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Where Minds Pollinate

Class 11
Mathematics

Chapter 2
Exercise 2.2
Notes

National Book Foundation

Exercise : 2.2

Q No. 1: Construct a matrix $A = [a_{ij}]$ of order 2×2 for which:

(1) $a_{ij} = \frac{i+3j}{2}$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

put $i=1, j=1$

$a_{11} = \frac{1+3}{2} = 2$

put $i=2$ $j=1$

$a_{21} = \frac{2+3}{2} = 5/2$

put $i=1$ $j=2$

$a_{12} = \frac{1+6}{2} = 7/2$

put $i=2$ $j=2$

$a_{22} = \frac{2+6}{2} = 4$

$A = \begin{bmatrix} 2 & 7/2 \\ 5/2 & 4 \end{bmatrix}$

(2) $a_{ij} = \frac{i \times j}{2}$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

put $i=1$ $j=1$

$a_{11} = \frac{1\times1}{2} = 1/2$

put $i=1$ $j=2$

$a_{12} = \frac{1\times2}{2} = 1$

put $i=2$ $j=1$

$a_{21} = \frac{2\times1}{2} = 1$

put $i=2$ $j=2$

$a_{22} = \frac{2\times2}{2} = 2$

$A = \begin{bmatrix} 1/2 & 1 \\ 1 & 2 \end{bmatrix}$

(3) $a_{ij} = \frac{i}{j}$

put $i=1$ $j=1$

$a_{11} = \frac{1}{1} = 1$

put $i=1$ $j=2$

$a_{12} = \frac{1}{2}$

put $i=2$ $j=1$

$a_{21} = 2$

put $i=2$ $j=2$

$a_{22} = 1$

$A = \begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$

(4) $a_{ij} = \frac{2i - 3j}{3}$

put $i=1$ $j=1$

$a_{11} = \frac{2-3}{3} = -1/3$

put $i=1$ $j=2$

$a_{12} = \frac{2-6}{3} = -4/3$

put $i=2$ $j=1$

$a_{21} = \frac{4-3}{3} = \frac{1}{3}$

put $i=2$ $j=2$

$a_{22} = \frac{4-6}{3}$

$A = \begin{bmatrix} -1/3 & -4/3 \\ 1/3 & -2/3 \end{bmatrix}$

QNo.2: Construct a matrix $B = [a_{iJ}]$ of order 3x3 for

(1) $b_{iJ} = \frac{i^2 - J}{3}$

$$B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

put $i=1, J=1$

$a_{11} = 0/3 = 0$

put $i=1, J=2$

$a_{12} = -1/3$

put $i=1, J=3$

$a_{13} = -2/3$

put $i=2, J=1$

$a_{21} = 3/3 = 1$

put $i=2, J=2$

$a_{22} = 2/3$

put $i=2, J=3$

$a_{23} = 1/3$

put $i=3, J=1$

$a_{31} = 8/3$

put $i=3, J=2$

$a_{32} = 7/3$

put $i=3, J=3$

$a_{33} = 2$

$$B = \begin{bmatrix} 0 & -1/3 & -2/3 \\ 1 & 2/3 & 1/3 \\ 8/3 & 7/3 & 2 \end{bmatrix}$$

(2) $b_{iJ} = \frac{2}{2i+J}$

put $i=1$ $J=1$

$a_{11} = 2/3$

put $i=1$ $J=2$

$a_{12} = 1/2$

put $i=1$ $J=3$

$a_{13} = 2/5$

put $i=2$ $J=1$

$a_{21} = 2/5$

put $i=2$ $J=2$

$a_{22} = 1/3$

put $i=2$ $J=3$

$a_{23} = 2/7$

put $i=3$ $J=1$

$a_{31} = 2/7$

put $i=3$ $J=2$

$a_{32} = 1/4$

put $i=3$ $J=3$

$a_{33} = 2/9$

$$B = \begin{bmatrix} 2/3 & 1/2 & 2/5 \\ 2/5 & 1/3 & 2/7 \\ 2/7 & 1/4 & 2/9 \end{bmatrix}$$

