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Where Minds Pollinate

Class 11
Mathematics

Chapter 2
Exercise 2.4
Notes

National Book Foundation

Exercise : 2.4

Properties of Determinants:

(1) For any square matrix A

$|A| = |A^t|$

(2) If any two rows (or) any two columns of squ... matrix are interchange then resulting matrix is

$|B| = -|A|$

e.g: $A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \quad R_2 \longleftrightarrow R_1 \quad - \begin{vmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{vmatrix}$

2 time interchange (+)

(3) If any two rows/two columns are identical/sa... then the value of determinant is zero.

e.g: $A = \begin{vmatrix} 2 & 1 & 6 \\ 3 & 2 & 0 \\ 3 & 2 & 0 \end{vmatrix} \quad |A| = \begin{vmatrix} \; \end{vmatrix} = 0$

R_2, R_3 is identical so $|A| = 0$

(4) If we multiply each element of a row/column with non-zero scalar number (K) then resulting matrix is

e.g: $A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

let multiply k of row 1, then the result is:

$\begin{vmatrix} Ka_{11} & Ka_{12} \\ a_{21} & a_{22} \end{vmatrix} = K|A|$

6

2(3)

$\downarrow$

K

(5) If all element of row/column of square matrix A are zero then $|A| = 0$

e.g.: $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{bmatrix}$

If $|A| = 0$ (of any matrix)

$= 0$ inverse not exists.

(6)

QNo.1: Without expansion show that:

(1)
$$\begin{vmatrix} 9 & 27 & 36 \\ 18 & 54 & 24 \\ 27 & 81 & 28 \end{vmatrix} = 0$$

L.H.S:

Taking (9) common from C_1:

$$= 9\begin{vmatrix} 1 & 27 & 36 \\ 2 & 54 & 24 \\ 3 & 81 & 28 \end{vmatrix}$$

Taking (27) common from C_2:

$$= 9\times 27\begin{vmatrix} 1 & 1 & 36 \\ 2 & 2 & 24 \\ 3 & 3 & 28 \end{vmatrix}$$

If any two columns are identical then $|A| = 0$

C_1 and C_2 are identical so, $|A| = 0$

$= 9\times 27\,(0)$

$= 0$

R.H.S: 0 Hence Proved

(2)
$$\begin{vmatrix} 1/a & bc & b+c \\ 1/b & ac & a+c \\ 1/c & ab & a+b \end{vmatrix} = 0$$

L.H.S:

Multiply column 1 by (abc)

$$= \frac{1}{abc}\begin{vmatrix} 1/a & bc & b+c \\ 1/b & ac & a+c \\ 1/c & ab & a+b \end{vmatrix}$$

$\frac{1}{a}\times\frac{a\times bc}{abc}$

$\frac{1}{b}\times\frac{abc}{abc}$

$\frac{1}{c}\times\frac{abc}{abc}$

$$= \frac{1}{abc}\begin{vmatrix} bc & bc & b+c \\ ac & ac & a+c \\ ab & ab & a+b \end{vmatrix}$$

If any two columns are identical then $|A| = 0$

$$= \frac{1}{abc}(0)$$

R.H.S $= 0$

$$= 0$$

Hence Proved

(3) $$\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

L.H.S:

Multiply C-1 by (c)

Multiply C-2 by (-b)

$$= \frac{1}{abc}\begin{vmatrix} 0 & -a & -b \\ ac & 0 & -c \\ bc & c & 0 \end{vmatrix} = \frac{1}{abc}(-1)\begin{vmatrix} 0 & +ab & -b \\ ac & 0 & -c \\ bc & -bc & 0 \end{vmatrix}$$

$C_1 + (C_2 + C_3)$

Multiply C-3 by (a)

(Combine these 3 steps)

$$= \frac{1}{abc}(-1)\begin{vmatrix} 0 & +ab & -ab \\ ac & 0 & -ac \\ bc & -bc & 0 \end{vmatrix} = \begin{vmatrix} 0+ab-ab & ab & -b \\ \cancel{ac}+0-\cancel{ac} & 0 & -ac \\ \cancel{bc}-\cancel{bc}+0 & -bc & 0 \end{vmatrix} = \frac{-1}{abc}$$

$$= \frac{-1}{abc}\begin{vmatrix} 0 & ab & -ab \\ 0 & 0 & -ac \\ 0 & -bc & 0 \end{vmatrix}$$

If all elements of a column are 0 the $|A| = 0$

$$= -\frac{1}{abc}(0)$$

$$= 0$$

R.H.S:

0

Hence Proved.

(4) $\begin{vmatrix} \sin^2\alpha & 1 & \cos^2\alpha \\ \tan^2\alpha & \sec^2\alpha & 1 \\ -\csc^2\alpha & -\cot^2\alpha & 1 \end{vmatrix} = 0$

$\sin^2\alpha + \cos^2\alpha = 1$
$1 + \tan^2\alpha = \sec^2\alpha$
$1 + \cot^2\alpha = \csc^2\alpha$
$1 - \csc^2\theta = \cot^2\theta$

L·H·S:

$= \begin{vmatrix} \sin^2\alpha + \cos^2\alpha & 1 & \cos^2\alpha \\ 1 + \tan^2\alpha & \sec^2\alpha & 1 \\ 1 - \csc^2\alpha & -\cot^2\alpha & 1 \end{vmatrix}$ $C_1 + C_3$

$= \begin{vmatrix} 1 & 1 & \cos^2\alpha \\ \sec^2\alpha & \sec^2\alpha & 1 \\ -\cot^2\alpha & -\cot^2\alpha & 1 \end{vmatrix}$ If two columns are same than $|A| = 0$

$= 0$

R·H·S:

0

Hence Proved.

