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Where Minds Pollinate

Class 11
Mathematics

Chapter 2
Exercise 2.3
Notes

National Book Foundation

Exercise : 2.3

Determinants:

Square Matrix

Order 2x2

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Minor of an element:

let $A = [a_{iJ}]_{n\times n}$

The minor of an element a_{iJ} of matrix is determinant of order $(n-1)\times(n-1)$ obtained by deleting with the ith row and Jth column. Minor of a_{iJ} is denoted by M_{iJ}

e.g: let $A = \begin{bmatrix} \overset{a_{11}}{1} & \overset{a_{12}}{2} \\ \overset{a_{21}}{3} & \overset{a_{22}}{4} \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ Minor of $a_{22} = \begin{bmatrix} 1 & 3 \\ 7 & 9 \end{bmatrix}$

Minor of $a_{11} = [4]$

Cofactor of an element:

The co-factor of element a_{iJ} is denoted by

$A_{iJ} = (-1)^{i+J} \underset{\text{Minor}}{M_{iJ}}$

$A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$ $\text{Adj}A = \begin{bmatrix} 6 & -3 \\ -5 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ Adj of $A = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}$

$A_{11} = ?$

$A_{iJ} = (-1)^{i+J} M_{iJ}$

$A_{11} = (-1)^{1+1} M_{11}$

$A_{11} = (-1)^2 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix}$

$= 1(3)$

$= 3$

$A_{12} = ?$

$A_{iJ} = (-1)^{1+2} M_{12}$

$A_{12} = (-1)^3 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix}$

$A_{12} = -1(36-42)$

$A_{12} = 6$

QNo.1: Evaluate the determinants of following matrices

(1) $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ $\quad |A| = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{vmatrix}$

Expanding by Row-1

$$2\begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix}$$

$2(-2-2) - 3(2-8) + 1(1+4)$

$-8 + 18 + 5 = 15$

(2) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad |A| = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Expanding by R-1

$$\cos\theta\begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} - (\sin\theta)\begin{vmatrix} \sin\theta & 0 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} \sin\theta & \cos\theta \\ 0 & 0 \end{vmatrix}$$

$\cos\theta(\cos\theta - 0) + \sin\theta(\sin\theta + 0) + 0(0+0)$

$\cos^2\theta + \sin^2\theta = 1$

(3) $\begin{bmatrix} i & 3 & -2i \\ 1 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $\quad |A| = \begin{vmatrix} i & 3 & -2i \\ 1 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix}$

Expanding by Column 2

$$-3\begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} i & -2i \\ 0 & 2 \end{vmatrix} - 1\begin{vmatrix} i & -2i \\ 1 & 4 \end{vmatrix}$$

$-3(2-4) + 3(2i+0) - 1(4i+2i)$

$6 + 6i - 6i = 6$

(4) $\begin{bmatrix} 2+i & 1 & i \\ 0 & 2 & 1 \\ -3i & 1 & 6 \end{bmatrix}$ $|A| = \begin{vmatrix} 2+i & 1 & i \\ 0 & 2 & 1 \\ -3i & 1 & 6 \end{vmatrix}$

Expanding by R-2

$0 \begin{vmatrix} 1 & i \\ 1 & 6 \end{vmatrix} + 2 \begin{vmatrix} 2+i & i \\ -3i & 6 \end{vmatrix} - 1 \begin{vmatrix} 2+i & 1 \\ -3i & 1 \end{vmatrix}$

$0(6-i) + 2(12+6i+3) - 1(2+i+3i)$

$0 + 12i + 18 - 2 - 4i = 16 + 8i$

QNo.2: Evaluate the determinants of following matrices using co-factor method.

(1) $\begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ Adj of $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{23} \end{bmatrix}$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$A_{iJ} = (-1)^{i+J} M_{iJ}$ $A_{12} = (-1)^{1+2} M_{12}$ $A_{13} = (-1)^{1+3} M_{13}$

$A_{11} = (-1)^{1+1} M_{11}$ $A_{12} = (-1)^3 \begin{vmatrix} 4 & 1 \\ 2 & 0 \end{vmatrix}$ $A_{13} = (-1)^4 \begin{vmatrix} 4 & 5 \\ 2 & 1 \end{vmatrix}$

$A_{11} = (-1)^2 \begin{vmatrix} 5 & 1 \\ 1 & 0 \end{vmatrix}$ $A_{12} = -1(0-2)$ $A_{13} = 1(4-10)$

$A_{11} = 1(0-1)$ $A_{12} = 2$ $A_{13} = -6$

$A_{11} = -1$ $|A| = 3(-1) + 2(2) + 3(-6) = -17$

(2) $\begin{bmatrix} 2 & 3 & -1 \\ -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ Adj of $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$ $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$|A| = 2(-2) + 3(10) - 1(-1)$