(3) $b_{iJ} = \frac{i^2 + j^2}{i+j}$

put $i=1$ $J=1$

$a_{11} = 1$

put $i=1$ $J=2$

$a_{12} = 5/3$

put $i=1$ $J=3$

$a_{13} = 5/2$

put $i=2$ $J=1$ — $a_{21} = 5/3$

put $i=2$ $J=2$ — $a_{22} = 2$

put $i=2$ $J=3$ — $a_{23} = 13/5$

put $i=3$ $J=1$ — $a_{31} = 5/2$

put $i=3$ $J=2$ — $a_{32} = 13/5$

put $i=3$ $J=3$ — $a_{33} = 3$

$$B = \begin{bmatrix} 1 & 5/3 & 5/2 \\ 5/3 & 2 & 13/5 \\ 5/2 & 13/5 & 3 \end{bmatrix}$$

C.

Q No. 3: If $A = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 6 & 1 \\ -1 & 0 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 & 7 \\ 0 & 2 & -1 \\ -3 & 4 & 2 \end{bmatrix}$ than find matrix C such that: $A+B+C=0$

$$\begin{bmatrix} 3 & -1 & 2 \\ 0 & 6 & 1 \\ -1 & 0 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 7 \\ 0 & 2 & -1 \\ -3 & 4 & 2 \end{bmatrix} + C = \begin{bmatrix} 5 & 0 & 9 \\ 0 & 8 & 0 \\ -4 & 4 & -1 \end{bmatrix} + C = 0$$

$$C = -\begin{bmatrix} 5 & 0 & 9 \\ 0 & 8 & 0 \\ -4 & 4 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -9 \\ 0 & -8 & 0 \\ 4 & -4 & 1 \end{bmatrix}$$

Q No. 5: If $X = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then prove that $X^2 - 4X - 5I = 0$

$$X^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$X^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} \qquad 4X = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} \qquad I =$$

$$\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$0 = 0$$

proved.

Q No. 6: If $A = \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$ then find α and β such that $A^2 + \alpha I = \beta A$

$$A^2 = \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 4+3 & 2-3 \\ 6-9 & 3+9 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ -3 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -1 \\ -3 & 12 \end{bmatrix} + \begin{bmatrix} \alpha & 0 \\ \alpha & \alpha \end{bmatrix} = \begin{bmatrix} 2\beta & \beta \\ 3\beta & -3\beta \end{bmatrix}$$

$$\begin{bmatrix} 7+\alpha & -1 \\ -3 & 12+\alpha \end{bmatrix} = \begin{bmatrix} 2\beta & \beta \\ 3\beta & -3\beta \end{bmatrix}$$

$\beta = -1$

$7 + \alpha = 2\beta$

$\alpha = 5$

Q No. 10: If A and B are two matrices such that $AB = B$ and $BA = A$. Find $A^2 + B^2$.

$A^2 + B^2 = (BA)^2 + (AB)^2$

$= (BA)(BA)\ (AB)(AB)$

$= B(AB)A + A(BA)B$

$= B(B)A + A(A)B$

$= B(BA) + A(AB)$

$= B(A) + A(B)$

$= BA + AB$

$= A + B$

$\therefore A^2 + B^2 = A + B$

Q No. 11: If $A = [a_{iJ}]$ is a matrix of order 3x3 and $a_{iJ} = i^2 - J^2$. Check whether A is symmetric or skew symmetric.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$i=1$ $J=1$
$a_{11} = 0$

$i=1$ $J=2$
$a_{12} = -3$

$i=1$ $J=3$
$a_{13} = -8$

$i=2$ $J=1$
$a_{21} = 3$

$i=2$ $J=2$
$a_{22} = 0$

$i=2$ $J=3$
$a_{23} = -5$

$i=3$ $J=1$
$a_{31} = 8$

$i=3$ $J=2$
$a_{32} = 5$

$i=3$ $J=3$
$a_{33} = 0$

$$A = \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix} \quad A^t = \begin{bmatrix} 0 & 3 & 8 \\ -3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}$$

Skew-symmetric

Q No. 12: For any square matrix A prove that $(A^n)^t = (A^t)^n$

L.H.S:

$= (A \cdot A \cdot A \text{ ---- } n\text{times})^t$

$= (A^t \cdot A^t \cdot A^t \text{ ---- } n\text{times})$

$= (A^t)^n$ $= R.H.S$

Proved.