(5) $\begin{vmatrix} (a-b)^3 & a^3-b^3 & ab(a-b) \\ (c-d)^3 & c^3-d^3 & cd(c-d) \\ (e-f)^3 & e^3-f^3 & ef(e-f) \end{vmatrix} = 0$

$(a-b)^3 = a^3 - b^3 - 3a$ ... $(a-b)$

L·H·S:

Multiply C3 by (3)

$= \begin{vmatrix} (a-b)^3 & a^3-b^3 & 3ab(a-b) \\ (c-d)^3 & c^3-d^3 & 3cd(c-d) \\ (e-f)^3 & e^3-f^3 & 3ef(e-f) \end{vmatrix}$

$= \frac{1}{3}\begin{vmatrix} (a-b)^3 & a^3-b^3 & 3ab(a-b) \\ (c-d)^3 & c^3-d^3 & 3cd(c-d) \\ (e-f)^3 & e^3-f^3 & 3ef(e-f) \end{vmatrix}$

ubtracting C_3 From C_2:

$$\frac{1}{3}\begin{vmatrix} (a-b)^3 & a^3-b^3-3ab(a-b) & 3ab(a-b) \\ (c-d)^3 & c^3-d^3-3cd(c-d) & 3cd(c-d) \\ (e-f)^3 & e^3-f^3-3ef(e-f) & 3ef(e-f) \end{vmatrix}$$

$$\frac{1}{3}\begin{vmatrix} (a-b)^3 & (a-b)^3 & 3ab(a-b) \\ (c-d)^3 & (c-d)^3 & 3cd(c-d) \\ (e-f)^3 & (e-f)^3 & 3ef(e-f) \end{vmatrix}$$ If 2 Columns are same then $|A|=0$

$\frac{1}{3}(0)$

0

.H.S:

0

Hence proved.

)
$$\begin{vmatrix} x & -z & 0 \\ 0 & y & -x \\ -y & 0 & z \end{vmatrix} = 0$$

.H.S:

Multiply C_1 by Z, C_2 by X, C_3 by Y

$$\frac{1}{xyz}\begin{vmatrix} xz & -xz & 0 \\ 0 & xy & -xy \\ -yz & 0 & yz \end{vmatrix}$$

Adding:- $C_1 + (C_2 + C_3)$

$$\frac{1}{xyz}\begin{vmatrix} xz-xz+0 & -xz & 0 \\ 0+xy-xy & xy & -xy \\ -yz+0+yz & 0 & yz \end{vmatrix} : \frac{1}{xyz}\begin{vmatrix} 0 & -xz & 0 \\ 0 & xy & -xy \\ 0 & 0 & yz \end{vmatrix}$$

f any column = 0 then $|A|=0$.

$\frac{1}{xyz}(0) = 0$

R.H.S:

0

Hence Proved.

(7) $\begin{vmatrix} (a-b)^2 & (a+b)^2 & ab \\ (c-d)^2 & (c+d)^2 & cd \\ (e-f)^2 & (e+f)^2 & ef \end{vmatrix} = 0$

L.H.S:

$$= \begin{vmatrix} a^2+b^2-2ab & a^2+b^2+2ab & ab \\ c^2+d^2-2cd & c^2+d^2+2cd & cd \\ e^2+f^2-2ef & e^2+f^2+2ef & ef \end{vmatrix}$$

Subtracting $C_2 - C_1$

$$= \begin{vmatrix} a^2+b^2-2ab & \cancel{a^2}+\cancel{b^2}+2ab-\cancel{a^2}-\cancel{b^2}+2ab & ab \\ c^2+d^2-2cd & \cancel{c^2}+\cancel{d^2}+2cd-\cancel{c^2}-\cancel{d^2}+2cd & cd \\ e^2+f^2-2ef & \cancel{e^2}+\cancel{f^2}+2ef-\cancel{e^2}-\cancel{f^2}+2ef & ef \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2-2ab & 4ab & ab \\ c^2+d^2-2cd & 4cd & cd \\ e^2+f^2-2ef & 4ef & ef \end{vmatrix}$$

Taking 4 common from C_2:

$$= 4\begin{vmatrix} a^2+b^2-2ab & ab & ab \\ c^2+d^2-2cd & cd & cd \\ e^2+f^2-2ef & ef & ef \end{vmatrix}$$

If two columns are same then $|A| = 0$

$= 4(0)$

$= 0$

R.H.S:

$= 0$

Hence Proved.

