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Where Minds Pollinate

Class 11
Mathematics

Chapter 1
Exercise 1.4
Notes

National Book Foundation

Exercise # 1.4

Q. Write the complex no. in polar form

i) $2+i2\sqrt{2}$

Sol:-

$$= 2+i2\sqrt{3} \rightarrow a=2 \ , \ b=2\sqrt{2}$$

In polar form $z = r(\cos\theta + i\sin\theta)$

$$\because \ r=\sqrt{a^2+b^2} \Rightarrow r=\sqrt{(2)^2+(2\sqrt{3})^2}$$

$$r=\sqrt{4+4(3)} \Rightarrow r = 4\sqrt{3}$$

and $\theta = \tan^{-1}\left[\frac{b}{a}\right] \Rightarrow \theta = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)$

$$\theta = 60° \Rightarrow 60 \times \frac{\pi}{180} \text{ radians} \Rightarrow \theta = \frac{\pi}{3}$$

Hence, $z = 2+i2\sqrt{3}$

$$z = 2\,4\left[\cos(\pi/3) + i\sin(\pi/3)\right]$$

$$z = 4\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right]$$

ii) $3-i\sqrt{3}$

Sol:-

$$z = 3-i\sqrt{3} \rightarrow x=3 \ , \ y=-\sqrt{3}$$

In polar form $z = r(\cos\theta + i\sin\theta)$

$\because \quad r = \sqrt{x^2+y^2} \Rightarrow r = \sqrt{(3)^2 + (-\sqrt{3})^2}$

$r = \sqrt{9+3} \Rightarrow r = 2\sqrt{3}$

and $\theta = \tan^{-1}\left(\frac{y}{x}\right) \Rightarrow \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\theta = 30^\circ \Rightarrow \theta = 30 \times \frac{\pi}{180}$ radians

$\therefore \quad \theta = \frac{\pi}{6}$ {IV-Quadr.} $\Rightarrow \theta = -\frac{\pi}{6}$

Hence, $z = 3 - i\sqrt{3}$

$$z = 2\sqrt{3}\left[\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right]$$

iii) $-2-i2$

Sol:-

$z = -2 - i2 \rightarrow x = -2, \; y = -2$

In polar form $z = r(\cos\theta + i\sin\theta)$

$\because \quad r = \sqrt{x^2+y^2} \Rightarrow r = \sqrt{(-2)^2 + (-2)^2}$

$r = \sqrt{4+4} \Rightarrow r = 2\sqrt{2}$

and $\theta = \tan^{-1}\left|\frac{y}{x}\right| \Rightarrow \tan^{-1}\left(\frac{2}{2}\right)$

$\theta = 45^\circ$ (III-Quad.)

$\therefore \theta = 45^\circ - \pi$

$\theta = \dfrac{45 \times \pi}{18} - \pi$

$\theta = \dfrac{-3\pi}{4}$

Hence, $z = -2 - i\sqrt{2}$

$$z = 2\sqrt{2}\left[\cos\left(\frac{-3\pi}{4}\right) + i\sin\left(\frac{-3\pi}{4}\right)\right]$$

iv) $\dfrac{i-1}{\cos(\pi/3) + i\sin(\pi/3)}$

Sol:-

$$z = \frac{i-1}{\cos(\pi/3) + i\sin(\pi/3)}$$

$$z = \frac{i-1}{\cos(60^\circ) + i\sin(60^\circ)}$$

$$z = \frac{i-1}{1/2 + \sqrt{3}/2\, i}$$

$$z = \frac{2(i-1)}{1+\sqrt{3}i} \Rightarrow z = \frac{2i-2}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$$

$$z = \frac{(2i-2)(1-\sqrt{3}i)}{(1)^2 - (\sqrt{3}i)^2}$$

$$z = \frac{-2+2\sqrt{3}+(2+2\sqrt{3})i}{1-3(-1)}$$

$$z = \frac{-2+2\sqrt{3}}{4} + \frac{2+2\sqrt{3}}{4} i$$

$$z = \frac{\sqrt{3}-1}{2} + \frac{1+\sqrt{3}}{2} i \rightarrow x = \frac{\sqrt{3}-1}{2}, y = \frac{1+\sqrt{3}}{2}$$