$= 27$

$A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix}$ $A_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix}$ $A_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix}$

$A_{11} = (-1)^2 (0-2)$ $A_{12} = -1(-4-6)$ $A_{13} = 1(-1-0)$

$A_{11} = 1(-2)$ $A_{12} = 10$ $A_{13} = -1$

$A_{11} = -2$

(3) $\begin{bmatrix} 2i & 6 & 1 \\ 1 & -i & 2 \\ 0 & 1 & 3i \end{bmatrix}$ Adj of $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$A_{11} = (-1)^{1+1}\begin{vmatrix} -i & 2 \\ 1 & 3i \end{vmatrix}$ $A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 2 \\ 0 & 3i \end{vmatrix}$ $A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -i \\ 0 & 1 \end{vmatrix}$

$A_{11} = 1(3-2)$ $A_{12} = -1(3i-0)$ $A_{13} = 1(1+0)$

$A_{11} = 1$ $A_{12} = -3i$ $A_{13} = 1$

$|A| = 2i(1) + 6(-3i) + 1(1)$

$= 2i - 18i + 1 = 1 - 16i$

(4) $\begin{bmatrix} 1-i & 2 & 1+i \\ 3 & 1 & 4 \\ 0 & 2 & 3 \end{bmatrix}$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix}$ $A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 4 \\ 0 & 3 \end{vmatrix}$ $A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$

$A_{11} = 1(3-8)$ $A_{12} = -1(9-0)$ $A_{13} = 1(6-0)$

$A_{11} = -5$ $A_{12} = -9$ $A_{13} = 6$

$|A| = 1-i(-5) + 2(-9) + 1+i(6)$

$= -5 + 5i - 18 + 6 + 6i$

$= -17 + 11i$

QNo. 3: Determine which of the following matrices are singular or non-singular.

Singular matrix:

If $|A| = 0$ then 'A' is called singular matrix

Non-Singular matrix:

If $|A| \neq 0$ then 'A' is called non-singular.

(1) $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ -4 & 1 & -3 \end{bmatrix}$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix}$ $\quad$ $A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ -4 & -3 \end{vmatrix}$ $\quad$ $A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix}$

$A_{11} = 1(-9-1)$ $\quad$ $A_{12} = -1(-6+4)$ $\quad$ $A_{13} = 1(2+12)$

$A_{11} = -10$ $\quad$ $A_{12} = 2$ $\quad$ $A_{13} = 14$

$|A| = 3(-10) + 1(2) + 2(14)$

$= -30 + 2 + 28 = -30 + 30 = 0$ $\quad$ $|A| \neq 0$ $\quad$ Singular

$|A| = \begin{vmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ -4 & 1 & -3 \end{vmatrix} = 3\begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ -4 & -3 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix}$

$= 3(-9-1) - 1(-6+4) + 2(2+12)$

$= -30 + 2 + 28$

$= 0$ $\quad$ $|A| = 0$ $\quad$ Singular

(2) $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 0 & 1 \\ -1 & 5 & 1 \end{bmatrix}$ $\quad$ $|A| = \begin{vmatrix} 3 & -1 & 2 \\ 2 & 0 & 1 \\ -1 & 5 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} 0 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 0 \\ -1 & 5 \end{vmatrix}$

$3(0-5) + 1(2+1) + 2(10+0)$

$-15 + 3 + 20 = 8$ $\quad$ Non Singular

(3) $\begin{bmatrix} 3i & 1 & 2 \\ -4 & 1 & i \\ 2 & 0 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 3i & 1 & 2 \\ -4 & 1 & i \\ 2 & 0 & 1 \end{vmatrix} = 3i \begin{vmatrix} 1 & i \\ 0 & 1 \end{vmatrix} - 1 \begin{vmatrix} -4 & i \\ 2 & 1 \end{vmatrix} + 2 \begin{vmatrix} -4 & 1 \\ 2 & 0 \end{vmatrix}$

$= 3i(1-0) - 1(-4-2i) + 2(0-2)$

$= 3i + 4 + 2i - 4 = 0 + 5i$ Non Singular

(4) $\begin{bmatrix} 2 & -i & 1 \\ i & 3 & -2 \\ -2+i & i+3 & -3 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -i & 1 \\ i & 3 & -2 \\ -2+i & i+3 & -3 \end{vmatrix} = 2 \begin{vmatrix} 3 & -2 \\ i+3 & -3 \end{vmatrix} + i \begin{vmatrix} i & -2 \\ -2+i & -3 \end{vmatrix} + 1 \begin{vmatrix} i & 3 \\ -2+i & i+3 \end{vmatrix}$

$= 2(-9 + 2i + 6) + i(-3i - 4 + 2i) + 1(-1 \cancel{+3i} + 6 \cancel{-3i})$

$= -\cancel{6} \cancel{+4i} \cancel{-4i} + 1 - 1 \cancel{+6}$

$= 0$ Singular matrix.