QNo.2: Using properties of determinant solve following:

(i)
$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2(x^3+y^3)$$

L.H.S:

Adding $C_1+(C_2+C_3)$

$$= \begin{vmatrix} x+y+x+y & y & x+y \\ y+x+y+x & x+y & x \\ x+y+x+y & x & y \end{vmatrix} = \begin{vmatrix} 2(x+y) & y & x+y \\ 2(x+y) & x+y & x \\ 2(x+y) & x & y \end{vmatrix}$$

Taking $2(x+y)$ common:

$$= 2(x+y)\begin{vmatrix} 1 & y & x+y \\ 1 & x+y & x \\ 1 & x & y \end{vmatrix}$$

Subtract R_2-R_1, R_3-R_1:

$$= 2(x+y)\begin{vmatrix} 1 & y & x+y \\ 0 & x & -y \\ 0 & x-y & -x \end{vmatrix}$$

Expand by C-1

$$= 2(x+y)1\begin{vmatrix} x & -y \\ x-y & -x \end{vmatrix} - 0 + 0$$

$$= 2(x+y)\left[-x^2+y(x-y)\right]$$

$$= 2(x+y)(-x^2+xy-y^2)$$

$$= -2(x+y)(x^2-xy+y^2)$$

$$= -2(x^3+y^3)$$

R.H.S:

$$= -2(x^3+y^3)$$

Hence proved.

(2) $\begin{vmatrix} a & b-c & b+c \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

L.H.S:

Multiply $C1$ by (a), C_2 by (b), C_3 by (c)

$$= \frac{1}{abc}\begin{vmatrix} a^2 & b^2-bc & bc+c^2 \\ a^2+ac & b^2 & c^2-ac \\ a^2-ab & ab+b^2 & c^2 \end{vmatrix}$$

Adding: $C_1+(C_2+C_3)$

$$= \frac{1}{abc}\begin{vmatrix} a^2+b^2-\cancel{bc}+\cancel{bc}+c^2 & b^2-bc & bc+c^2 \\ a^2+\cancel{ac}+b^2+c^2-\cancel{ac} & b^2 & c^2-ac \\ a^2-\cancel{ab}+\cancel{ab}+b^2+c^2 & ab+b^2 & c^2 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^2+b^2+c^2 & b^2-bc & bc+c^2 \\ a^2+b^2+c^2 & b^2 & c^2-ac \\ a^2+b^2+c^2 & ab+b^2 & c^2 \end{vmatrix}$$ Take $(a^2+b^2+c^2)$ common:

$$= \frac{a^2+b^2+c^2}{abc}\begin{vmatrix} 1 & b^2-bc & bc+c^2 \\ 1 & b^2 & c^2-ac \\ 1 & ab+b^2 & c^2 \end{vmatrix}$$ Subtract, R_3-R_2, R_2-R_1

$$= \frac{a^2+b^2+c^2}{abc}\begin{vmatrix} 1 & b^2-bc & bc+c^2 \\ 0 & bc & -ab-bc \\ 0 & ab & ac \end{vmatrix}$$ Expanding by C_1

$$= \frac{a^2+b^2+c^2}{abc}\begin{vmatrix} bc & -ac-bc \\ ab & ac \end{vmatrix} - 0 + 0$$ Taking bc common

$$= \frac{a^2+b^2+c^2}{a\cancel{bc}}(\cancel{bc})\begin{vmatrix} c & -a-b \\ a & a \end{vmatrix}$$

$$= \frac{a^2+b^2+c^2}{a}\left(ca - a(-a-b)\right.$$

$= \frac{a^2+b^2+c^2}{a1}\left(ca+a^2+ab\right)$

$= \frac{a^2+b^2+c^2}{\cancel{a}} \times \cancel{a}\left(c+a+b\right) = (a+b+c)(a^2+b^2+c^2)$

R.H.S: $(a+b+c)(a^2+b^2+c^2)$

Hence Proved.

(3) $\begin{vmatrix} na_1+b_1 & na_2+b_2 & na_3+b_3 \\ nb_1+c_1 & nb_2+c_2 & nb_3+c_3 \\ nc_1+a_1 & nc_2+a_2 & nc_3+a_3 \end{vmatrix} = (n^3+1)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

L.H.S.

$= \begin{vmatrix} na_1+b_1+0c_1 & na_2+b_2+0c_2 & na_3+b_3+0c_3 \\ 0a_1+nb_1+c_1 & 0a_2+nb_2+c_2 & 0a_3+nb_3+c_3 \\ a_1+0b_1+nc_1 & a_2+0b_2+nc_2 & a_3+0b_3+nc_3 \end{vmatrix}$

$= \begin{vmatrix} n & 1 & 0 \\ 0 & n & 1 \\ 1 & 0 & n \end{vmatrix} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Expanding by R_1

$= n\begin{vmatrix} n & 1 \\ 0 & n \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 1 & n \end{vmatrix} + 0\begin{vmatrix} 0 & n \\ 1 & 0 \end{vmatrix} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= n(n^2-0) - 1(0-1)$

$= (n^3+1)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \text{R.H.S}$

Hence Proved.

Board Questions:

1) $$\begin{vmatrix} x+y & -y+z & -z-x \\ -x-y & y-z & z+x \\ x-y & z-x & y-z \end{vmatrix} = 0$$

$C_2 + (C_1 + C_3)$

$$= \begin{vmatrix} x+y & -y+z+x+y-z-x & -z-x \\ -x-y & y-z-x-y+z+x & z+x \\ x-y & z-x+x-y+y-z & y-z \end{vmatrix} = \begin{vmatrix} x+y & 0 & -z-x \\ -x-y & 0 & z+x \\ x-y & 0 & y-z \end{vmatrix}$$

If any column is 0 then $|A|=0$

$= 0$

R.H.S $= 0$

Hence Proved.