In polar form $z = r(\cos\theta + i\sin\theta)$

$$\therefore r = \sqrt{x^2+y^2} \Rightarrow r = \sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{1+\sqrt{3}}{2}\right)^2}$$

$$r = \sqrt{\frac{(\sqrt{3})^2+(1)^2-2(1)(\sqrt{3})}{4} + \frac{(1)^2+(\sqrt{3})^2+2(1)(\sqrt{3})}{4}}$$

$$r = \sqrt{\frac{3+1\cancel{-2\sqrt{3}}+1+3\cancel{+2\sqrt{3}}}{4}}$$

$$r = \sqrt{\frac{\cancel{8}^{2}}{\cancel{4}_{1}}} \Rightarrow r = \sqrt{2}$$

$$\text{and } \theta = \tan^{-1}\left|\frac{y}{x}\right| \Rightarrow \theta = \tan^{-1}\left(\frac{1+\sqrt{3}}{2} \div \frac{\sqrt{3}-1}{2}\right)$$

$$\theta = \tan^{-1}\left(\frac{1+\sqrt{3}}{\cancel{2}} \times \frac{\cancel{2}}{\sqrt{3}-1}\right)$$

$$\theta = \tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}-1}\right) \Rightarrow \theta = 75^{\circ} \ \{\text{I - Quad.}\}$$

$$\therefore \theta = 75 \times \frac{\pi}{180} \text{ rad.} \Rightarrow \theta = \frac{5\pi}{12}$$

Hence, $z = \dfrac{i-1}{\cos(\pi/3) + i\sin(\pi/3)}$

OR $\quad z = \sqrt{2}\left[\cos\left(\dfrac{5\pi}{12}\right) + i\sin\left(\dfrac{5\pi}{12}\right)\right]$

Q2. Write the following in rectangular form

i) $\left[\cos\left(\dfrac{\pi}{6}\right) + i\sin\left(\dfrac{\pi}{6}\right)\right]\left[\cos\left(\dfrac{\pi}{3}\right) + i\sin\left(\dfrac{\pi}{3}\right)\right]$

Sol:-

$$z = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$z = (\cos 30° + i\sin 30°)(\cos 60° + i\sin 60°)$$

$$z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$z = \left(\frac{\sqrt{3}+i}{2}\right)\left(\frac{1+\sqrt{3}i}{2}\right)$$

$$z = \frac{(\sqrt{3}+i)(1+\sqrt{3}i)}{2\times 2}$$

$$z = \frac{\sqrt{3} + \sqrt{3\times 3}i + i + \sqrt{3}i^2}{4}$$

$$z = \frac{\cancel{\sqrt{3}} - \cancel{\sqrt{3}} + 3i + i}{4}$$

$$z = \frac{0 + 4i}{4} \Rightarrow z = i$$

OR $\quad z = 0 + i$ (Rectangular form)

ii) $\dfrac{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

Soln.

$$z = \frac{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$$

$$z = \frac{\cos 30° - i\sin 30°}{2(\cos 60° + i\sin 60°)}$$

$$z = \frac{\sqrt{3}/2 - i/2}{2\left(1/2 + \sqrt{3}/2\, i\right)}$$

$$z = \frac{\sqrt{3} - i}{2} \div 2\left[\frac{1 + \sqrt{3}i}{2}\right]$$

$$z = \frac{\sqrt{3} - i}{\cancel{2}} \times \frac{\cancel{2}}{2(1 + \sqrt{3}i)}$$

$$z = \frac{\sqrt{3} - i}{2 + 2\sqrt{3}i} \Rightarrow z = \frac{\sqrt{3} - i}{2 + 2\sqrt{3}i} \times \frac{2 - 2\sqrt{3}i}{2 - 2\sqrt{3}i}$$

$$z = \frac{(\sqrt{3} - i)(2 - 2\sqrt{3}i)}{(2)^2 - (2\sqrt{3}i)^2} \Rightarrow z = \frac{2\sqrt{3} - 2\sqrt{3\times 3}\,i - 2i + 2\sqrt{3}i^2}{4 - 4(3)(-1)}$$

$$z = \frac{\cancel{2\sqrt{3}} - \cancel{2\sqrt{3}} - 6i - 2i}{4 + 12}$$

$$z = \frac{-8i}{16} \Rightarrow z = 0 - \frac{1}{2}i$$

Q3. If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\ldots(x_n+iy_n) = a+ib$. the show that

i) $(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\ldots(x_n^2+y_n^2) = a^2+b^2$

Sol:-

Given $\to$ $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\ldots(x_n+iy_n) = a+bi$