QNo.4: Find the value of λ so that given matrices are singular. IMP

(1) $\begin{bmatrix} \lambda & 1 & 3 \\ 2 & 1 & 8 \\ 0 & 3 & 1 \end{bmatrix}$ $\begin{vmatrix} \lambda & 1 & 3 \\ 2 & 1 & 8 \\ 0 & 3 & 1 \end{vmatrix} = 0$

$\lambda \begin{vmatrix} 1 & 8 \\ 3 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 8 \\ 0 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 0$

$\lambda(1-24) - 1(2-0) + 3(6-0) = 0$

$-23\lambda - 2 + 18 = 0$

$-23\lambda + 16 = 0$

$\lambda = \frac{-16}{-23}$

$\lambda = 16/23$

(2) $\begin{bmatrix} \lambda & 2 & 0 \\ 2 & 1 & 3 \\ \lambda & 2 & 1 \end{bmatrix} = \begin{vmatrix} \lambda & 2 & 0 \\ 2 & 1 & 3 \\ \lambda & 2 & 1 \end{vmatrix}$

$\lambda \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 3 \\ \lambda & 1 \end{vmatrix} + 0 \begin{vmatrix} 2 & 1 \\ \lambda & 2 \end{vmatrix} = 0$

$\lambda(1-6) - 2(2-3\lambda) + 0(4-\lambda) = 0$

$-5\lambda + 6\lambda - 4 = 0$

$\lambda = 4$

(3) $\begin{bmatrix} \lambda & i & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{vmatrix} \lambda & i & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{vmatrix}$

$\lambda \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} - i \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = \lambda(2-3) - i(4-9) + 1(2-3)$

$= -\lambda + 9 - 4i - 1 = 0$

$-\lambda - 4i = -8 \qquad \lambda = 8 - 4i$

$-\lambda = -8 + 4i$

(4) $\begin{bmatrix} 2+i & 1 & 6 \\ 2 & \lambda & 1 \\ 3 & 0 & 2 \end{bmatrix} = \begin{vmatrix} 2+i & 1 & 6 \\ 2 & \lambda & 1 \\ 3 & 0 & 2 \end{vmatrix} = 2+i \begin{vmatrix} \lambda & 1 \\ 0 & 2 \end{vmatrix} - 1 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 2 & \lambda \\ 3 & 0 \end{vmatrix} = 0$

$2+i(2\lambda) - 1(1) + 6(-3\lambda) = 0$

$4\lambda + 2\lambda i - 1 - 18\lambda = 0$

$\lambda(4 + 2i - 18) = 1$

$\lambda(-14 + 2i) = 1$

$\lambda = \frac{1}{-14+2i} \times \frac{-14-2i}{-14-2i}$

$\lambda = \frac{-14-2i}{196+4} = \frac{-7}{100} - \frac{1}{100} i$

$\lambda = \frac{2(-7-i)}{200}$

Q No. 5: Find the multiplicative inverse of following matrices if exsists by adjoint method.

(1) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & -1 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj of } A}{|A|} = \frac{\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t}{|A|}$$

$$|A| = 1\begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 1(-3) + 1(-1) + 1(-5) = -9$$

$$A_{11} = (-1)^2 \begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix} = (-1)^2(-3) = -3$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = (-1)^3(-1) = 1$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = (-1)^4(-5) = -5$$

$$A_{21} = (-1)^3 \begin{vmatrix} -1 & 1 \\ -2 & -1 \end{vmatrix} = -1(3) = -3$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1(-2) = -2$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} = -1(-1) = 1$$

$$A_{31} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 1(0) = 0$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1(-3) = 3$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1(3) = 3$$

$$= \begin{bmatrix} -3 & 1 & -5 \\ -3 & -2 & 1 \\ 0 & 3 & 3 \end{bmatrix}^t$$

$$A^{-1} = \frac{\begin{bmatrix} -3 & -3 & 0 \\ 1 & -2 & 3 \\ -5 & 1 & 3 \end{bmatrix}}{-9}$$

$$A^{-1} = \begin{bmatrix} 1/3 & 1/3 & 0 \\ -1/9 & 2/9 & [illegible] \\ 5/9 & -1/9 & [illegible] \end{bmatrix}$$