$C_1 + (C_2 + C_3)$

$$= \begin{vmatrix} x+y-y+z-z-x & -y+z & -z-x \\ -x-y+y-z-z+x & y-z & z+x \\ x-y+z-x+y-z & z-x & y-z \end{vmatrix} = \begin{vmatrix} 0 & -y+z & -z-x \\ 0 & y-z & z+x \\ 0 & z-x & y-z \end{vmatrix} = 0$$

R.H.S $= 0$

Hence proved.

2) $$\begin{vmatrix} b+c & c+a & -a+b \\ c+a & b+c & -b+a \\ a+b & c+b & a-c \end{vmatrix} = 0$$

L.H.S:

$C_3 + C_2$

$$= \begin{vmatrix} b+c & c+a & -\cancel{a}+b+\cancel{a}+c \\ c+a & b+c & -\cancel{b}+a+\cancel{b}+c \\ a+b & c+b & a-\cancel{c}+\cancel{c}+b \end{vmatrix} = \begin{vmatrix} b+c & c+a & b+c \\ c+a & b+c & a+c \\ a+b & c+b & a+b \end{vmatrix}$$

If two columns are same $|A|=0$

$= 0$

R.H.S $= 0$

Hence Proved

$C_1 - C_3$

$$= \begin{vmatrix} b+c+a-b & c+a & -a+b \\ c+a+b-a & b+c & -b+a \\ a+b-a+c & c+b & a-c \end{vmatrix} = \begin{vmatrix} c+a & c+a & -a+b \\ b+c & b+c & -b+a \\ c+b & c+b & a-c \end{vmatrix}$$

If 2 columns are same $|A| = 0$

$= 0$

R.H.S $= 0$ Hence Proved.

$C_1 - (C_2 + C_3)$

$$= \begin{vmatrix} b+c-b-c & c+a & -a+b \\ c+a-c-a & b+c & -b+a \\ a+b-a-b & c+b & a-c \end{vmatrix} = \begin{vmatrix} 0 & c+a & -a+b \\ 0 & b+c & -b+a \\ 0 & c+b & a-c \end{vmatrix}$$

IF any column $= 0$ the $|A| = 0$

$= 0$

R.H.S $= 0$ Hence Proved.

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$$\begin{vmatrix} a+k & a & a \\ a & a+k & a \\ a & a & a+k \end{vmatrix} = (3a+k)k^2$$

L.H.S. $C_1 + (C_2 + C_3)$ Taking $3a+k$ common from C_1

$$= \begin{vmatrix} 3a+k & a & a \\ 3a+k & a+k & a \\ 3a+k & a & a+k \end{vmatrix} = 3a+k \begin{vmatrix} 1 & a & a \\ 1 & a+k & a \\ 1 & a & a+k \end{vmatrix}$$

$$= 3a+k \begin{vmatrix} 1 & a & a \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix} \quad \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$= 3a+k\,(1 \times k \times k)$

$= 3a+k\,(k^2)$

$= (3a+k)k^2 =$ R.H.S

UTM Hence proved.

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

L.H.S: $R_2 - R_1$. $R_3 - R_1$

$$\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$ Expanding by C_1

$$= 1 \begin{vmatrix} b-a & (a+b)(a-b) \\ c-a & (c+a)(c-a) \end{vmatrix}$$

$$= 1(b-a)(c-a) \begin{vmatrix} 1 & a+b \\ 1 & c+a \end{vmatrix}$$
common

$= (b-a)(c-a)(c+\cancel{a}-\cancel{a}-b) = (b-a)(c-a)(c-b)$

$= \cancel{bc-ab-ac+a^2(c-b)}$

$= bc^2$

IMP

(4) $$\begin{vmatrix} x & x^2 & 1+\alpha x^3 \\ y & y^2 & 1+\alpha y^3 \\ z & z^2 & 1+\alpha z^3 \end{vmatrix} = \cancel{(1+\alpha xyz)(x-y)(y-z)(z-x)}$$

L.H.S:

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & \alpha x^3 \\ y & y^2 & \alpha y^3 \\ z & z^2 & \alpha z^3 \end{vmatrix}$$ Interchanging column (-1)

$$= -1 \begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix} + \alpha \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$$ Taking α common from C_3

Interchanging (-1)

$$= (-1)(-1) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \alpha xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$ Take common x from C_1, y from C_2, z from C_3

$$= (1+\alpha xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad R_2 - R_1 \;,\; R_3 - R_1$$

$$= (1+\alpha xyz)\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} \quad \text{Expanding by } C_1$$

$$= (1+\alpha xyz) \times 1 \begin{vmatrix} y-x & y^2-x^2 \\ z-x & z^2-x^2 \end{vmatrix} = (1+\alpha xyz)\begin{vmatrix} y-x & (y+x)(y-x) \\ z-x & (z+x)(z-x) \end{vmatrix}$$

Take $(y-x)$ common from $R1$, $(z-x)$ from $R2$

$$= \cancel{(1+\alpha xyz)(y-x)(z-x)}\begin{vmatrix} 1 & y+x \\ 1 & z+x \end{vmatrix}$$

$$= (1+\alpha xyz)(y-x)(z-x)(z+\cancel{x}-y-\cancel{x})$$ Take $(-)$ common from $(y-x)$ and $(z-y)$ $(-)(-)=+$

$$= (1+\alpha xyz)(x-y)(y-x)(z-x)$$

R.H.S $(1+\alpha xyz)(x-y)(y-x)(z-x)$

Hence proved.