Taking modulus on b/s

$\therefore |(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\ldots(x_n+iy_n)| = |a+ib|$

OR $|(x_1+iy_1)||(x_2+iy_2)||(x_3+iy_3)|\ldots|(x_n+iy_n)| = |a+ib|$

$\therefore \sqrt{(x_1)^2+(y_1)^2} \cdot \sqrt{(x_2)^2+(y_2)^2} \cdot \sqrt{(x_3)^2+(y_3)^2} \ldots \sqrt{(x_n)^2+(y_n)^2}$
$= \sqrt{a^2+b^2}$

OR $\sqrt{[(x_1)^2+(y_1)^2][(x_2)^2+(y_2)^2][(x_3)^2+(y_3)^2]\ldots[(x_n)^2+(y_n)^2]}$
$= \sqrt{a^2+b}$

Squaring on b/s, we get

so, $(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\ldots(x_n^2+y_n^2) = a^2+b^2$

(Hence proved)

ii) $\sum_{r=1}^{n} \tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi \;, k \in Z$

Sol:-

Given that,

$(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \ldots (x_n + iy_n) = a + ib$

Taking Principal Argument both sides.

$\therefore \arg\left[(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\ldots(x_n+iy_n)\right] = \arg(a+ib)$

using propert of Principal argument.

$\arg(x_1+iy_1) + \arg(x_2+iy_2) + \arg(x_3+iy_3) + \ldots + \arg(x_n+iy_n)$
$= \arg(a+ib)$

$\tan^{-1}\left(\frac{y_1}{x_1}\right) + \tan^{-1}\left(\frac{y_2}{x_2}\right) + \tan^{-1}\left(\frac{y_3}{x_3}\right) + \ldots + \tan^{-1}\left(\frac{y_n}{x_n}\right)$
$= \tan^{-1}\left(\frac{b}{a}\right)$

OR

$$\sum_{r=1}^{n} \tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, \quad k \in Z$$

(Hence proved).

Q6. Write the following complex no. in algebric form:

i) $\sqrt{2}(\cos 315^\circ + i\sin 315^\circ)$

Sol:-

$$= \sqrt{2}(\cos 315^\circ + i\sin 315^\circ)$$

$$= \sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$

$$= \sqrt{2}\left(\frac{\sqrt{2}-\sqrt{2}i}{2}\right) \Rightarrow \frac{\sqrt{(2)^2}-\sqrt{(2)^2}\,i}{2}$$

$$= \frac{2-2i}{2} \Rightarrow 1-i$$

ii) $5(\cos 210° + i\sin 210°)$

Sol:-

$$= 5(\cos 210° + i\sin 210°)$$

$$= 5\left(\frac{-\sqrt{3}}{2} - \frac{i}{2}\right)$$

$$= 5\left(\frac{-\sqrt{3}-i}{2}\right)$$

$$= \frac{-5\sqrt{3}}{2} - \frac{5}{2}i$$

iii) $2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

Sol:-

$$= 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$$

$$= 2\left[\cos\left(\frac{3\pi}{2}\times\frac{180}{\pi}\right) + i\sin\left(\frac{3\pi}{2}\times\frac{180}{\pi}\right)\right]$$

$$= 2(\cos 270° + i\sin 270°)$$

$$= 2(0-i)$$

$$= 0-2i \Rightarrow -2i$$

iv) $4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

Sol:-

$$= 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

$$= 4\left[\cos\left(\frac{5\pi}{6}\times\frac{180}{\pi}\right) + i\sin\left(\frac{5\pi}{6}\times\frac{180}{\pi}\right)\right]$$

$$= 4(\cos 150^\circ + i\sin 150^\circ)$$

$$= 4\left(\frac{-\sqrt{3}}{2} + \frac{i}{2}\right)$$

$$= \cancel{4}^{2}\left(\frac{-\sqrt{3}+i}{\cancel{2}}\right)$$

$$= -2\sqrt{3} + 2i$$

v) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

Sol:-

$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$= 2\left[\cos\left(\frac{\pi}{6}\times\frac{180}{\pi}\right) + i\sin\left(\frac{\pi}{6}\times\frac{180}{\pi}\right)\right]$$

$$= 2(\cos 30^\circ + i\sin 30^\circ)$$

$$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= \cancel{2}\left(\frac{\sqrt{3}+1i}{\cancel{2}}\right)$$