(2) $\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$ $A^{-1} = \dfrac{\text{Adj of } A}{|A|}$ $= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$|A| = 3\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} + 4\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = 3(3)+4(-3)+2(-3)$

$= 9-12-6 = -9$

$A_{11} = (-1)^2 \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 1(3) = 3$

$A_{12} = (-1)^3 \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = -1(-3) = -3$

$A_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = 1(-3) = -3$

$A_{21} = (-1)^3 \begin{vmatrix} -4 & 2 \\ 0 & 1 \end{vmatrix} = -1(-4) = 4$

$A_{22} = (-1)^4 \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 1(1) = 1$

$A_{23} = (-1)^5 \begin{vmatrix} 3 & -4 \\ 1 & 0 \end{vmatrix} = -1(4) = -4$

$A_{31} = (-1)^4 \begin{vmatrix} -4 & 2 \\ 3 & 5 \end{vmatrix} = 1(-26) = -26$

$A_{32} = (-1)^5 \begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} = -1(11) = -11$

$A_{33} = (-1)^6 \begin{vmatrix} 3 & -4 \\ 2 & 3 \end{vmatrix} = 1(17) = 17$

$= \begin{bmatrix} 3 & -3 & -3 \\ 4 & 1 & -4 \\ -26 & -11 & 17 \end{bmatrix}^t$

$= \dfrac{\begin{bmatrix} 3 & 4 & -26 \\ -3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}}{-9}$

$= \begin{bmatrix} -1/3 & -4/9 & -26/9 \\ -1/3 & -1/9 & 11/9 \\ 1/3 & 4/9 & -17/9 \end{bmatrix}$

(3) $\begin{bmatrix} i & 0 & 1 \\ 2i & -1 & -i \\ 1 & 0 & 4i \end{bmatrix}$ $\quad A^{-1} = \dfrac{\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t}{|A|}$

$$|A| = i \begin{vmatrix} -1 & -i \\ 0 & 4i \end{vmatrix} - 0 \begin{vmatrix} 2i & -i \\ 1 & 4i \end{vmatrix} + 1 \begin{vmatrix} 2i & -1 \\ 1 & 0 \end{vmatrix} = i(-4i) + 1(1) = -4i^2 + 1 = 5$$

$$A_{11} = (-1)^2 \begin{vmatrix} -1 & -i \\ 0 & 4i \end{vmatrix} = 1(-4i) = -4i$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2i & -i \\ 1 & 4i \end{vmatrix} = -1(-8+i) = 8-i$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2i & -1 \\ 1 & 0 \end{vmatrix} = 1(1) = 1$$

$$A_{21} = (-1)^3 \begin{vmatrix} 0 & 1 \\ 0 & 4i \end{vmatrix} = -1(0) = 0$$

$$A_{22} = (-1)^4 \begin{vmatrix} i & 1 \\ 1 & 4i \end{vmatrix} = 1(-5) = -5$$

$$A_{23} = (-1)^5 \begin{vmatrix} i & 0 \\ 1 & 0 \end{vmatrix} = -1(0) = 0$$

$$A_{31} = (-1)^4 \begin{vmatrix} 0 & 1 \\ -1 & i \end{vmatrix} = 1(1) = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} i & 1 \\ 2i & -i \end{vmatrix} = -1(1-2i) = -1+2i$$

$$A_{33} = (-1)^6 \begin{vmatrix} i & 0 \\ 2i & -1 \end{vmatrix} = 1(-i) = -i$$

$$= \begin{bmatrix} -4i & 8-i & 1 \\ 0 & -5 & 0 \\ 1 & -1+2i & -i \end{bmatrix} = \dfrac{\begin{bmatrix} -4i & 0 & 1 \\ 8-i & -5 & -1+2i \\ 1 & 0 & -i \end{bmatrix}}{5}$$

$$= \begin{bmatrix} -4i/5 & 0 & 1/5 \\ 8-i/5 & -1 & 1+2i/5 \\ 1/5 & 0 & -i/5 \end{bmatrix}$$

(4) $\begin{bmatrix} 3 & -i & i \\ 2 & 1 & -3i \\ 4i & 2 & 6 \end{bmatrix} = 3\begin{vmatrix} 1 & -3i \\ 2 & 6 \end{vmatrix} + i\begin{vmatrix} 2 & -3i \\ 4i & 6 \end{vmatrix} + i\begin{vmatrix} 2 & 1 \\ 4i & 2 \end{vmatrix}$

$|A| = 18+18i+12i-12i+4i+4 = 22+22i$

$A_{11} = (-1)^2\begin{vmatrix} 1 & -3i \\ 2 & 6 \end{vmatrix} = 1(6+6i) = 6(1+i)$