Q No. 13: Find matrices X and Y such that

$$2X - Y = \begin{bmatrix} 1 & 6 & -3 \\ 2 & 1 & 7 \end{bmatrix} \text{ — (1)} \qquad X + 3Y = \begin{bmatrix} 4 & 3 & 2 \\ 1 & -3 & 0 \end{bmatrix} \text{ — (2)}$$

Multiply (1) by 3 and + in (2)

$$6X - 3Y = \begin{bmatrix} 3 & 18 & -9 \\ 6 & 3 & 21 \end{bmatrix}$$

$$X + 3Y = \begin{bmatrix} 4 & 3 & 2 \\ 1 & -3 & 0 \end{bmatrix}$$

$+$

$$7X = \begin{bmatrix} 7 & 21 & -7 \\ 7 & 0 & 21 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 3 & -1 \\ 1 & 0 & 3 \end{bmatrix} \quad \text{put in eq (1)}$$

$$\begin{bmatrix} 9 & 6 & -2 \\ 9 & 0 & 6 \end{bmatrix} - Y = \begin{bmatrix} 1 & 6 & -3 \\ 2 & 1 & 7 \end{bmatrix}$$

$$Y = \begin{bmatrix} 9 & 6 & -2 \\ 9 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 6 & -3 \\ 2 & 1 & 7 \end{bmatrix} = \begin{bmatrix} +1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

Q No. 4 :(i) Find A; $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a+c+4b+2d & 6a+3c+8b+4d \\ 3a+2c+6b+4d & 9a+6c+12b+8d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$2a+c+4b+2d = 1 \rightarrow (1)$

$6a+3c+8b+4d = 0 \rightarrow (2)$

$3a+2c+6b+4d = 0 \longrightarrow (3)$

$9a+6c+12b+8d = 1 \longrightarrow (4)$

xing eq (1) by 3 and -ing by (2)

$$6a+3c+12b+6d = 3$$
$$\underline{\underset{-}{}6a \underset{-}{+} 3c \underset{-}{+} 8b \underset{-}{+} 4d = 0}$$
$$4b+2d = 3 \quad — (5)$$

xing (3) by 3 and -ing by (4)

$$9a+6c+18b+12d = 0$$
$$\underline{\underset{-}{}9a \underset{-}{+} 6c \underset{-}{+} 12b \underset{-}{+} 8d = 1}$$
$$6b+4d = -1 \quad — (6)$$

xing (5) by 2 and -ing by (6)

$8b + 4d = 6$

$-6b \pm 4d = -1$

$2b = 7 \quad b = 7/2$ put in eq (5)

$4(7/2) + 2d = 3$

$2d = -11$

$d = -11/2$

$b = 7/2$

$d = -11/2$

put in eq(1)

$2a + c + 14 - 11 = 1$

$2a + c = -2$

$c = -2 - 2a$ put in eq(2)

$6a + (-6 - 6a) + 28 + 2d = 0$

$\cancel{6a} - 6 - \cancel{6a} + 28 + 2d = 0$

$2d = -22$

$d = -1$

$2a + c = -2$

$c = -2 - 2a$ put in (4

$9a - 12a - 12 + 42 - 44 = 1$

$-3a = 15$

$a = -5$

$a = -5$ put in eq. $c = -2 - 2a$

$c = -2 - 2(-5)$

$c = -2 + 10$

$c = 8$

$$A = \begin{bmatrix} -5 & 7/2 \\ 8 & -11/2 \end{bmatrix}$$

(2) Find X $\begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 2 & 0 \end{bmatrix} X = \begin{bmatrix} 7/2 & 11 & 2 \\ 2 & 4 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

$$\begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3a+2d & 3b+2e & 3c+2f \\ d & +e & f \\ 2a & 2b & 2c \end{bmatrix}$$

$$\begin{bmatrix} 3a+2d & 3b+2e & 3c+2f \\ d & e & f \\ 2a & 2b & 2c \end{bmatrix} = \begin{bmatrix} 7/2 & 11 & 2 \\ 2 & 4 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$d = 2$ $\quad e = 4$ $\quad f = 1$ $\quad a = 1/2$ $\quad b = 1$ $\quad c = 0$