$$= \sqrt{3} + i$$

vi) $\cos 315^\circ + i \sin 315^\circ$

Sol:-

$$= \cos 315^\circ + i \sin 315^\circ$$
$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$$

vii) $10(\cos 50^\circ + i \sin 50^\circ)$

Sol:-

$$= 10(\cos 50^\circ + i \sin 50^\circ)$$
$$= 10(0.642 + 0.766 i)$$
$$= 6.428 + 7.660 i$$

viii) $\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

Sol:-

$$= \sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$$
$$= \sqrt{2}\left[\cos\left(\frac{3\cancel{\pi}}{4} \times \frac{180}{\cancel{\pi}}\right) + i \sin\left(\frac{3\cancel{\pi}}{4} \times \frac{180}{\cancel{\pi}}\right)\right]$$
$$= \sqrt{2}(\cos 135^\circ + i \sin 135^\circ)$$
$$= \sqrt{2}\left(\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$$
$$= \sqrt{2}\left(\frac{-\sqrt{2} + \sqrt{2} i}{2}\right)$$
$$= \frac{-\sqrt{(2)^2} + \sqrt{(2)^2} i}{2} \Rightarrow \frac{-2 + 2i}{2}$$
$$= -1 + i$$

ix) $4\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

Sol:-

$$= 4\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

$$= 4\left[\cos\left(\frac{2\cancel{\pi}}{3}\times\frac{180}{\cancel{\pi}}\right)+i\sin\left(\frac{2\cancel{\pi}}{3}\times\frac{180}{\cancel{\pi}}\right)\right]$$

$$= 4(\cos 120^\circ + i\sin 120^\circ)$$

$$= 4\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$= \overset{2}{\cancel{4}}\left(\frac{-1+\sqrt{3}i}{\cancel{2}}\right)$$

$$= -2+2\sqrt{3}i$$

x) $7\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$

Sol:-

$$= 7\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$$

$$= 7\sqrt{2}\left[\cos\left(\frac{5\cancel{\pi}}{4}\times\frac{180}{\cancel{\pi}}\right)+i\sin\left(\frac{5\cancel{\pi}}{4}\times\frac{180}{\cancel{\pi}}\right)\right]$$

$$= 7\sqrt{2}(\cos 225^\circ + i\sin 225^\circ)$$

$$= 7\sqrt{2}\left(\frac{-\sqrt{2}}{2}+\frac{-\sqrt{2}}{2}i\right)$$

$$= 7\sqrt{2}\left(\frac{-\sqrt{2}-\sqrt{2}i}{2}\right)$$

$$= \frac{-7\sqrt{(2)^{\cancel{2}}}-7\sqrt{(2)^{\cancel{2}}}i}{2}$$

$$= \frac{-7(2)-7(2)i}{2}$$

$$= \frac{-14-14i}{2} \Rightarrow \frac{\cancel{2}(-7-7i)}{\cancel{2}}$$

$$= -7-7i$$

xi) $10\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

Sol:-

$$= 10\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$= 10\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\times\frac{180}{\pi}\right) + i\sin\left(\frac{7\pi}{4}\times\frac{180}{\pi}\right)\right)$$

$$= 10\sqrt{2}(\cos 315^\circ + i\sin 315^\circ)$$

$$= 10\sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$

$$= \overset{5}{\cancel{10}}\sqrt{2}\left(\frac{\sqrt{2}-\sqrt{2}i}{\cancel{2}}\right)$$

$$= 5\sqrt{2}(\sqrt{2}-\sqrt{2}i)$$

$$= 5\sqrt{(2)^2} - 5\sqrt{(2)^2}i$$

$$= 5(2) - 5(2)i$$

$$= 10 - 10i$$

xii) $2\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right)$

Sol:-

$$= 2\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right)$$

$$= 2\left[\cos\left(\frac{5\pi}{2}\times\frac{180}{\pi}\right) + i\sin\left(\frac{5\pi}{2}\times\frac{180}{\pi}\right)\right]$$

$= 2(\cos 450° + i\sin 450°)$
$= 2(0 + i)$
$= 2(0) + 2i$
$= 2i$

xiii) $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Sol:-

$= \frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{4} \times \frac{180}{\pi}\right) + i\sin\left(\frac{\pi}{4} \times \frac{180}{\pi}\right)\right]$

$= \frac{1}{\sqrt{2}}(\cos 45° + i\sin 45°)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2} + \sqrt{2}i}{2}\right)$