$A_{12} = (-1)^3\begin{vmatrix} 2 & -3i \\ 4i & 6 \end{vmatrix} = -1(0) = 0$

$A_{13} = (-1)^4\begin{vmatrix} 2 & 1 \\ 4i & 2 \end{vmatrix} = 1(4-4i) = 4(1-i)$

$A_{21} = (-1)^3\begin{vmatrix} -i & i \\ 2 & 6 \end{vmatrix} = -1(-8i) = 8i$

$A_{22} = (-1)^4\begin{vmatrix} 3 & i \\ 4i & 6 \end{vmatrix} = 1(22) = 22$

$A_{23} = (-1)^5\begin{vmatrix} 3 & -i \\ 4i & 2 \end{vmatrix} = -1(2) = -2$

$A_{31} = (-1)^4\begin{vmatrix} -i & i \\ 1 & -3i \end{vmatrix} = 1(-3-i) = -3-i$

$A_{32} = (-1)^5\begin{vmatrix} 3 & i \\ 2 & -3i \end{vmatrix} = -1(-11i) = 11i$

$A_{33} = (-1)^6\begin{vmatrix} 3 & -i \\ 2 & 1 \end{vmatrix} = 1(3+2i) = 3+2i$

$= \begin{bmatrix} 6(1+i) & 0 & 4(1-i) \\ 8i & 22 & -2 \\ -3-i & 11i & 3+2i \end{bmatrix}$

$= \dfrac{\begin{bmatrix} 6(1+i) & 8i & -3-i \\ 0 & 22 & 11i \\ 4(1-i) & -2 & 3+2i \end{bmatrix}}{22+22i \text{ or } 22(1+i)}$

$= \dfrac{1(1-i)}{22\times 2} \rightarrow$ rationalize $\dfrac{(1-i)}{(1-i)}$ $\dfrac{1-i}{2}$

$= \dfrac{1(1-i)}{44}$

$A^{-1} = \dfrac{1}{44}(1-i)\begin{bmatrix} \text{"} \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{22}(1+i)(1-i) & \frac{2}{11}i(1-i) & \frac{1}{44}(-3-i)(1-i) \\ 0 & \frac{1}{2}(1-i) & \frac{1}{4}i(1-i) \\ \frac{1}{11}(1-i)^2 & -\frac{1}{22}(1-i) & \frac{1}{44}(3+2i)(1-i) \end{bmatrix}$

$\frac{3}{22}(1+i)(1-i) = \frac{3}{22}(1+1) = 3/11$

$\frac{2}{11}i(1-i) = \frac{2}{11}(i+1) = \frac{2+2i}{11}$

$\frac{1}{44}(-3-i)(1-i) = \frac{1}{44}(-3+2i-1) = \frac{2}{44}(-2+i) = \frac{1}{22}(-2+i)$

$\frac{1}{4}i(1-i) = \frac{1}{4}(i-i^2) = \frac{1}{4}(i+1) = \frac{1+i}{4}$

$\frac{1}{11}(1-i)^2 = \frac{1}{11}(1+i^2-2i) = \frac{1}{11}(\cancel{1}\cancel{-1}-2i) = \frac{-2i}{11}$

$\frac{-1}{22}(1-i) = \frac{-1+i}{22}$

$\frac{1}{44}(3+2i)(1-i) = \frac{1}{44}(3-3i+2i-2i^2) = \frac{1}{44}(5-i) = \frac{5-i}{44}$

By putting Values.

$$A^{-1} = \begin{bmatrix} 3/11 & 2+2i/11 & -2+i/22 \\ 0 & 1-i/2 & 1+i/4 \\ -2i/11 & -1+i/22 & 5-i/44 \end{bmatrix}$$

QNo.6: If $A = \begin{vmatrix} 2 & 1 & -3 \\ 0 & 1 & 0 \\ 2 & 1 & 6 \end{vmatrix}$ then find A^{-1} and show that $AA^{-1} = A^{-1}A = I_3$

$|A| = 2\begin{vmatrix} 1 & 0 \\ 1 & 6 \end{vmatrix} - 1\begin{vmatrix} 0 & 0 \\ 2 & 6 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = 12+6 = 18$

$A_{11} = (-1)^2\begin{vmatrix} 1 & 0 \\ 1 & 6 \end{vmatrix} = 1(6) = 6$

$A_{12} = (-1)^3\begin{vmatrix} 0 & 0 \\ 2 & 6 \end{vmatrix} = -1(0) = 0$

$A_{13} = (-1)^4\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = 1(-2) = -2$