(5) $$\begin{vmatrix} 2ab & 1+a^2-b^2 & 2b \\ 2a & -2b & 1-a^2-b^2 \\ 1-a^2+b^2 & 2ab & -2a \end{vmatrix} = (1+a^2+b^2)^3$$

L.H.S = Multiply $C3$ by (a)

$$= \frac{1}{a}\begin{vmatrix} 2ab & 1+a^2-b^2 & 2ab \\ 2a & -2b & a-a^3-ab^2 \\ 1-a^2+b^2 & 2ab & -2a^2 \end{vmatrix}$$

Subtract C_3 From C_1

$$= \frac{1}{a}\begin{vmatrix} 0 & 1+a^2-b^2 & 2ab \\ a(1+a^2+b^2) & -2b & a-a^3-ab^2 \\ 1+a^2+b^2 & 2ab & -2a^2 \end{vmatrix}$$

Take (a) common from C_3

$$= \frac{a}{a}\begin{vmatrix} 0 & 1+a^2-b^2 & 2b \\ a(1+a^2+b^2) & -2b & 1-a^2-b^2 \\ 1+a^2+b^2 & 2ab & -2a \end{vmatrix}$$

Taking common (1+
from C_1

$$= 1+a^2+b^2\begin{vmatrix} 0 & 1+a^2-b^2 & 2b \\ a & -2b & 1-a^2-b^2 \\ 1 & 2ab & -2a \end{vmatrix}$$

Multiply C_3 by (b)

$$= \frac{(1+a^2+b^2)}{b}\begin{vmatrix} 0 & 1+a^2-b^2 & 2b^2 \\ a & -2b & b-a^2b-b^3 \\ 1 & 2ab & -2ab \end{vmatrix}$$

Add C_2+C_3

$$= \left(\frac{1+a^2+b^2}{b}\right)\begin{vmatrix} 0 & 1+a+b^2 & 2b^2 \\ a & -b(1+a^2+b^2) & b-a^2b-b^3 \\ 1 & 0 & -2ab \end{vmatrix}$$

Take $(1+a^2+b^2)$ common from C_2 and b from C_3

$$= \frac{(1+a^2+b^2)(1+a^2+b^2)b}{b}\begin{vmatrix} 0 & 0 & 2b \\ a & -b & 1-a^2-b^2 \\ 1 & 0 & -2a \end{vmatrix}$$

Multiply R3 by

$$= \frac{(1+a^2+b^2)^2}{a}\begin{vmatrix} 0 & 1 & 2b \\ a & -b & 1-a^2-b^2 \\ a & 0 & -2a^2 \end{vmatrix}$$

$R_2 - R_3$

$$= \frac{(1+a^2+b^2)^2}{a}\begin{vmatrix} 0 & 1 & 2b \\ 0 & -b & 1+a^2-b^2 \\ a & 0 & -2a \end{vmatrix}$$

Taking a common
from R_3

$$= \frac{(1+a^2+b^2)^2 . a}{a}\begin{vmatrix} 0 & 1 & 2b \\ 0 & -b & 1+a^2-b^2 \\ 1 & 0 & -2a \end{vmatrix}$$

Expanding by C_1

Take (a) common from C_3

$$= \frac{a}{a}\begin{vmatrix} 0 & 1+a^2-b^2 & 2b \\ a(1+a^2+b^2) & -2b & 1-a^2-b^2 \\ 1+a^2+b^2 & 2ab & -2a \end{vmatrix}$$

Taking common $(1+a^2+b^2)$ from C_1

$$= 1+a^2+b^2\begin{vmatrix} 0 & 1+a^2-b^2 & 2b \\ a & -2b & 1-a^2-b^2 \\ 1 & 2ab & -2a \end{vmatrix}$$

Multiply C_3 by (b)

$$= \frac{(1+a^2+b^2)}{b}\begin{vmatrix} 0 & 1+a^2-b^2 & 2b^2 \\ a & -2b & b-a^2b-b^3 \\ 1 & 2ab & -2ab \end{vmatrix}$$

Add C_2+C_3

$$= \left(\frac{1+a^2+b^2}{b}\right)\begin{vmatrix} 0 & 1+a+b^2 & 2b^2 \\ a & -b(1+a^2+b^2) & b-a^2b-b^3 \\ 1 & 0 & -2ab \end{vmatrix}$$

Take $(1+a^2+b^2)$ common from C_2 and b from C_3

$$= \frac{(1+a^2+b^2)(1+a^2+b^2)b}{b}\begin{vmatrix} 0 & 0 & 2b \\ a & -b & 1-a^2-b^2 \\ 1 & 0 & -2a \end{vmatrix}$$

Multiply R_3 by a

$$= \left(\frac{1+a^2+b^2}{a}\right)^2\begin{vmatrix} 0 & 1 & 2b \\ a & -b & 1-a^2-b^2 \\ a & 0 & -2a^2 \end{vmatrix}$$

$R_2 - R_3$

$$= \frac{(1+a^2+b^2)^2}{a}\begin{vmatrix} 0 & 1 & 2b \\ 0 & -b & 1+a^2-b^2 \\ a & 0 & -2a \end{vmatrix}$$

Taking a common from R_3

$$= \frac{(1+a^2+b^2)^2 \times a}{a}\begin{vmatrix} 0 & 1 & 2b \\ 0 & -b & 1+a^2-b^2 \\ 1 & 0 & -2a \end{vmatrix}$$

Expanding by C_1

$= (1+a^2+b^2)^2 \quad (0-0+1) \begin{vmatrix} 1 & 2b \\ -b & 1+a^2-b^2 \end{vmatrix}$

$= (1+a^2+b^2)^2 \; 1 \; (1+a^2-b^2+2b^2)$

$= (1+a^2+b^2)^2 \; (1+a^2+b^2) = (1+a^2+b^2)^3 = R.H.S$

Hence Proved.