$= \frac{(\sqrt{2})^2 + (\sqrt{2})^2 i}{(2)^2}$

$= \frac{2 + 2i}{4} \Rightarrow \frac{2(1 + i)}{\cancel{4}\,2}$

$= \frac{1}{2} + \frac{1}{2}i$

xiv) $7(\cos 180° + i\sin 180°)$

Sol:-

$= 7(\cos 180° + i\sin 180°)$
$= 7(-1 + 0i)$
$= -7$

xv) $2e^{i\pi/4}$

Sol:-

$$= 2e^{i\pi/4}$$

OR

$$= 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$= 2(\cos 45^\circ + i\sin 45^\circ)$$

$$= 2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$$

$$= \cancel{2}\left(\frac{\sqrt{2}+\sqrt{2}i}{\cancel{2}}\right)$$

$$= \sqrt{2} + \sqrt{2}i$$

xvi) $3e^{i\pi/2}$

Sol:-

$$= 3e^{i\pi/2}$$

$$= 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$= 3(\cos 90^\circ + i\sin 90^\circ)$$

$$= 3(0 + i)$$

$$= 3i$$

xvii) $5e^{i\pi/3}$

Sol:-

$$= 5e^{i\pi/3}$$

$$= 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= 5(\cos 60^\circ + i\sin 60^\circ)$$

$$= 5\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

$$= 5\left(\frac{1 + \sqrt{3} i}{2}\right)$$

$$= \frac{5}{2} + \frac{5\sqrt{3}}{2} i$$

Q7. Change the following in Cartesin form.

i) $arg(z-1) = -\frac{\pi}{4}$

Sol:-

$$arg(z-1) = \frac{-\pi}{4}$$

let $z = x+iy$

$$\therefore \; arg(x+iy-1) = -\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y}{x-1}\right) = \frac{-\pi}{4}$$

Taking tan on b/s

$$\therefore \; \frac{y}{x-1} = \tan\left(\frac{-\pi}{4}\right)$$

$$\frac{y}{x-1} = \tan(-45^\circ)$$

$$\frac{y}{x-1} = -1 \Rightarrow y = 1 - x$$

$$x+y-1=0 \Rightarrow x+y=1$$

ii) $z\bar{z} = 4|e^{i\theta}|$

Sol:-

$$z\bar{z} = 4|e^{i\theta}|$$

Let $z = x+iy \Rightarrow \bar{z} = x-iy$

$$\therefore (x+iy)(x-iy) = 4|(\cos\theta + i\sin\theta)|$$

$$(x)^2 - (iy)^2 = 4\left(\sqrt{(\cos\theta)^2 + (\sin\theta)^2}\right)$$

$$x^2 - i^2y^2 = 4(1) \rightarrow \{\cos^2\theta + \sin^2\theta = 1\}$$

$$x^2 + y^2 = 4 \qquad \{i^2 = -1\}$$

iii) $-\frac{\pi}{3} \le \arg(z-4) \le \frac{\pi}{3}$

Sol:-

$$-\frac{\pi}{3} \le \arg(z-4) \le \frac{\pi}{3}$$

$$-\frac{\pi}{3} \le \arg(x+iy-4) \le \frac{\pi}{3}$$

$$-\frac{\pi}{3} \le \tan^{-1}\left(\frac{y}{x-4}\right) \le \frac{\pi}{3}$$

Taking tan on b/s

$$\tan\left(\frac{-\pi}{3}\right) \le \frac{y}{x-4} \le \tan\left(\frac{\pi}{3}\right)$$

$$\tan^{-1}(-60^\circ) \le \frac{y}{x-4} \le \tan(60^\circ)$$

$$-\sqrt{3} \le \frac{y}{x-4} \le \sqrt{3}$$

OR $\quad -\sqrt{3}(x-4) \le y \le \sqrt{3}(x-4)$

iv) $0 \le \arg\left(\frac{z-4}{1+i}\right) \le \frac{\pi}{6}$

Sol:-

$$0 \le \arg\left(\frac{z-4}{1+i}\right) \le \frac{\pi}{6}$$

$$0 \le \arg\left(\frac{z-4}{1+i} \times \frac{1-i}{1-i}\right) \le \frac{\pi}{6}$$