$A_{21} = (-1)^3\begin{vmatrix} 1 & 0 \\ 1 & 6 \end{vmatrix} = -1(6) = -6$

$A_{22} = (-1)^4\begin{vmatrix} 2 & -3 \\ 2 & 6 \end{vmatrix} = 1(12+6) = 18$

$A_{23} = (-1)^5\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -1(-2)$

$A_{31} = (-1)^4\begin{vmatrix} 1 & -3 \\ 1 & 0 \end{vmatrix} = 1(3)$

$A_{32} = (-1)^5\begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix} = -1(0)$

$A_{33} = (-1)^6\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 1(2)$

$$A^{-1} = \frac{\begin{bmatrix} 6 & -9 & 3 \\ 0 & 18 & 0 \\ -2 & 0 & 2 \end{bmatrix}}{18} = \frac{1}{18}\begin{bmatrix} 6 & -9 & 3 \\ 0 & 18 & 0 \\ -2 & 0 & 2 \end{bmatrix}$$

(1) AA^{-1}

$$\begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 0 \\ 2 & 1 & 6 \end{bmatrix} \frac{1}{18}\begin{bmatrix} 6 & -9 & 3 \\ 0 & 18 & 0 \\ -2 & 0 & 2 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 6 & -9 & 3 \\ 0 & 18 & 0 \\ -2 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 0 \\ 2 & 1 & 6 \end{bmatrix}$$

$$\frac{1}{18}\begin{bmatrix} 12-0+6 & 6-9+3 & -18-6+18 \\ 0+0+0 & 0+18+0 & 0+0+0 \\ -4+0+4 & -2+0+2 & 6+0+12 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $A^{-1}A$

$$\frac{1}{18}\begin{bmatrix} 6 & -9 & 3 \\ 0 & 18 & 0 \\ -2 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 0 \\ 2 & 1 & 6 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) I_3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Hence Proved $\because$

QNo.7: Verify that $(AB)^{-1} = B^{-1}A^{-1}$ in following

(1) $A = \begin{bmatrix} 2 & 1 \\ 8 & 6 \end{bmatrix}$ $\quad B = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$

L.H.S:

$AB = \begin{bmatrix} 6 & 6 \\ 24 & 28 \end{bmatrix}$ $\quad |AB| = \begin{vmatrix} 6 & 6 \\ 24 & 28 \end{vmatrix} = 168 - 144 = 24$

$\text{Adj of } AB = \begin{bmatrix} 28 & -6 \\ -24 & 6 \end{bmatrix}$

$(AB)^{-1} = \dfrac{\begin{bmatrix} 28 & -6 \\ -24 & 6 \end{bmatrix}}{24} = \begin{bmatrix} \frac{7}{6} & \frac{1}{4} \\ -1 & +\frac{1}{4} \end{bmatrix}$

R.H.S

$B^{-1} = \dfrac{\begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix}}{6} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{2} \end{bmatrix}$ $\quad A^{-1} = \dfrac{\begin{bmatrix} 6 & -1 \\ -8 & 2 \end{bmatrix}}{4} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{4} \\ -2 & \frac{1}{2} \end{bmatrix}$

$B^{-1}A^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{3}{2} & -\frac{1}{4} \\ -2 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{2}{3} & -\frac{1}{12} - \frac{1}{6} \\ 0 - 1 & 0 + \frac{1}{4} \end{bmatrix}$

$B^{-1}A^{-1} = \begin{bmatrix} \frac{7}{6} & -\frac{1}{4} \\ -1 & \frac{1}{4} \end{bmatrix}$

L.H.S = R.H.S

(2) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$

L.H.S:

$AB = \begin{bmatrix} 3+2+4 & -2+1-3 & 3-1+2 \\ 6-2+4 & -4-1-3 & 6+1+2 \\ 6+2-12 & -4+1+9 & 6-1-6 \end{bmatrix} = \begin{bmatrix} 9 & -4 & 4 \\ 8 & -8 & 9 \\ -4 & 6 & -1 \end{bmatrix}$

$$|AB| = 9\begin{vmatrix} -8 & 9 \\ 6 & -1 \end{vmatrix} \; 4\begin{vmatrix} 8 & 9 \\ -4 & -1 \end{vmatrix} + 4\begin{vmatrix} 8 & -8 \\ -4 & 6 \end{vmatrix} = 9(8-54)+4(-8+36)+4(48-$$

$$= -238$$

$$A_{11} = (-1)^2\begin{vmatrix} -8 & 9 \\ 6 & -1 \end{vmatrix} = 1(-46) = -46$$

$$A_{12} = (-1)^3\begin{vmatrix} 8 & 9 \\ -4 & -1 \end{vmatrix} = -1(28) = -28$$

$$A_{13} = (-1)^4\begin{vmatrix} 8 & -8 \\ -4 & 6 \end{vmatrix} = 1(16) = 16$$

$$A_{21} = (-1)^3\begin{vmatrix} -4 & 4 \\ 6 & -1 \end{vmatrix} = -1(-20) = 20$$

$$A_{22} = (-1)^4\begin{vmatrix} 9 & -4 \\ -4 & 6 \end{vmatrix} = 1(7\cancel{3}) = 7$$