(6) $\begin{vmatrix} 3a & 1 & 2a+1 \\ 2a+1 & 1 & a+2 \\ 3 & 1 & 2 \end{vmatrix} = (a-1)(a-2)$

L.H.S: $R_2 - R_1, \quad R_3 - R_1$

$= \begin{vmatrix} 3a & 1 & 2a+1 \\ -a+1 & 0 & -a+1 \\ 3-3a & 0 & 1-2a \end{vmatrix}$ Expanding by C_2

$= -1 \begin{vmatrix} -a+1 & -a+1 \\ 3-3a & 1-2a \end{vmatrix}$ ~~to-o~~ $= -1 \underset{\text{common}}{(-a+1)} \begin{vmatrix} 1 & 1 \\ 3-3a & 1-2a \end{vmatrix}$

$= a-1 \, (1-2a-3+3a) = a-1 \, (-2+a) = (a-1)(a-2) = R.H.S$

Hence proved

(7) IMP

$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

L.H.S: $R_1 - R_2 - R_3$

$= \begin{vmatrix} b+c-b-c & a-c-a-c & a-b-a-b \\ b & c+a & \cancel{a+}b \\ a & c & a+b \end{vmatrix} = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Expanding by C_1

$$= 0 - b\begin{vmatrix} -2c & -2b \\ c & a+b \end{vmatrix} + c\begin{vmatrix} -2c & -2b \\ c+a & b \end{vmatrix}$$

$$= -b\left(-2c(a+b) + 2bc\right) + c\left(-2bc + \cancel{2bc} + 2ab\right)$$

$$= -b\left(-2ac - 2bc + 2bc\right) + c\,(2ab)$$

$$= +2abc + 2abc \quad = 4abc \quad = R.H.S$$

Hence Proved

(8) $$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^{3}$$

L.H.S =

Multiply R_1 by a, R_2 by b, R_3 by c

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

Take common a from C_1, b from C_2, c from C_3

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$R_1 + (R_2 + R_3)$

$$= \begin{vmatrix} -\cancel{bc}+ab+\cancel{bc}+ac+bc & ab+\cancel{ac}-\cancel{ac}+bc+ac & ac+ab+bc-ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Taking common $(ab+bc+ca)$ from R_1

$$= (ab+bc+ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$C_2 - C_1$, $C_3 - C_1$

$$= (ab+bc+ca)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -ac-ab-bc & bc+ab-ab-bc \\ ac+bc & bc+ac-ac-bc & -ab-ac-bc \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ac+ab+bc) & 0 \\ ac+bc & 0 & -(ab+ac+bc) \end{vmatrix}$$

Expanding by R_1

$$= (ab+bc+ca)\ 1\begin{vmatrix} -(ab+bc+ac) & 0 \\ 0 & -(ab+bc+ca) \end{vmatrix} - 0 + 0$$

$$= (ab+bc+ca)\,(ab+bc+ca)^2 - 0 = (ab+bc+ca)^3 = R.H.S$$

Hence proved.

(9) $$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

L.H.S: Multiplying a by R_1, b by R_2, c by R_3

$$= \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & a^2b & ca^2 \\ ab^2 & b(a+c)^2 & b^2c \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$$

Taking (abc) common, a from C_1, b from C_2, c from C_3

$$= \frac{abc}{abc}\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

$C_2 - C_1\ ,\ C_3 - C_1$

$$= \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (a+c)^2-b^2 & \cancel{b^2}-\cancel{b^2} \\ c^2 & \cancel{c^2}-\cancel{c^2} & (a+b)^2-c^2 \end{vmatrix} = \begin{vmatrix} (b+c)^2 & (a-b-c)(a+b+c) & (a-b-c)(a+b+c) \\ b^2 & (a+b+c)(a-b+c) & 0 \\ c^2 & 0 & (a+b+c)(a+b-c) \end{vmatrix}$$

Taking $(a+b+c)$ common from C_2 and C_3

$$= (a+b+c)^2 \begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & a-b+c & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$R_1 - R_2 - R_3$

$$= (a+b+c)^2 \begin{vmatrix} \cancel{b^2}+\cancel{c^2}+2bc-\cancel{b^2}-\cancel{c^2} & \cancel{a}-\cancel{b}-c-\cancel{a}+\cancel{b}-c & \cancel{a}-b-\cancel{c}-\cancel{a}-b+\cancel{c} \\ b^2 & a-b+c & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Taking 2 common from R_1

$$= 2(a+b+c)^2 \begin{vmatrix} bc & -c & -b \\ b^2 & a-b+c & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Expand by C_3

$$= 2(a+b+c)^2 \left[-b \begin{vmatrix} b^2 & a-b+c \\ c^2 & 0 \end{vmatrix} - 0 + (a+b-c) \begin{vmatrix} bc & -c \\ b^2 & a-b+c \end{vmatrix} \right]$$

$$= 2(a+b+c)^2 \left[-b(0 - c^2(a-b+c)) + (a+b-c)(abc - \cancel{b^2c} + bc^2 + \cancel{b^2c}) \right]$$

$$= 2(a+b+c)^2 \left(bc^2(a-b+c) + (a+b-c)\, bc(a+c) \right)$$

$$= 2(a+b+c)^2\, bc \left(c(a-b+c) + (a+c)(a+b-c) \right)$$

$$= 2bc(a+b+c)^2 \left(\cancel{ac} - \cancel{bc} + \cancel{c^2} + a^2 + ab - \cancel{ac} + ac + \cancel{bc} - \cancel{c^2} \right)$$

$$= 2bc(a+b+c)^2 (a^2 + ab + ac)$$

$$= 2bc(a+b+c)^2\, a(a+b+c)$$

$$= 2abc(a+b+c)^3$$

R.H.S:

$$= 2abc(a+b+c)^3$$

Hence Proved.