$$0 \le \arg\left(\frac{(z-4)(1-i)}{(1)^2-(i)^2}\right) \le \frac{\pi}{6}$$

$$0 \le \arg\left[\frac{z-zi-4+4i}{1+1}\right] \le \frac{\pi}{6}$$

$$0 \le \arg\left(\frac{(z-4)+(4-z)i}{2}\right) \le \frac{\pi}{6}$$

$$0 \le \arg\left(\frac{(x+iy-4)+(4-x-iy)i}{2}\right) \le \frac{\pi}{6}$$

$$0 \le \arg\left(\frac{x-4+iy+4i-xi+y}{2}\right) \le \frac{\pi}{6}$$

$$0 \le \arg\left(\frac{(x+y+4)}{2}+\frac{(y-x-4)i}{2}\right) \le \frac{\pi}{6}$$

$$0 \le \tan^{-1}\left[\frac{\frac{(y-x+4)}{\cancel{2}}}{\frac{(x+y-4)}{\cancel{2}}}\right] \le \frac{\pi}{6}$$

$$0 \leq \tan^{-1}\left(\frac{y-x+4}{x+y-4}\right) \leq \frac{\pi}{6}$$

Taking tan on b/s

$$\therefore \quad \tan(0) \leq \frac{y-x+4}{x+y-4} \leq \tan\left(\frac{\pi}{6}\right)$$

$$0 \leq \frac{y-x+4}{x+y-4} \leq \tan(30^\circ)$$

$$0 \leq \frac{y-x+4}{x+y-4} \leq \frac{\sqrt{3}}{3}$$

v) $\arg\left(\frac{1-iz}{1-z}\right) = \frac{\pi}{4}, \ z \neq i$

Sol:-

$$\arg\left(\frac{1-iz}{1-z}\right) = \frac{\pi}{4}$$

$$\arg\left(\frac{1-i(x+iy)}{1-(x+iy)}\right) = \frac{\pi}{4}$$

$$\arg\left(\frac{1-xi+y}{1-x-yi}\right) = \frac{\pi}{4}$$

$$\arg\left(\frac{1-xi+y}{(1-x)-yi}\right) = \frac{\pi}{4}$$

$$\arg\left(\frac{1-xi+y}{(1-x)-yi} \times \frac{(1-x)+yi}{(1-x)+yi}\right) = \frac{\pi}{4}$$

$$\arg\left[\frac{(1+y-xi)(1-x+yi)}{(1-x)^2-(yi)^2}\right] = \frac{\pi}{4}$$

$$\arg\left[\frac{1-x+yi+y-xy+y^2i-xi+x^2i-xyi^2}{(1)^2+(x)^2-2(1)(x)-y^2i^2}\right] = \frac{\pi}{4}$$

$$\arg\left[\frac{(y - x - \cancel{xy} + xy - 1) + (y + y^2 - x + x^2)i}{x^2 + 1 - 2x + y^2}\right] = \frac{\pi}{4}$$

$$\arg\left[\frac{(y - x - 1) + (y + y^2 - x + x^2)i}{x^2 + y^2 - 2x + 1}\right] = \frac{\pi}{4}$$

$$\tan^{-1}\left[\frac{\frac{(y + y^2 - x + x^2)}{\cancel{x^2 + y^2 - 2x + 1}}}{\frac{(y - x - 1)}{\cancel{x^2 + y^2 - 2x + 1}}}\right] = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y + y^2 - x + x^2}{y - x - 1}\right) = \frac{\pi}{4}$$

OR $$\frac{y + y^2 - x + x^2}{y - x - 1} = \tan(45°)$$

$$\frac{y + y^2 - x + x^2}{y - x - 1} = 1$$

OR $$\cancel{y} + y^2 - \cancel{x} + x^2 = \cancel{y} - \cancel{x} - 1$$

$$\therefore \quad x^2 + y^2 = 1$$

vi) $\frac{1}{2}\arg(z - i) = \frac{\pi}{3} - \frac{1}{2}\arg(z + i)$

Sol:-

$$\frac{1}{2}\arg(z - i) = \frac{\pi}{3} - \frac{1}{2}\arg(z + i)$$

$$\frac{1}{2}\arg(x+iy-i) = \frac{\pi}{3} - \frac{1}{2}\arg(x+iy+i)$$

$$\frac{\arg(x+yi-i)}{2} = \frac{\pi}{3} - \frac{\arg(x+iy+i)}{2}$$

$$\frac{\arg(x+yi-i)}{2} + \frac{\arg(x+iy+i)}{2} = \frac{\pi}{3}$$

$$\arg(x+yi-i) + \arg(x+yi+i) = \frac{2\pi}{3}$$

$$\tan^{-1}\left(\frac{y-1}{x}\right) + \tan^{-1}\left(\frac{y+1}{x}\right) = \frac{2\pi}{3}$$