$$A_{23} = (-1)^5\begin{vmatrix} 9 & -4 \\ -4 & 6 \end{vmatrix} = -1(38) = -38$$

$$A_{31} = (-1)^4\begin{vmatrix} -4 & 4 \\ -8 & 9 \end{vmatrix} = 1(-4) = -4$$

$$A_{32} = (-1)^5\begin{vmatrix} 9 & 4 \\ 8 & 9 \end{vmatrix} = -1(49) = -49$$

$$A_{33} = (-1)^6\begin{vmatrix} 9 & -4 \\ 8 & -8 \end{vmatrix} = 1(-40) = 40$$

$$(AB)^{-1} = \frac{\begin{bmatrix} -48 & 20 & -4 \\ -28 & 7 & -49 \\ 16 & -38 & -40 \end{bmatrix}}{-238} = \begin{bmatrix} 23/119 & -10/119 & 2/119 \\ 2/17 & -1/34 & 7/34 \\ -8/119 & 19/119 & 20/119 \end{bmatrix}$$

R.H.S:

$$|A| = -1\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 1(2)-1(-8)+1(4) = 14$$

$$A_{11} = (-1)^2\begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} = 1(2) = 2$$

$$A_{12} = (-1)^3\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = -1(-8) = 8$$

$$A_{13} = (-1)^4\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 1(4) = 4$$

$$A_{21} = (-1)^3$$

(3) $A = \begin{bmatrix} 2 & -i & 6 \\ 1 & 2 & i \\ -i & 1 & 6 \end{bmatrix}$. $B = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

L.H.S:

$AB = \begin{bmatrix} 6-i & 8 & 10-i \\ 5 & 1+i & 4+i \\ 1-3i & 6-i & 7-2i \end{bmatrix}$

$(AB)^{-1} = \dfrac{\text{Adj of } AB}{|AB|}$

$|AB| = \begin{vmatrix} 6-i & 8 & 10-i \\ 5 & 1+i & 4+i \\ 1-3i & 6-i & 7-2i \end{vmatrix}$

$|AB| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= (6-i)(-16+3i) + 8(-28-i) + (10-i)(26-3i)$

$= -60 - 30i$

$= -30(2+i)$

$a_{13} = (-1)^4 \begin{vmatrix} 5 & 1+i \\ 1-3i & 6-i \end{vmatrix}$

$= 1(30-5i-4+2i)$

$= 26-3i$

Adj of AB:

$a_{11} = (-1)^2 \begin{vmatrix} 1+i & 4+i \\ 6-i & 7-2i \end{vmatrix}$

$= 1((1+i)(7-2i) - (6-i)(4+i))$

$= 9+5i - 25-2i$

$= -16+3i$

$a_{21} = (-1)^3 \begin{vmatrix} 8 & 10-i \\ 6-i & 7-2i \end{vmatrix}$

$= -1(56-\cancel{16i}-59+16i)$

$= -1(-3)$

$= 3$

$a_{12} = (-1)^3 \begin{vmatrix} 5 & 4+i \\ 1-3i & 7-2i \end{vmatrix}$

$= -1(35-10i-7+11i)$

$= -(28+i)$

$= -28-i$

$a_{22} = (-1)^4 \begin{vmatrix} 6-i & 10-i \\ 1-3i & 7-2i \end{vmatrix}$

$= 1(40-19i-7+31i)$

$= 33+12i$

$A_{23} = (-1)^5 \begin{vmatrix} 6-i & 8 \\ 1-3i & 6-i \end{vmatrix}$ ... $A_{31} = (-1)^4 \begin{vmatrix} 8 & 10-i \\ 1+i & 4+i \end{vmatrix}$

$= -1(36-1-2i-8+24i)$ ... $= +(32+8i-11-9i)$

$= -(27+12i) = -27-12i$ ... $= 21-i$

$A_{32} = (-1)^5 \begin{vmatrix} 6-i & 8 \\ 1-3i & 6-i \end{vmatrix}$ ... $A_{33} = (-1)^6 \begin{vmatrix} 6-i & 8 \\ 5 & 1+i \end{vmatrix}$

