\overset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 1 & 8/3 & 1/2 & 1/6 & 0 \\ 0 & -6 & -15 & -4 & 0 & 1 \end{array}\right] \frac{1}{6}R_2$$

$$\overset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 0 & -1/3 & 0 & -1/3 & 0 \\ 0 & 1 & 8/3 & 1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{matrix} R_1-2R_2 \\ R_3+6R_2 \end{matrix}$$

$$\overset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 0 & 1/3 \\ 0 & 1 & 0 & 19/6 & -5/2 & -8/3 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{matrix} R_1+\frac{1}{3}R_3 \\ R_2-\frac{8}{3}R_3 \end{matrix}$$

$$A^{-1} = \begin{bmatrix} -1/3 & 0 & 1/3 \\ 19/6 & -5/2 & -8/3 \\ -1 & 1 & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -2 & 0 & 2 \\ 19 & -15 & -16 \\ -6 & 6 & 6 \end{bmatrix}$$

$$AA^{-1} = \frac{1}{6}\begin{bmatrix} +1 & 2 & 5 \\ -3 & 0 & 1 \\ 4 & 2 & 5 \end{bmatrix}\begin{bmatrix} -2 & 0 & 2 \\ 19 & -15 & -16 \\ -6 & 6 & 6 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \rightarrow (1)$$

$A^{-1}A:$

$$\frac{1}{6}\begin{bmatrix} -2 & 0 & 2 \\ 19 & -15 & -16 \\ -6 & 6 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 5 \\ -3 & 0 & 1 \\ 4 & 2 & 5 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \rightarrow (2)$$

From (1), (2) it is proved: $AA^{-1} = A^{-1}A = I$

(4) $\begin{bmatrix} 0 & 1 & 3 \\ 3 & 2 & 4 \\ 6 & -1 & 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 3 & 1 & 0 & 0 \\ 3 & 2 & 4 & 0 & 1 & 0 \\ 6 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 0 & 1 & 3 & 1 & 0 & 0 \\ 1 & 2/3 & 4/3 & 0 & 1/3 & 0 \\ 6 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \quad \frac{1}{3}R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 2/3 & 4/3 & 0 & 1/3 & 0 \\ 0 & 1 & 3 & 1 & 0 & 0 \\ 6 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \quad R_{12}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 2/3 & 4/3 & 0 & 1/3 & 0 \\ 0 & 1 & 5 & 1 & 0 & 0 \\ 0 & -5 & -6 & 0 & -2 & 1 \end{array}\right] \quad R_3 - 6R_1$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 0 & -2/3 & -2/3 & 1/3 & 0 \\ 0 & 1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 9 & 5 & -2 & 1 \end{array}\right] \quad \begin{matrix} R_1 - \frac{2}{3}R_2 \\ R_3 + 5R_2 \end{matrix}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 0 & -2/3 & -2/3 & 1/3 & 0 \\ 0 & 1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 9 & 5/9 & -2/9 & 1/9 \end{array}\right] \quad \frac{1}{9}R_3$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -8/27 & 5/27 & 2/27 \\ 0 & 1 & 0 & -2/3 & 2/3 & -1/3 \\ 0 & 0 & 1 & 5/9 & -2/9 & 1/9 \end{array}\right] \quad \begin{array}{l} R_1 + \frac{2}{3}R_3 \\ R_2 - 3R_2 \end{array}$$

$$A^{-1} = \begin{bmatrix} -8/27 & 5/27 & 2/27 \\ -2/3 & 2/3 & -1/3 \\ 5/9 & -2/9 & 1/9 \end{bmatrix} = \frac{1}{27}\begin{bmatrix} -8 & 5 & 2 \\ -18 & 18 & -9 \\ 15 & -6 & 3 \end{bmatrix}$$

3	27, 3, 9
3	9, 1, 3
3	3, 1, 1
	1, 1, 1

$$AA^{-1} = \frac{1}{27}\begin{bmatrix} 0 & 1 & 3 \\ 3 & 2 & 4 \\ 6 & -1 & 2 \end{bmatrix}\begin{bmatrix} -8 & 5 & 2 \\ -18 & 18 & -9 \\ 15 & -6 & 3 \end{bmatrix}$$

$$= \frac{1}{27}\begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \rightarrow (1)$$

$$A^{-1}A = \frac{1}{27}\begin{bmatrix} -8 & 5 & 2 \\ -18 & 18 & -9 \\ 15 & -6 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 3 \\ 3 & 2 & 4 \\ 6 & -1 & 2 \end{bmatrix}$$

$$= \frac{1}{27}\begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \rightarrow (2)$$

From eq (1), (2) it is proved : $AA^{-1} = A^{-1}A = I$