Taking tan on b/s

$$\therefore \tan\left[\tan^{-1}\left(\frac{y-1}{x}\right) + \tan^{-1}\left(\frac{y+1}{x}\right)\right] = \tan\left(\frac{2\pi}{3}\right)$$

we know $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

$$\therefore \frac{\tan[\tan^{-1}(y-1/x)] + \tan[\tan^{-1}(y+1/x)]}{1 - \tan[\tan^{-1}(y-1/x)] \cdot \tan[\tan^{-1}(y+1/x)} = \tan(120^\circ)$$

OR $$\frac{y-1/x + y+1/x}{1-(y-1/x) \times (y+1/x)} = -\sqrt{3}$$

$$\frac{\frac{Y-\cancel{1}+Y+\cancel{1}}{\cancel{x}}}{\frac{x^2-(Y^2-1)}{\cancel{x^2}}} = -\sqrt{3}$$

$$\therefore \frac{2Y}{\frac{x^2-(y^2-1)}{x}} = -\sqrt{3} \Rightarrow \frac{2xy}{x^2-y^2+1} = -\sqrt{3}$$

$$2xy = -\sqrt{3}(x^2-y^2+1)$$

$$\sqrt{3}(x^2-y^2+1)+2xy=0$$

Q8 Calculate the position of a particle from mean position when amplitude is 0.004mm and angle is:-

i) $\frac{\pi}{4}$

Sol:-

We know that

$$x = x_{max}(\cos\theta + i\sin\theta)$$

Given $\theta = \frac{\pi}{4} = 45^\circ$, $x_{max} = 0.004m$

So, $x = 0.004(\cos 45^\circ + i\sin 45^\circ)$

$$x = 0.004\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$$

$$x = 0.004\left(\frac{\sqrt{2}+\sqrt{2}i}{2}\right)$$

$$x = \frac{1}{250}\left(\frac{\sqrt{2}+\sqrt{2}i}{2}\right)$$

$$x = \frac{\sqrt{2}}{500}(1+i)$$

ii) $\frac{\pi}{3}$

Sol:-

$\because x = x_{max}(\cos\theta + i\sin\theta)$

Given, $\theta = \frac{\pi}{3} = 60°$, $x_{max} = 0.004\,nm$

$\therefore x = 0.004(\cos 60° + i\sin 60°)$

$$x = 0.004\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$x = 0.004\left(\frac{1+\sqrt{3}i}{2}\right)$$

$$x = \frac{1}{250}\left(\frac{1+\sqrt{3}i}{2}\right)$$

$$x = \frac{1}{500}(1+\sqrt{3}i)$$

iii) $\frac{\pi}{6}$

Sol:-

$\because x = x_{max}(\cos\theta + i\sin\theta)$

Given, $\theta = \frac{\pi}{6} = 30^\circ$, $x_{max} = 0.004 nm$

$\therefore$ $x = 0.004(\cos 30^\circ + i \sin 30^\circ)$

$x = \frac{1}{250}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)$

$x = \frac{1}{250}\left(\frac{\sqrt{3}+i}{2}\right)$

$x = \frac{1}{500}(\sqrt{3}+i)$

Q9 When a particle is at position of $x = 2+3i$ from its mean position and $x_{max} = 1+4i$ is the position at max. distance from mean position.

i) Calculate the angle at $t=0$ and find the position of the particle.

Sol:-

Given, $x = 2+3i$, $x_{max} = 1+4i$
$t = 0$

$\because$ $x = x_{max}(\cos\theta + i\sin\theta)$

$\Rightarrow$ $2+3i = 1+4i(\cos\theta + i\sin\theta)$

$\frac{2+3i}{1+4i} = \cos\theta + i\sin\theta$

$\frac{2+3i}{1+4i} \times \frac{1-4i}{1-4i} = \cos\theta + i\sin\theta$

$$\frac{(2+3i)(1-4i)}{(1)^2-(4i)^2} = \cos\theta + i\sin\theta$$

$$\frac{14-5i}{1-16(-1)} = \cos\theta + i\sin\theta$$

$$\frac{14}{17} - \frac{5}{17}i = \cos\theta + i\sin\theta$$

Comparing on b/s

$$\therefore \quad \cos\theta = \frac{14}{17}, \quad \sin\theta = -\frac{5}{17}$$