$3 + 2\cancel{e} = 11$

$\cancel{e} = 4$

$X = \begin{bmatrix} 1/2 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix}$

(3) If $A = \begin{bmatrix} 3 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 14 \end{bmatrix}$ then find non-zero matrix C such that AC = BC

let $C = \begin{bmatrix} a \\ b \end{bmatrix}$

$$\begin{bmatrix} 3 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 & 14 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$3a + 7b = 2a + 14b$

$3a - 2a = 14b - 7b$

$a = 7b$

let $b = x$

$a = 7x$

$C = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7x \\ x \end{bmatrix}$

(4) $\begin{bmatrix} xy & 4 \\ 0 & x+y \end{bmatrix} = \begin{bmatrix} 8 & z \\ t & 6 \end{bmatrix}$ then find value of z, t an x^2+y^2

$xy = 8 \qquad z = 4 \qquad t = 0 \qquad x+y = 6$

$x^2 + 2xy + y^2 = 36$

$x^2 + y^2 = 36 - 16$

$x^2 + y^2 = 20$

(5) If $A = \begin{bmatrix} 3 & 4 \\ 7 & 6 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find α and β so that $A^2 + \alpha I = \beta A$

$A^2 = \begin{bmatrix} 3 & 4 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 9+28 & 12+24 \\ 21+42 & 28+36 \end{bmatrix} = \begin{bmatrix} 37 & 36 \\ 63 & 64 \end{bmatrix}$

$A^2 + \alpha I = \beta A$

$\begin{bmatrix} 37 & 36 \\ 63 & 64 \end{bmatrix} + \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} = \begin{bmatrix} 3\beta & 4\beta \\ 7\beta & 6\beta \end{bmatrix}$

$\begin{bmatrix} 37+\alpha & 36 \\ 63 & 64+\alpha \end{bmatrix} = \begin{bmatrix} 3\beta & 4\beta \\ 7\beta & 6\beta \end{bmatrix}$

$36 = 4\beta \qquad 37 + \alpha = 3\beta$

$\beta = 9 \qquad \alpha = 18 - 37$

$\alpha = -19$

(6) Find the value of x if

$\begin{bmatrix} x & -4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = 0$

$\begin{bmatrix} x+0+4 & 0-4+0 & 3x+0+8 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = 0$

$\begin{bmatrix} x+4 & -4 & 3x+8 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = 0$

$$x^2+4x-4-3x-8 = 0$$

$$x^2+x-12 = 0$$

$$x^2+x-12=0 \quad \therefore \quad x^2+4x-3x-12 \qquad x(x+4)-3(x+4)=0$$

$$(x+4)(x-3)=0$$

$$x=-4 \qquad x=3$$

QNo.7: If $A = \begin{bmatrix} x & 0 \\ y & 1 \end{bmatrix}$ then

(1) Prove that for positive integers n, $A^n = \begin{bmatrix} x^n & 0 \\ \frac{y(x^n-1)}{x-1} & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} x & 0 \\ y & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ y & 1 \end{bmatrix} = \begin{bmatrix} x^2+0 & 0+0 \\ xy+y & 0+1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ xy+y & 1 \end{bmatrix}$$

$$\begin{bmatrix} x^2 & 0 \\ \frac{y(x+1)(x-1)}{x-1} & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ \frac{y(x^2-1)}{x-1} & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} x^2 & 0 \\ \frac{y(x^2-1)}{x-1} & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ y & 1 \end{bmatrix} = \begin{bmatrix} x^3+0 & 0+0 \\ \frac{y(x^2-1)}{x-1}\cdot x+y & 0+1 \end{bmatrix} = \begin{bmatrix} x^3 & 0 \\ \frac{y(x^3-x)}{x-1}+y & 1 \end{bmatrix}$$

$$\begin{bmatrix} x^3 & 0 \\ y\left(\frac{x^3-x+x-1}{x-1}\right) & 1 \end{bmatrix} \qquad \begin{bmatrix} x^3 & 0 \\ \frac{y(x^3-1)}{x-1} & 1 \end{bmatrix}$$

By continuing in this way we get,

$$A^n = \begin{bmatrix} x^n & 0 \\ \frac{y(x^n-1)}{x-1} & 1 \end{bmatrix}$$

(2) If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then prove that for all +ive integ...