Assignment 2

$$\begin{vmatrix} 1 & a^2 & a \\ a & 1 & a^2 \\ a^2 & a & 1 \end{vmatrix} = (1-a^3)^2 / (1+a+a^2)^2 (1-a)^2$$

L.H.S. $R_1 \to (R_2 + R_3)$

$$= \begin{vmatrix} 1+a+a^2 & 1+a+a^2 & 1+a+a^2 \\ a & 1 & a^2 \\ a^2 & a & 1 \end{vmatrix}$$ Take $(1+a+a^2)$ common from R_1

$$= (1+a+a^2)\begin{vmatrix} 1 & 1 & 1 \\ a & 1 & a^2 \\ a^2 & a & 1 \end{vmatrix}$$ $C_2 - C_1$, $C_3 - C_1$

$$(1+a+a^2)\begin{vmatrix} 1 & 0 & 0 \\ a & 1-a & a^2-a \\ a^2 & a-a^2 & 1-a^2 \end{vmatrix}$$ Expanding by R1

$$= (1+a+a^2)\, 1 \begin{vmatrix} 1-a & a^2-a \\ a-a^2 & 1-a^2 \end{vmatrix}$$

$$= (1+a+a^2)\begin{vmatrix} 1-a & -a(1-a) \\ a(1-a) & (1-a)(1+a) \end{vmatrix} \quad \begin{vmatrix} 1-a & -a(1-a) \\ a(1-a) & (1-a)(1+a) \end{vmatrix}$$

$= (1+a+a^2)\left((1-a)(1-a)(1+a) + a^2(1+a)^2\right)$ Verify

$(1+a+a^2)\left((1-a)^2(1+a) + a^2(1+a)\right)$

$$(1+a+a^2)\begin{vmatrix} 1-a & -a(1-a) \\ a(1-a) & (1-a)(1+a) \end{vmatrix}$$ Take $(1-a)$ common

$$(1+a+a^2)(1-a)(1-a)\begin{vmatrix} 1 & -a \\ a & (1+a) \end{vmatrix} \to (1+a)$$

$(1+a+a^2)$

$[(1+a+a^2)(1-a)][(1-a)(1+a+a^2)] = (1-a^3)^2$ or

$(1+a+a^2)^2(1-a)^2$

Test chapter.2 "1st half"

MCQ's:

(1) If B is a row matrix of order $1\times n$ then ord of B^tB is:

(a) $1\times n$

(b) 1×1

(c) $n\times 1$

(d) $n\times n$ ✓

(2) Which matrix satifies the condition $a_{ij}=0\ \forall\ i>j$

(a) lower Δ matrix

(b) upper Δ matrix ✓

(c) symmetric

(d) Identity.

(3) $A=\begin{bmatrix}4 & 0 & 0\\ 0 & 0 & 0\\ 0 & 5 & 6\end{bmatrix}$ which condition is voilated so this isnt diagonal matrix?

(a) $a_{ij}=0\ \forall\ i\neq j$

(b) $a_{32}=5$ ✓

(c) rectangular matrix

(d) B and C

(4) $E=\begin{bmatrix}0 & 2 & -1\\ -2 & 0 & -4\\ 1 & 4 & 0\end{bmatrix}$ which rule is broken for skew symmetric?

(a) $a^t=-A$

(b) diagonal matrix

(c) identity matrix

✓(d) $a_{32}=4\neq -a_{23}$

When a_{12} and a_{21} have same sign = symmetric

" opposite sign = skew symmetric

(5) $\begin{bmatrix}1\end{bmatrix}$ is called

(a) square matrix

(b) Diagonal matrix

(c) Identity matrix

(d) All ✓

(6) $(ABEF)^{-1} = ?$

(a) $A^{-1}B^{-1}E^{-1}F^{-1}$ (b) $F^{-1}B^{-1}E^{-1}$

(c) $B^{-1}A^{-1}E^{-1}$ (d) None ✓

Section: C

Q No.3 :- Find A^{-1} if $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 0 \\ 2 & 1 & 6 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj of } A}{|A|}$$

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$= 12(6-0) - 1(0) - 3(-2) = 12+6 = 18$$

$A_{11} = (-1)^2(6) = 6$ $A_{12} = (-1)^3(0) = 0$ $A_{13} = (-1)^4(-2) = -2$

$A_{21} = (-1)^3(9) = -9$ $A_{22} = (-1)^4(18) = 18$ $A_{23} = (-1)^5(0) = 0$

$A_{31} = (-1)^4(3) = 3$ $A_{32} = (-1)^5(0) = 0$ $A_{33} = (-1)^6(2) = 2$

$$A^{-1} = \frac{\begin{bmatrix} 6 & -9 & 3 \\ 0 & -18 & 0 \\ -2 & 0 & 2 \end{bmatrix}}{18} = \begin{bmatrix} 6/18 & -9/18 & 3/18 \\ 0 & -18/18 & 0 \\ -2/18 & 0 & 2/18 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/3 & -1/2 & 1/6 \\ 0 & 1 & 0 \\ -1/9 & 0 & 1/9 \end{bmatrix}$$