$= -1(24+6i-4i+1-50+5i)$ ... $= 6+6i-i-i^2-40$

$= -(-25+7i) = 25-7i$ ... $= -33+5i$

$$(AB)^{-1} = \frac{\begin{bmatrix} -16+3i & 3 & 21-i \\ -28-i & 33+12i & 25-7i \\ 26-3i & -27-12i & -33+5i \end{bmatrix}}{-30(2+i)} \rightarrow \text{also } \frac{1}{-30(2+i)}\begin{bmatrix} " \end{bmatrix}$$

Consider,

$$\frac{1}{2+i} \times \frac{2-i}{2-i} = \frac{2-i}{2^2-i^2} = \frac{2-i}{5} = \frac{2-i}{5}$$

$$\rightarrow \frac{-2+i}{-30(5)}\begin{bmatrix} " \end{bmatrix} = \frac{1}{150}\begin{bmatrix} (-2-i)(-16+3i) & 3(-2-i) & (-2-i)(21-i) \\ (-2-i)(-28-i) & (-2-i)(33+12i) & (-2-i)(25-7i) \\ (-2-i)(26-3i) & (-2-i)(-27-12i) & (-2-i)(-33+5i) \end{bmatrix}$$

$$\frac{1}{150}\begin{bmatrix} 29-22i & -6+3i & -41+23i \\ 57-26i & -78+9i & -43+39i \\ -49+32i & 66-3i & 61-43i \end{bmatrix} \rightarrow \text{L.H.S}$$

R.H.S $= B^{-1}A^{-1}$

$|A| = 2\begin{vmatrix} 2 & i \\ 1 & 6 \end{vmatrix} + i\begin{vmatrix} 1 & 1 \\ -i & 6 \end{vmatrix} + 6\begin{vmatrix} 1 & 2 \\ i & 1 \end{vmatrix}$

$= 24-2i+6i-i+6+12i = 30+15i$

$= 15(2+i)$

$A_{11} = 12-i$, $A_{12} = -5$, $A_{13} = 1+2i$

$A_{21} = 6+6i$, $A_{22} = 12+6i$, $A_{23} = -3$

$A_{31} = -11$, $A_{32} = 6-2i$, $A_{33} = 4+i$

$$A^{-1} = \frac{1}{15(2+i)} \begin{bmatrix} 12-i & 6+6i & -11 \\ -5 & 12+6i & 6-2i \\ 1+2i & -3 & 4+i \end{bmatrix}$$

Consider,

$$\frac{1}{2+i} \times \frac{2-i}{2-i} = \frac{2-i}{4+1} = \frac{2-i}{5}$$

$$= \frac{2-i}{15(5)} \begin{bmatrix} & " & \\ & & \end{bmatrix} = \frac{1}{75} \begin{bmatrix} (2-i)(12-i) & (2-i)(6+6i) & -11(2-i) \\ -5(2-i) & (2-i)(12+6i) & (2-i)(6-2i) \\ (2-i)(1+2i) & (2-i)(-3) & (2-i)(4+i) \end{bmatrix}$$

$$A^{-1} = \frac{1}{75} \begin{bmatrix} 23-14i & 18+6i & -22+11i \\ -10+5i & 30 & 10-10i \\ 4+3i & -6+3i & 9-2i \end{bmatrix} \rightarrow A^{-1}$$

$$|B| = 3(0-1) - 1(1-0) + 2(1-0)$$

$$= -3 - 1 + 2 = -2$$

$B_{11} = -1$, $B_{12} = -1$, $B_{13} = 1$

$B_{21} = 1$, $B_{22} = 3$, $B_{23} = -3$

$B_{31} = 1$, $B_{32} = -1$, $B_{33} = -1$.

$$B^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ -1 & 3 & -1 \\ +1 & -3 & -1 \end{bmatrix} \longrightarrow B^{-1}$$

$$B^{-1} A^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ -1 & 3 & -1 \\ 1 & -3 & -1 \end{bmatrix} \frac{1}{75} \begin{bmatrix} 23-14i & 18+6i & -22+11i \\ -10+5i & 30 & 10-10i \\ 4+3i & -6+3i & 9-2i \end{bmatrix}$$

$$= \frac{1}{150}\begin{bmatrix} 23-14i+10-5i-4-3i & 18+6i-30+6-3i & -22+11i-10+10i-9+2i \\ 23-14i+30-15i+4+3i & 18+6i-90-6-3i & -22+11i-30+30i+9-2i \\ -23+14i-30+15i+4+3i & -18-6i+90-6+3i & 22-11i+30-30i+9-2i \end{bmatrix}$$

$$= \frac{1}{150}\begin{bmatrix} 29-22i & -6+3i & -41+23i \\ 57-26i & -78+9i & -43+39i \\ -49+32i & 66-3i & 61-43i \end{bmatrix} \rightarrow \text{R.H.S}$$

L.H.S = R.H.S

Hence Proved.