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4 & -12+4 \\ 3-1 & -4+1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 1+2(2) & -4(2) \\ 2 & 1-2(2) \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 15-8 & -20+8 \\ 6-3 & -8+3 \end{bmatrix} = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1+2(3) & -4(3) \\ 3 & 1-2(3) \end{bmatrix}$$

By continuing in this way we get

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$

QNo.8: Consider any two particular matrices A and B of your choice of order 2x3 and 3x2 respectively and show that $(AB)^t = A^t B^t$.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+1+0 & 1+0+0 \\ 0+2+0 & 0+0+1 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}^t = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} \qquad B^t = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+1+0 & 0+2+0 \\ 1+0+0 & 0+0+1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Test Chapter: 1 Solution

Q no. 1: $z_1 = 4+5i \quad z_2 = \alpha - 2i \quad \alpha = ? \quad Re(z_1 z_2) = 20$

$$Re\left((4+5i)(\alpha-2i)\right) = 20$$

$$Re\left(4\alpha + 5i\alpha + 10\right) = 20 \qquad 4\alpha + 10 = 20 \qquad \alpha = 5/2$$

Q no. 2: $z = 3-2i \quad \overline{z_2} = 5+2i \quad z_3 = 1+2i^2 = -1 \quad Im\left[\frac{\overline{z_1} - \overline{z_3}}{z_1+z_2}\right] = ?$

$$Im\ \frac{3-2i-(-1)}{3-2i+5+2i} = \frac{4-2i}{8} = \frac{2(2-i)}{8_4} = \frac{2-i}{4} = \frac{-1}{4}$$

Q no. 3: $z_1 = 10(\cos 100^\circ + i\sin 100^\circ) \quad z_2 = 5(\cos 40^\circ + i\sin 40^\circ)$

find $z_1 \cdot z_2$, $\frac{z_1}{z_2}$

$$z_1 z_2 = 50(\cos 140^\circ + i\sin 140^\circ)$$

$x = r\cos\theta \qquad y = r\sin 140$

$x = 50\cos 140^\circ$

$\frac{z_1}{z_2} = 10(\cos 60^\circ + i\sin 60^\circ) \qquad x = r\cos\theta \qquad y = r\sin\theta$

$x = 5 \qquad y = 5\sqrt{3}$

Q no. 4: $-2 \leq Im(-z+i) \leq 3$

$-2 \leq Im(-a-bi+i) \leq 3$

$-2 \leq Im((1-b)i) \leq 3$

$-3 \leq -b \leq 2$

$3 \geq b \geq 2$

Q no. 3: $5z-(3+i)w = 7-i$ — (1) $\quad (2-i)z+2iw = -1+i$ —

Multiply eq (1) by $(2-i)$

$5(2-i)z-(3+i)w(2-i) = 7-i(2-i)$

$5(2-i)z-(6-i+1)w = 13-9i$

$5(2-i)z-(7-i)w = 13-9i$ — (3)

Multiply eq (2) by (5)

$5(2-i)z+10iw = -5+5i$ — (3a)

Subtract eq (3) and (3a)

$$\begin{array}{r} \cancel{5(2-i)z} + 10iw = -5+5i \\ -\cancel{5(2-i)z} \mp 7+iw = -13-9i \\ \hline 10iw+(7-i)w = -18+14i \end{array}$$

$(7+9i)w = -18+14i$ $\quad w = \dfrac{-18+14i}{7+9i} \times \dfrac{7-9i}{7-9i}$

$w = \dfrac{260i}{130} = 2i$ put in eq (1)

$5z-(3+i)(2i) = 7-i$

$5z-6i+2 = 7-i$

$5z = 7-i+6i-2$

$5z = 5+5i$

$z = \dfrac{5+5i}{5} = 1+i$

$w = 2i$

$z = 1+i$