(10) $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

L.H.S: $R_1+(R_2+R_3)$

$$= \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

Take 2 common from R_1:

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & z+y \end{vmatrix}$$

R_2-R_1 , R_3-R_1 :

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$

$R_1+R_2+R_3$:

$$= 2\begin{vmatrix} a & p & x \\ -b & -q & -y \\ -c & -r & z \end{vmatrix}$$

Take (-1) common from R_2 and R_3 :

$$= 2(-1)(-1)\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = R.H.S$$

Hence Proved.

(11) If $|AB| = |A| \cdot |B|$ and $A^{-1} = 1/|A|$ then for a square matrix of 3×3 prove that $|\text{adj } A| = |A|^2$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$A^{-1}|A| = \text{adj} A$, $\text{adj} A = A^{-1}|A|$

$A.\text{adj} A = AA^{-1}|A|$ ∴ Apply A on B/S =

$A \cdot \text{adj} A = I |A|$

$$A \cdot \text{adj } A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} |A|$$

$$|A \cdot \text{Adj} A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} |A|$$

$$|A| \, |\text{Adj} A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$|A| \, |\text{Adj} A| = |A|^3$ → we can also find by expanding

$$|\text{adj} A| = \frac{|A|^3}{|A|}$$

$|\text{adj} A| = |A|^2$ proved.

(12) If A is order of 3×3 such that $|\text{adj} A| = 64$ then find $|A^{-1}|$

As we know,

$|A|^2 = |\text{adj } A|$

$|A|^2 = 64$

Take Sq. root on B/S :

$\sqrt{|A|^2} = \sqrt{64}$

$|A| = 8$

As, $|A^{-1}| = \frac{1}{|A|}$

$\frac{1}{|A|} = \frac{1}{8}$

$A^{-1} = \frac{1}{8}$

(11) If $|AB| = |A|\cdot|B|$ and $A^{-1} = 1/|A|$ then for a square matrix of 3×3 prove that $|\text{adj } A| = |A|^2$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$A^{-1}|A| = \text{adj} A$, $\text{adj } A = A^{-1}|A|$

$A.\text{adj}A = AA^{-1}|A|$ ∴ Apply A on B/S :

$A\cdot\text{adj}A = I\,|A|$

$$A\cdot\text{adj } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} |A|$$

$$|A\cdot \text{Adj} A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} |A$$

$$|A|\,|\text{Adj}A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$|A||\text{Adj}A| = |A|^3$ → We can also find by expanding

$$|\text{adj}A| = \frac{|A|^{\cancel{3}}}{\cancel{|A|}}$$

$|\text{adj}A| = |A|^2$ proved.

(12) If A is order of 3×3 such that $|\text{adj}A| = 64$ then find $|A^{-1}|$

As we know,

$|A|^2 = |\text{adj } A|$

$|A|^2 = 64$

Take Sq. root on B/S :

$\sqrt{|A|^2} = \sqrt{64}$

$|A| = 8$

As, $|A^{-1}| = \frac{1}{|A|}$

$\frac{1}{|A|} = \frac{1}{8}$

$A^{-1} = \frac{1}{8}$

(13) If a, b, c are real numbers and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$ Show that either $a+b+c=0$ or $a=b=c$.

$R_1 + (R_2 + R_3)$

$$\begin{vmatrix} 2a+2b+2c & 2a+2b+2c & 2a+2b+2c \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Take $(2a+2b+2c)$ common from R_1

$$2a+2b+2c \begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

$C_2 - C_1$, $C_3 - C_1$:

$$2a+2b+2c \begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix} = 0$$

Expanding by R_1 :

$$(2a+2b+2c)\, 1 \begin{vmatrix} b-c & b-a \\ c-a & c-b \end{vmatrix} = 0$$

$$(2a+2b+2c)\left[bc - b^2 - c^2 + \cancel{bc} - \cancel{bc} + ac + ab - a^2\right] = 0$$

$$-(2a+2b+2c)\,(a^2 + b^2 + c^2 - ab - bc - ac) = 0$$

$$-2\,(a+b+c)\,(a^2+b^2+c^2 - ab - bc - ac) = 0$$

$$-(a+b+c)\,(2a^2+2b^2+2c^2 - 2ab - 2bc - 2ac) = 0$$

$$-(a+b+c)\,(a^2+a^2+b^2+b^2+c^2+c^2 - 2ab - 2bc - 2ac) = 0$$

$$-(a+b+c)\left[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(a^2+c^2-2ac)\right]$$

$$-(a+b+c)\left[(a-b)^2 + (b-c)^2 + (a-c)^2\right] = 0$$

$-(a+b+c) = 0$ $\qquad$ $(a-b)^2 + (b-c)^2 + (a-c)^2 = 0$

$a+b+c = 0$ $\qquad$ $\sqrt{(a-b)^2} = 0$, $\sqrt{(b-c)^2} = 0$, $\sqrt{(a-c)^2} = 0$

$a = b$, $b = c$