$$\because \tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{-5/\cancel{17}}{14/\cancel{17}}$$

$$\theta = \tan^{-1}\left[\frac{-5}{14}\right] \rightarrow \text{Angle.}$$

Position $\rightarrow$ $x = x_{max}(\cos\theta + i\sin\theta)$

$$x = (1+4i)\left(\frac{14}{17} + i\left(\frac{-5}{17}\right)\right)$$

$$x = (1+4i)\left(\frac{14-5i}{17}\right)$$

$$x = \frac{34}{17} + \frac{51}{17}i$$

$$x = 2 + 3i$$

ii) If $x = 2+3i$ and $x_{max} = 1+4i$. Calculate the frequency when $t = 2$.

Sol:-

Given, $x = 2+3i$, $x_{max} = 1+4i$

$t = 2$.

Since, $x = x_{max} e^{i\omega t}$

OR $x = x_{max}(\cos\omega t + i\sin\omega t)$

$\Rightarrow \quad 2+3i = 1+4i\left[\cos(\omega \times 2) + i\sin(\omega \times 2)\right]$

$$\frac{2+3i}{1+4i} = \cos 2\omega + i\sin 2\omega$$

$$\frac{2+3i}{1+4i} \times \frac{1-4i}{1-4i} = \cos 2\omega + i\sin 2\omega$$

$$\frac{14-5i}{(1)^2-(14i)^2} = \cos 2\omega + i\sin 2\omega$$

$$\frac{14}{17} - \frac{5}{17}i = \cos 2\omega + i\sin 2\omega$$

Comparing on b/s

$\therefore \quad \cos 2\omega = \frac{14}{17}$, $\sin 2\omega = \frac{-5}{17}$

$\because \quad \omega = \frac{2\pi}{f}$

$\therefore \quad \cos\left(2 \times \frac{2\pi}{f}\right) = \frac{14}{17} \quad , \sin\left(2 \times \frac{2\pi}{f}\right) = \frac{-5}{17}$

$\cos \frac{4\pi}{f} = \frac{14}{17} \quad , \quad \sin \frac{4\pi}{f} = \frac{-5}{17}$

Let $\theta = \frac{4\pi}{f}$

$\therefore \quad \cos\theta = \frac{14}{17} \quad , \quad \sin\theta = \frac{-5}{17}$

also $\tan\theta = \frac{\sin\theta}{\cos\theta}$

OR $\tan\left(\frac{4\pi}{f}\right) = \frac{-5/\cancel{17}}{14/\cancel{17}}$

$\frac{4\pi}{f} = \tan^{-1}\left(\frac{-5}{14}\right)$

$\frac{4\pi}{f} = 19.65 \longrightarrow$ only magnitude

$f = \frac{4\pi}{+19.65}$

$f = 0.6$ units

Q16. Find the impendance Z for the following

i) $E = (-50+100i)$ volts,
$I = (-6-2i)$ amps

Sol:-

$\because \quad E = I \times Z$

$$\Rightarrow \quad Z = \frac{E}{I} \Rightarrow Z = \frac{-50+100i}{-6-2i}$$

$$Z = \frac{-50+100i}{-6-2i} \times \frac{-6+2i}{-6+2i}$$

$$Z = \frac{(-50+100i)(-6+2i)}{(-6)^2-(2i)^2}$$

$$Z = \frac{300-100i-600i+200i^2}{36-4(-1)}$$

$$Z = \frac{100-700i}{40} \Rightarrow Z = \frac{100}{40} - \frac{700}{40}i$$

$$Z = \left(\frac{5}{2} - \frac{35}{2}i\right) \text{ units}$$

ii) $E = (100+10i)$ volts,
$I = (-8+3i)$ amps

Sol:-

$\because E = I \times Z \Rightarrow Z = \frac{E}{I}$

$\because Z = \frac{100+10i}{-8+3i} \Rightarrow Z = \frac{100+10i}{-8+3i} \times \frac{-8-3i}{-8-3i}$

$Z = \frac{(100+10i)(-8-3i)}{(-8)^2-(3i)^2}$

$Z = \frac{-800-300i-80i-30i^2}{64-9(-1)}$

$Z = \frac{-770-380i}{73}$

$Z = \left(-\frac{770}{73} - \frac{380}{73}i\right)$ units